## Definition and Symbol Index

 $A^{\mathbb{N}}$ , infinite sequences of elements of A, 7  $\mathcal{EM}(S,\mathcal{A})$ , category of Eilenberg-Moore  $A^{\S}$ , space of polynomials on a vector space A, coalgebras of a comonad S that satisfy axioms  $\mathcal{A}$ , 423  $\mathcal{EM}(T)$ , category of algebras for the monad T,  $A^{\infty}$ , both finite and infinite sequences of elements of A, 7  $\mathcal{EM}(T,\mathcal{A})$ , category of Eilenberg-Moore  $A^*$ , finite sequences of elements of A, 7, 31 algebras of a monad T that satisfy  $\mathcal{L}(A)$ , set  $\mathcal{P}(A^*)$  of languages over A, 144 axioms  $\mathcal{A}$ , 406  $\mathcal{R}(A)$ , set of regular languages over A, 144  $\mathcal{K}(S)$ , Kleisli category of comonad S, 266  $\mathcal{K}(T)$ , Kleisli category of monad T, 263 DA, category of deterministic automata, 101  $\mathcal{K}_{\mathbb{N}}(T)$ , finitary Kleisli category of a monad T DB, category of deterministic behaviours, 101 on **Sets**, with  $n \in \mathbb{N}$  as objects, 393 Dcpo, category of directed complete partial  $\mathbf{Mnd}(\mathbb{C})$ , category of monads on  $\mathbb{C}$ , 257 orders, 26, 301  $\mathbf{Model}(T)$ , category of functorial models of a Grp, category of groups, 26 monad T, 394 Hilb, category of Hilbert spaces, 195 **Model** $(T, \mathcal{A})$ , category of functorial models of **JSL**, the category of join semilattices, 301 a monad T that satisfy axioms  $\mathcal{A}$ , 406 MSL, the category of meet semilattices, 188,  $\mathbb{C}/I$ , slice category over I, 31 311, 383  $\mathbb{C} \times \mathbb{D}$ , product category of  $\mathbb{C}$  and  $\mathbb{D}$ , 28 Mon, category of monoids, 26  $\mathbb{C}^{op}$ , opposite category of  $\mathbb{C}$ , 28 **PreOrd**, category of preorders, 26  $\operatorname{Pred}(\mathbb{C})$ , category of predicates from  $\mathfrak{M}$  in  $\mathbb{C}$ , **Pred**, category of predicates, 338 Rel, category of binary relations, 124  $Rel(\mathbb{C})$ , category of relations in  $\mathbb{C}$ , 186 Sets, category of sets and functions, 26 SetsRel, category of sets and relations, 27  $F \dashv G$ , F is left adjoint of G, 95 Sp, category of topological spaces, 26  $F^*$ , free monad on a functor F, 257 Vect, category of vector spaces, 66  $F^{\infty}$ , cofree comonad on a functor F, 263  $F_{\#}$ , functor associated with arity #, 51 Alg(F), category of F-algebras, 78  $T/\mathcal{A}$ , quotient monad obtained from T via  $\mathbf{Alg}(F,\mathcal{A})$ , category of algebras of a functor F axioms  $\mathcal{A}$ , 412 that satisfy axioms  $\mathcal{A}$ , 406 D, discrete probability distribution functor, CoAlg(F), category of F-coalgebras, 30  $CoAlg(F, \mathcal{A})$ , category of coalgebras of a  $\mathcal{EM}(G)$ , lifting of a functor G to an functor F that satisfy axioms  $\mathcal{A}$ , 423 Eilenberg-Moore category, 306

 $\mathcal{EM}(S)$ , category of coalgebras for the

comonad S, 297

 $\mathcal{K}(F)$ , lifting of a functor F to a Kleisli

category, 267, 306

```
\mathcal{M}_M, multiset functor, counting in monoid M,
                                                       ev, evaluation morphism, 42
                                                       id_X, identity morphism on an object X in a
\alpha: H \Rightarrow K, \alpha is a natural transformation from
                                                               category, 25
        H to K. 99
                                                       int<sub>b</sub>, interpretation map for coalgebra b, 79
Q, quotient functor, 410
                                                       \kappa_1, first coprojection morphism, 37
Pred(F), predicate lifting
                                                       \kappa_2, second coprojection morphism, 37
  for a polynomial functor, 336
                                                       \pi_1, first projection morphism, 35
  with respect to a factorisation system, 342
                                                       \pi_2, second projection morphism, 35
Pred(F), left adjoint to predicate lifting, 340
                                                       st, strength natural transformation, 107, 272
                                                       \langle f, g \rangle, tuple of morphisms f and g, 35
Rel(F), relation lifting
                                                       d \odot c, composition of coalgebras: c followed
  with respect to a factorisation system, 202
                                                               by d, 249
  for a polynomial functor, 116
                                                       f[U], direct image, 44
\mathcal{D}_{\leq 1}, discrete sub-probability distribution
                                                       f^{\$}, Kleisli extension of f, 266
        functor, 164
                                                       g \odot f, composition of Kleisli maps: f
\{S \mid \mathcal{A}\}\, subset comonad obtained from S via
                                                               followed by g, 263
        axioms A. 428
                                                       g \circ f, composition g after f in a category, 25
\theta, universal map F \Rightarrow F^* from an endofunctor
        F to the free monad F^* on F, 257
{-}, comprehension functor, 188
                                                       BT, Böhm tree function, 367
                                                       FV, free variables in a Böhm tree, 369
                                                       hnf, head normal form function, 367
Th(Ax), set of equations derivable from Ax,
                                                       comp, on sequences, 17
                                                       evens, on sequences, 12
\square, before operator, 373
merge, on sequences, 15
                                                       nextdec, 10
♦, eventually operator
  on sequences, 20
                                                       next, final coalgebra for sequences, 7
□, henceforth operator, 354
                                                       odds, on sequences, 14
  for a factorisation system, 361
                                                       tail, on sequences, 14
  on sequences, 19
O, lasttime operator, 373
                                                       L_a, a-derivative of language L, 72
\overline{\neg}U, negation (or complement) of U, 43
                                                       [-]_R, quotient map, 135
O, nexttime operator, 364
                                                       [\![ - ]\!]_{\rho} interpretation in an algebra, for
                                                               valuation \rho, 393
  on sequences, 19
S, since operator, 373
                                                       \delta^*, iteration of transition function \delta, 57
Ax + t_1 = t_2, t_1 = t_2 is derivable from Ax, 403
                                                       \mu, least fixed point operator, 365
U, until operator
                                                       v, greatest fixed point operator, 365
                                                       0, null process, 152
  on sequences, 24
U, until operator, 365
                                                       supp, support, 161
                                                       b \cdot z, prefix of action b to process z, 152
!, unique map
                                                       c/R, coalgebra on quotient by R, 135, 222
  to a final object, 35
                                                       c_P, subcoalgebra on greatest invariant in
                                                               subset P. 355
  from an initial object, 38
X' \rightarrow X, epimorphism, 108
                                                       t[\vec{s}/\vec{v}], term t with terms \vec{s} substituted for
                                                               variables \vec{v}, 395
X' \rightarrowtail X, monomorphism, 108
                                                       z + w, sum of two processes, 152
\Lambda(f), abstraction morphism, 42
beh<sub>c</sub>, behaviour function for coalgebra c, 8, 68
≅, isomorphism, 27
                                                       V, join, 24
[f, g], cotuple of morphisms f and g, 37
                                                       \wedge, meet, 24
dst, double strength for a commutative monad,
                                                       ⊥, bottom element, 46
        273
                                                       T, top element, 46
```

 $(\cdot \neq x)$ , predicate of elements unequal to x, 370  $(\cdot = x)$ , predicate of elements equal to x, 370  $R^{\dagger}$ , reverse relation, 43  $S \circ R$ , relation composition, 27 U(x), predicate U holds for x, 43  $\operatorname{Graph}(f)$ , the graph of a function f, 29  $c \hookrightarrow d$ , bisimilarity with respect to coalgebras c and d, 121  $\hookrightarrow$ , bisimilarity, 120  $\coprod_f (U)$ , direct image, 44  $\operatorname{Eq}(X)$ , equality relation on a set X, 27  $\coprod_f (U)$ , product predicate, 47

observations b, b', 56, 59  $x \nrightarrow$ , state x halts, 11  $x \downarrow b, b$  can be observed about x, 56, 59  $a \in x$ , a occurs in the behaviour sequence of x, 21  $x \stackrel{a}{\rightarrow}$ , there is no a-step from x, 59  $x \stackrel{\sigma}{\rightarrow} * y$ , multiple  $\sigma$ -steps from x to y, 57  $x \stackrel{a}{\rightarrow} x'$ , a-step from x to x', 11, 56, 59

 $(x \downarrow b) \xrightarrow{a} (x' \downarrow b')$ , a-transition with

 $L^{\triangle}$ , initial algebra (of parsed words) for the functor  $(-+L)^*$ , 284  $\mathcal{B}$ , final coalgebra of Böhm trees, 367 BinTree(A), initial algebra of binary trees, 83

 $X^n$ , n-fold product (power), 35  $n \cdot X$ , n-fold coproduct (copower) of X, 38 0, empty set, 38 0, initial object, 38 1, final object, 35 1, singleton set, 36 2, two-element set  $\{0,1\}$ , 72 X + Y, coproduct of objects X, Y, 37  $X \times Y$ , product of objects X, Y, 42  $\mathcal{P}_{\text{fin}}(X)$ , set of finite subsets/predicates on X, 44 Ker(f), kernel of function f, 125  $\mathcal{P}(X)$ , set of subsets/predicates on X, 43  $\prod_{i \in I} X_i$ , set-indexed product, 36 Eq(f,g), pullback of f, g, 126, 171