**CS763, Assignment 1**

**Team Members**

* Ashray Malhotra (11D070002)
* Akshay Sarode (110260010)
* Rohan Prinja (110050011)

**Question 3**

The vanishing point of a line depends only on its direction vector. So we will ignore the precise location of the three coplanar lines and instead only look at their direction vectors. Let the direction vectors be v1 = (x1, y1, z1), v2 = (x2, y2, z2) and v3 = (x3, y3, z3) respectively. Then, since the three lines are coplanar,

v1 · (v2 × v3) = 0  
⇔ (x1, y1, z1) · (y2z3 – y3z2, x3z2 – x2z3, x2y3 – x3y2) = 0  
⇒ x1(y2z3 – y3z2) + y1(x3z2 – x2z3) + z1(x2y3 – x3y2) = 0 [1]

The vanishing point of a line with direction vector (x, y, z) is (fx/z, fy/z, f). So, the vanishing points are (fx1/z1, fy1/z1, f), (fx2/z2, fy2/z2, f) and (fx3/z3, fy3/z3, f) respectively. To prove that these points are collinear is equivalent to proving that the points wi = (xi/zi, yi/zi, 1) are collinear for i = 1, 2, 3 (we are simply scaling each point by 1/f). Now, w1, w2 and w3 are collinear if

(w3 – w2) × (w2 – w1) = 0.  
 ⇔ (x3z2 – x2z3, y3z2 – y2z3, 0)/(z2z3) × (x2z1 – x1z2, y2z1 – y1z2, 0)/(z1z2) = 0

Expanding the above cross product and simplifying, we get the following equation,

⇔ z2(x1y3 – x3y1) + z3(x2y1 – x1y2) + z1(x3y2 – x2y3) = 0 [2]

It can be seen that [2] is just a rearranged version of [1]. Thus, the coplanarity of the three lines in the real world implies [2] which is equivalent to [1], which is equivalent to the collinearity of the vanishing points of the three lines. Hence proved.