CS420 Parallel Programming, Spring 2007

Isoefficiency Analysis Notes from Febuary 9, 2007

Communication/Computation Ratio

Notation: N = problem size (N by N matrix)
m = problem size = kN^2
P = number of processors
a, a', b, b' = constants

1). Row Decomposition of Jacobi Relaxation Problem

Each processor has $a\frac{N^2}{P}$ computation amount

$T_{compute} = a'\frac{N^2}{P} = \frac{am}{P}$

$T_{communication} = b'N = b\sqrt{m}$

$T_{parallel} = \frac{am}{P} + b\sqrt{m}$

$T_{seq} = am$

$Speedup = \frac{seq\ time}{parallel\ time} = \frac{am}{\frac{am}{P}+b\sqrt{m}} = \frac{amP}{am+bP\sqrt{m}} = \frac{a\sqrt{m}P}{a\sqrt{m}+bP}$

$Efficiency = \eta = \frac{speedup}{\#processors} = \frac{a\sqrt{m}}{a\sqrt{m}+bP}$

$\gamma = \frac{Communication}{Computation} = \frac{b\sqrt{m}P}{am} = \frac{bp}{a\sqrt{m}}$

$$\text{So, } \boxed{\eta = \frac{a\sqrt{m}}{a\sqrt{m}+bP} = \frac{1}{1+\gamma}}$$

If we double the number of processors η decreases, thus we need to quadruple m to get same η
Thus, $Isoeff_{row} = m^2$

2). Block Decomposition of Jacobi Relaxation Problem

$T_{compute} = \frac{am}{P}$ (same)

$T_{communication} = b\sqrt{\frac{m}{P}}$

$\gamma = \frac{b\sqrt{m}P}{a\sqrt{P}m} = \frac{b}{a}\frac{\sqrt{P}}{\sqrt{m}}$

So, $Iso_{block} = linear$