MATH484 - MIDTERM 2
Mar 7th 2017, 7pm – 8:30pm
223 Gregory Hall   Section: C13 C14   Name: ....

Question 1:
Give the definition of the following terms.
- A *posynomial*
- A *convex program*
- The *feasible region* of a convex program
- A *Slater point* for convex program
- A *superconsistent* convex program
- The *Lagrangian* of a convex program

Solution: See the definitions in the book.

Question 2:

(a) State the saddle point version of the Karush-Kuhn-Tucker Theorem. *You must write all of the relevant Karush-Kuhn-Tucker conditions here.*

Solution: See Theorem 5.2.13 on pages 182–183 of the book.

(b) Show that

$$\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{12}+\frac{w}{12}\right)^4 \le \frac{x^4}{2}+\frac{y^4}{3}+\frac{z^4}{12}+\frac{w^4}{12}$$

for all $x, y, z, w \in \mathbb{R}$.

Solution: The second derivative of x^4 is $12x^2$, which is nonnegative for all real numbers, so $f(x) = x^4$ is convex. Therefore, because $\frac{1}{2}+\frac{1}{3}+\frac{1}{12}+\frac{1}{12} = 1$,

$$\begin{aligned}\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{12}+\frac{w}{12}\right)^4 &= f\left(\frac{1}{2}x+\frac{1}{3}y+\frac{1}{12}z+\frac{1}{12}w\right)\\ &\le \frac{1}{2}f(x)+\frac{1}{3}f(y)+\frac{1}{12}f(z)+\frac{1}{12}f(w)\\ &= \frac{x^4}{2}+\frac{y^4}{3}+\frac{z^4}{12}+\frac{w^4}{12}\end{aligned}$$

(c) *Circle exactly one choice.* Consider a geometric program (GP) with object function $g(\mathbf{t})$ and its dual geometric program (DGP) with object function $v(\delta)$.

A. If (GP) has a solution $\mathbf{t}^*$, then (DGP) has a solution δ^* and $g(\mathbf{t}^*) = v(\delta^*)$.

B. If (GP) has a solution $\mathbf{t}^*$, then (DGP) must be consistent.

C. If $\mathbf{t}$ has positive components and δ is feasible for (DGP), then $g(\mathbf{t}) \ge v(\delta)$.

D. All of the above

Solution: All of the above, this follows from the primal-dual inequality for geometric programming and Theorem 2.4.4.

Question 3:
Solve the following problem using the Arithmetic–Geometric mean inequality. You must supply the solution, i.e. the point which attains the minimum or maximum and the value of the function at that point for full credit. You must use the AM–GM inequality to receive credit for this problem.

Maximize xy subject to $2x^3 + 12y^2 = 40$ and $x > 0$ and $y > 0$.

Solution:
For any $x > 0$, $y > 0$, $\delta_1 > 0$ and $\delta_2 > 0$ such that $\delta_1 + \delta_2 = 1$, we have that

$$40 = 2x^3 + 12y^2 = \delta_1\left(\frac{2}{\delta_1}x^3\right) + \delta_2\left(\frac{12}{\delta_2}y^2\right) \le \left(\frac{2}{\delta_1}\right)^{\delta_1}\left(\frac{12}{\delta_2}\right)^{\delta_2} x^{3\delta_1}y^{2\delta_2}.$$

Because we want the right-hand side to be xy raised to some power, we need $3\delta_1 = 2\delta_2$, so we use $\delta_1 = \frac{2}{5}$ and $\delta_2 = \frac{3}{5}$. This gives us that

$$40 = \frac{2}{5}(5y^3) + \frac{3}{5}(20y^2) \ge 5 \cdot 4^{3/5} \cdot (xy)^{6/5}.$$

with equality when $40 = 5x^3 = 20y^2$. Therefore, the solution is $x^* = 2$, $y^* = \sqrt{2}$, which yields $x^*y^* = 2\sqrt{2}$.

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Question 4:
Apply the gradient form of the Karush-Kuhn-Tucker Theorem to find all solutions of the following program. Solving means finding all solutions and writing the value of the objective function at your solutions. *(You must consider all possible solutions to get full credit on this problem).*

$$\begin{cases} \text{Minimize} & f(x_1,x_2) = -x_1 - 3x_2 \\ \text{subject to} & x_1 - x_2 - 1 \le 0 \\ & x_1 + x_2^2 - 3 \le 0 \end{cases}$$

Solution:
The functions, $-x_1 - 3x_2$, $x_1 - x_2 - 1$ and $x_1 + x_2^2 - 3$ are convex, and, because $(0,0)$ is a Slater point, the program is superconsistent, so we can apply the gradient form of the Karush-Kuhn-Tucker Theorem, which implies that $\mathbf{x} = (x_1, x_2)$ is a solution if and only if $\mathbf{x}$ is feasible and there exist $\lambda_1 \ge 0$ and $\lambda_2 \ge 0$ such that the following equations are satisfied:

$$0 = \frac{\partial L(\mathbf{x},\lambda)}{\partial x_1} = -1 + \lambda_1 + \lambda_2$$
$$0 = \frac{\partial L(\mathbf{x},\lambda)}{\partial x_2} = -3 - \lambda_1 + 2x_2\lambda_2$$
$$0 = \lambda_1(x_1 - x_2 - 1)$$
$$0 = \lambda_2(x_1 + x_2^2 - 3)$$

There are three cases to consider.
If $\lambda_2 = 0$, then the second equation implies $\lambda_1 = -3$, so we can assume $\lambda_2 > 0$.
If $\lambda_1 = 0$ and $\lambda_2 > 0$, then the first equation implies $\lambda_2 = 1$ and the second equation implies that $x_2 = \frac{3}{2}$. The fourth equation implies that $x_1 = \frac{3}{4}$. We check that $\frac{3}{4} - \frac{3}{2} - 1 = -7/4 < 0$, to verify that $(\frac{3}{4}, \frac{3}{2})$ is feasible. Therefore, $(\frac{3}{4}, \frac{3}{2})$ is a solution. Note that $f(\frac{3}{4}, \frac{3}{2}) = -\frac{21}{4}$.
If $\lambda_1 > 0$ and $\lambda_2 > 0$, then the third and fourth equation imply that $x_2^2 + x_2 - 1 = 0$, so $x_2 = -2$ or $x_2 = 1$. If $x_2 = -2$, then $x_1 = -1$ and if $x_2 = 1$, then $x_1 = 2$. Since $f(-1,-2) = 7 > -21/4$ and $f(2,1) = -5 > -21/4$, we can conclude that the unique solution is $\mathbf{x}^* = (\frac{3}{4}, \frac{3}{2})$ and the value of the object function at this point is $-\frac{21}{4}$.

Question 5:
Consider the following geometric program.

$$(GP)\begin{cases} \text{Minimize} & g(t_1,t_2) = \frac{2t_1^2}{t_2^2} + t_2^4 + \frac{4}{t_1} \\ \text{subject to} & t_1 > 0, t_2 > 0 \end{cases}$$

(a) Write the dual geometric program (DGP).

Solution:

$$(DGP)\begin{cases} \text{Maximize} & v(\delta) = \left(\frac{2}{\delta_1}\right)^{\delta_1}\left(\frac{1}{\delta_2}\right)^{\delta_2}\left(\frac{4}{\delta_3}\right)^{\delta_3} \text{ subject to} \\ & \delta_1 + \delta_2 + \delta_3 = 1 \\ & 2\delta_1 - \delta_3 = 0 \\ & -2\delta_2 + 4\delta_2 = 0 \\ & \delta_1 > 0,\ \delta_2 > 0,\ \delta_3 > 0 \end{cases}$$

(b) Solve (DGP) and use your solution to solve (GP). Solving means finding the solutions and writing the value of the object function the solutions.

Solution: Solving (DGP) gives $\delta^* = (2/7, 1/7, 4/7)$ and $v(\delta^*) = 7$. A solution $\mathbf{t}^*$ of (GP) must satisfy $v(\delta^*)\delta_i = u_i(\mathbf{t}^*)$ for $i \in [3]$, so, we must solve the following system:

$$7\frac{2}{7} = \frac{2t_1^2}{t_2^2};\ 7\frac{1}{7} = t_2^4;\ \text{and}\ 7\frac{4}{7} = \frac{4}{t_1}$$

This gives us that $\mathbf{t}^* = (1,1)$ and $g(\mathbf{t}^*) = 7$.

Question 6: *(C14 (four credit hours) required, C13 (three credit hours) optional)*
State and prove The Arithmetic–Geometric Mean Inequality. *You must include the complete theorem for full credit including when equality holds.*

Solution: See Theorem 2.4.1 on page 59.

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Math-484 Homework #6

Due 10am Mar 17
Write your name on your solutions and indicate if you are a C14 (4 credit hour) student.

1: Prove that if M is a subspace of $\mathbb{R}^n$ such that $M \neq \mathbb{R}^n$, then the interior of M is empty.

Solution:
To prove the statement, we pick an arbitrary $\mathbf{x} \in M$ and show that $\mathbf{x}$ is not in the interior of M by showing that, for an arbitrary $r > 0$, there exist $\mathbf{z} \in B(\mathbf{x}, r)$ that is not in M. To do this, we first note that there exists $\mathbf{y} \in \mathbb{R}^n \setminus M$, because $M \subseteq \mathbb{R}^n$ and $M \neq \mathbb{R}^n$. Note, for any $\alpha \in \mathbb{R} \setminus \{0\}$, $\mathbf{x} + \alpha\mathbf{y}$ is not in M, because $\mathbf{y} = \frac{1}{\alpha}(\mathbf{x} + \alpha\mathbf{y} - \mathbf{x}) \notin M$ and M is closed under vector addition and scale multiplication. In particular, $\mathbf{z} = \mathbf{x} + \frac{r}{2\|\mathbf{y}\|}\mathbf{y} \notin M$. We have that $\|\mathbf{y}\| \neq 0$ because $\mathbf{y} \notin M$ and $\mathbf{0} \in M$ for every vertex space. But $\|\mathbf{x} - \mathbf{z}\| = \frac{r}{2} < r$, so $\mathbf{z} \in B(\mathbf{x}, r)$. Therefore, $B(\mathbf{x}, r)$ is not a subset of M, so $\mathbf{x}$ is not in the interior of M.

2: Let A be an $m \times n$-matrix and let $C = \{A\mathbf{x} : \mathbf{x} \in \mathbb{R}^n \text{ and } \mathbf{x} \geq \mathbf{0}\} \subseteq \mathbb{R}^m$. Prove that if there exists $\mathbf{a} \in \mathbb{R}^m$ such that $\mathbf{a} \cdot \mathbf{c} \leq \alpha$ for all $\mathbf{c} \in C$, then $\mathbf{a} \cdot \mathbf{c} \leq 0$ for all $\mathbf{c} \in C$.

Solution:
If $\alpha \leq 0$, then there is nothing to prove, so we can assume $\alpha > 0$. Suppose for a contradiction that there exists $\mathbf{c} \in C$, such that $\mathbf{a} \cdot \mathbf{c} = \beta > 0$. Then there exists $\mathbf{x} \in \mathbb{R}^n$ such that $\mathbf{x} \geq \mathbf{0}$ and $\mathbf{c} = A\mathbf{x}$. Therefore, because $\frac{2\alpha}{\beta}\mathbf{x} \geq \mathbf{0}$, $A\left(\frac{2\alpha}{\beta}\mathbf{x}\right) = \frac{2\alpha}{\beta}\mathbf{c}$ is in C. But then $\mathbf{a} \cdot \left(\frac{2\alpha}{\beta}\right)\mathbf{c} = \frac{2\alpha}{\beta}\mathbf{a} \cdot \mathbf{c} = 2\alpha > \alpha$ gives a contradiction.

3: Let A be an $m \times n$-matrix, with columns $A_1, \ldots, A_n \in \mathbb{R}^m$ and let $\mathbf{b} \in \mathbb{R}^m$. You can assume that the set

$$C = \{A\mathbf{x} : \mathbf{x} \in \mathbb{R}^n \text{ and } \mathbf{x} \geq \mathbf{0}\} \subseteq \mathbb{R}^m$$

is closed and convex. Show that if there does not exist $\mathbf{x} \in \mathbb{R}^n$ such that $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x} \geq \mathbf{0}$, then there exists $\mathbf{a} \in \mathbb{R}^m$ such that

$$\mathbf{a} \cdot \mathbf{b} > 0 \text{ and } \mathbf{a} \cdot A_j \leq 0 \text{ for every } j \in \{1, \ldots, n\}.$$

Hint: Use the previous problem and the basic separation theorem with the set C and the vector $\mathbf{b}$. *Note that this is a version of Farkas Lemma from linear programming.*

Solution:
Since $\mathbf{b}$ is not in C, by the Basic Separation Theorem, there exists $\mathbf{a}$ and $\alpha \in \mathbb{R}$ such that $\mathbf{a} \cdot \mathbf{b} > \alpha$ and $\mathbf{a} \cdot \mathbf{c} \leq \alpha$ for every $\mathbf{c} \in C$. Since $\mathbf{0} \in C$, and $\mathbf{a} \cdot \mathbf{0} = 0$, we have that $\alpha \geq 0$, so $\mathbf{a} \cdot \mathbf{b} > 0$. Since $\mathbf{a} \cdot \mathbf{c} \leq \alpha$ for every $c \in C$, we have that $\mathbf{a} \cdot \mathbf{c} \leq 0$ for every $\mathbf{c} \in C$ by the previous problem. If we let $\mathbf{e}_j, \ldots, \mathbf{e}_n$, be the standard basis vectors in $\mathbb{R}^n$, then, because, for any $j \in \{1, \ldots, n\}$, $\mathbf{e}_j \geq \mathbf{0}$ and $A_j = A\mathbf{e}_j$, we have that $A_j \in C$. Therefore, $\mathbf{a} \cdot A_j \leq 0$ for every $j \in \{1, \ldots, n\}$.

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4: Let (P) be a convex program and let

$$S := \{\mathbf{x} \in C : f(\mathbf{x}) = MP \text{ and } g_i(\mathbf{x}) \leq 0 \text{ for all } i \in [m]\ \}.$$

Prove that S is a convex set.
Hint: First show that if f is convex on C, then for any $M \in \mathbb{R}$, the set $\{\mathbf{x} \in C : f(\mathbf{x}) \leq M\}$ is convex.

Solution:
First we prove the hint by using the definition. Let f be convex on C, and suppose that $\mathbf{x}^1, \mathbf{x}^2 \in \{\mathbf{x} \in C : f(\mathbf{x}) \leq M\}$ and $\lambda \in [0,1]$. We have that

$$f(\lambda\mathbf{x}^1 + (1-\lambda)\mathbf{x}^2) \leq \lambda f(\mathbf{x}^1) + (1-\lambda)f(\mathbf{x}^2) \leq \lambda M + (1-\lambda)M = M.$$

So $\{\mathbf{x} \in C : f(\mathbf{x}) \leq M\}$ is convex.

For every $i = 1, \ldots, m$, the hint implies that $\{\mathbf{x} \in C : g_i(\mathbf{x}) \leq 0\}$ is convex. Let F be the set of feasible points. Because $F = \bigcap_{i=1}^m \{\mathbf{x} \in C : g_i(\mathbf{x}) \leq 0\}$, F is the intersection of convex sets and is therefore convex. Now $S = \{\mathbf{x} \in F : f(\mathbf{x}) = MP\}$, which is equal to $S = \{\mathbf{x} \in F : f(\mathbf{x}) \leq MP\}$, because for every $\mathbf{x} \in F$, $f(\mathbf{x}) \geq MP$. Therefore, by the hint and the fact that F is convex, S is also convex.

5: Question #5 has been removed, because we haven't covered the necessary material. The question that was here will be asked on homework #7.

6: *(C14 only)* Let A be an $m \times n$ matrix and let $\mathbf{b} \in \mathbb{R}^m$ be a fixed vector. Suppose that the convex program

$$(P)\begin{cases} \text{Minimize} & ||\mathbf{x}||^2 \\ \text{subject to} & A\mathbf{x} \leq \mathbf{b} \end{cases}$$

is superconsistent and has a solution $\mathbf{x}^*$. Use Karush-Kuhn-Tucker Theorem to show that there is a vector $\mathbf{y} \in \mathbb{R}^m$ such that $\mathbf{x}^* = A^T\mathbf{y}$.

Solution:
Let us first write the program with more details

$$(P)\begin{cases} \text{Minimize} & x_1^2 + x_2^2 + \cdots + x_n^2 \\ \text{subject to} & a_{1,1}x_1 + a_{1,2}x_2 + \cdots + a_{1,n}x_n \leq b_1 \\ & a_{2,1}x_1 + a_{2,2}x_2 + \cdots + a_{2,n}x_n \leq b_2 \\ & \vdots \\ & a_{m,1}x_1 + a_{m,2}x_2 + \cdots + a_{m,n}x_n \leq b_m \end{cases}$$

Because (P) is superconsistent, for every solution $\mathbf{x}^*$ there exists λ^* such that $\mathbf{x}^*$ and λ^* satisfy the gradient version of the KKT conditions. We therefore have that for every $i = 1, \ldots, n$,

$$2x_i^* + \lambda_1^* a_{1,i} + \lambda_2^* a_{2,i} + \cdots \lambda_m^* a_{m,i} = 0.$$

We can write this whole system more compactly as, $2(\mathbf{x}^*)^T + (\lambda^*)^T A = \mathbf{0}^T$, which implies $2\mathbf{x}^* = -A^T\lambda^*$. So, if we let $\mathbf{y} = -\frac{1}{2}\lambda^*$, we have that $\mathbf{x}^* = A^T\mathbf{y}$.

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MATH484 - MIDTERM 3
April 4th 2017, 7pm – 8:30pm
223 Gregory Hall Section: C13 C14 Name: ....

Question 1:

(a) What is the definition of the closure $\overline{A}$ of a set $A \subseteq \mathbb{R}^n$.

(b) For a real valued function f defined on a set $C \subseteq \mathbb{R}^n$, what is the definition of epi(f).

(c) For a real valued function f defined on $C \subseteq \mathbb{R}^n$, what is the definition of a subgradient at a point $\mathbf{x}^0 \in C$.

The remaining parts of this question refer to the following convex program

$$(P)\begin{cases}\text{Minimize} & f(\mathbf{x})\\ \text{subject to} & g_1(\mathbf{x}) \leq 0, \ldots, g_m(\mathbf{x}) \leq 0\\ & \mathbf{x} \in C.\end{cases}$$

where $f, g_1, \ldots, g_m$ are convex function and $C \subseteq \mathbb{R}^n$ is a convex set.

(d) For $\mathbf{x}^* \in C$ and $\lambda^* \in \mathbb{R}^m$ such that $\lambda^* \geq \mathbf{0}$, what does it mean for $\mathbf{x}^*$ and λ^* to satisfy the complementary slackness condition?

(e) For any $\mathbf{z} \in \mathbb{R}^m$, write the program $(P(\mathbf{z}))$.

(f) For any $\mathbf{z} \in \mathbb{R}^m$, what is the definition of $\mathbf{MP(z)}$?

Question 2:

(a) *Complete the following statement*

Suppose that $C \subseteq \mathbb{R}^n$ is a convex set and that $\mathbf{y} \in \mathbb{R}^n$ is not in C. Then $\mathbf{x}^* \in C$ is the closest vector in C to $\mathbf{y}$ if and only if ...

Solution:

$(\mathbf{y} - \mathbf{x}^*)(\mathbf{x} - \mathbf{x}^*) \leq 0$ for every $\mathbf{x} \in C$.

(b) State the Support Theorem.

Solution:

See Theorem 5.1.9 on page 166

(c) State the **extended** arithmetic geometric mean inequality **with the condition for equality**.

Solution:

See Theorem 5.3.1 on page 188

Question 3:
Consider the following convex program.

$$(P)\begin{cases}\text{Minimize} & 8x_1 - 4x_2\\ \text{subject to} & 3x_1^2 - 2x_1x_2 + x_2^2 \ \leq \ 2\end{cases}$$

(a) Show that 0 is not feasible for the dual program (DP).

Solution:

Since $\inf_{\mathbf{x}\in\mathbb{R}^2}\{L(x, 0)\} = \inf_{\mathbf{x}\in\mathbb{R}^2}\{8x_1 - 4x_2\} = -\infty$, 0 is infeasible for (DP)

(b) Compute $h(\lambda)$ for any fixed $\lambda > 0$. *(Your answer should be in terms of λ)*

Solution:

We can solve the following two linear equations

$$0 = \frac{\partial L(\mathbf{x}, \lambda)}{\partial x_1} = 8 + \lambda(6x_1 - 2x_2)$$
$$0 = \frac{\partial L(\mathbf{x}, \lambda)}{\partial x_2} = -4 + \lambda(-2x_1 + 2x_2)$$

to get that $x_1 = -1/\lambda$ and $x_2 = 1/\lambda$, so $(-1/\lambda, 1/\lambda)$ is a critical point. Since

$$L(\mathbf{x}, \lambda) = 8x_1 - 4x_2 + \lambda(3x_1^2 - 2x_1x_2 + x_2^2 - 2)$$

is strictly convex as a function of $\mathbf{x}$ for any $\lambda > 0$, this is a unique global minimizer. Plugging this back into $L(\mathbf{x}, \lambda)$ we get that $h(\lambda) = \frac{-6}{\lambda} - 2\lambda$

(c) Use your answer from part (b) to compute MD and to find a solution λ^* to (DP). *(You must use your answer to the previous problem and the definition of MD to receive full credit, but you will get partial credit if you compute MD and the solution to (DP) another way).*

Solution:

We can take the derivative of $h(\lambda)$ to find that the only one critical point λ such that $\lambda > 0$ is $\lambda = \sqrt{3}$. Since then $h''(\lambda) = -12\lambda^3 < 0$ for all $\lambda > 0$, this is a global maximizer. Hence $\lambda^* = \sqrt{3}$ and $MD = h(\lambda^*) = -6/\sqrt{3} - 2\sqrt{3} = -4\sqrt{3}$.

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nsider the following linear program.

$$(LP)\begin{cases}\text{Minimize} & 3x & & +z & \\ \text{subject to} & x & -2y & +z & \geq 3 \\ & x & +y & -z & \geq -1 \\ & x \geq 0, & y \geq 0, & z \geq 0 & \end{cases}$$

a) Write the dual linear program.

Solution:

The dual program is

$$(DLP)\begin{cases}\text{Maximize} & 3x - y & & \\ & x + y & \leq & 3 \\ \text{subject to} & -2x + y & \leq & 0 \\ & x - y & \leq & 1 \\ & x \geq 0, \quad y \geq 0 & & \end{cases}$$

(b) Solve the dual by plotting the feasible region on the plane.

Solution:

By plotting the feasible region on the plane and checking the corner points, we find that $(2, 1)$ is the solution and the value of the objective function is 5.

(c) Use complementary slackness and the duality theorem for linear programming to solve the original linear program.

Solution:

Since both of the components in the dual are non-zero, complementary slackness implies that both $x - 2y + z = 3$ and $x + y - z = -1$. By the duality theorem for linear programming, we also have that $3x + z = 5$. If we use the Lagrangian, we have that $5 = 3x + z + 2(3 - x + 2y - z) + 1(-1 - x - y + z) = 5 + 3y$, so $y = 0$ in any solution. In either case, we get that $x = 1$, $y = 0$, and $z = 2$.

Question 5:
Consider the following constrained geometric program:

$$(GP)\begin{cases}\text{Minimize} & \dfrac{x^3 y}{z} \\ \text{subject to} & \dfrac{6z}{x^2} + \dfrac{1}{y} \leq 1, \text{ and} \\ & \dfrac{x}{z} \leq 1 \\ \text{where} & x > 0,\ y > 0,\ z > 0\end{cases}$$

(a) Write the dual geometric program (DGP).

Solution:

The dual program is

$$(GP)\begin{cases}\text{Maximize} & \left(\frac{1}{\delta_1}\right)^{\delta_1}\left(\frac{6}{\delta_2}\right)^{\delta_2}\left(\frac{1}{\delta_3}\right)^{\delta_3}\left(\frac{1}{\delta_4}\right)^{\delta_4}(\delta_2+\delta_3)^{\delta_2+\delta_3}(\delta_4)^{\delta_4} \\ \text{subject to} & \delta_1 = 1 \\ & 3\delta_1 - 2\delta_2 + \delta_4 = 0 \\ & \delta_1 - \delta_3 = 0 \\ & -\delta_1 + \delta_2 - \delta_4 = 0 \\ & \delta_1 > 0, \\ & \delta_2 > 0, \delta_3 > 0 \text{ or } \delta_2 = \delta_3 = 0, \\ & \delta_4 \geq 0\end{cases}$$

(b) Find a solution to (DGP) and write the value of the objective function at a solution.

Solution:

The solution is $\delta_1 = 1, \delta_2 = 2, \delta_3 = 1, \delta_4 = 1$. The objective function at this solution is

$$(1)^1(3)^2(1)^1(1)^1(2+1)^3(1)^2 = 3^5$$

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(c) Reconstruct the solution of (GP) from the solution of (DGP). Which means, find a solution (x^*, y^*, z^*) and value of the objective function at the solution (x^*, y^*, z^*).

Solution:

To get x, y, z, we need to compute

$$3^5 = \frac{x^3 y}{z}$$
$$\frac{2}{3} = \frac{6z}{x^2}$$
$$\frac{1}{3} = \frac{1}{y}$$
$$1 = \frac{x}{z}$$

The third equation gives us that $y = 3$ and the the fourth equation gives us that $z = x$. Using this in the second equation, we can deduce that $x = z = 9$. So, $(x^*, y^*, z^*) = (9, 3, 9)$. We can compute that $g_1(x^*, y^*, z^*) = 3^5$ as expected.

Question 6: *(C14 (four credit hours) required, C13 (three credit hours) optional)*
Prove that if M is a subspace of $\mathbb{R}^n$ and that $\mathbf{y} \in \mathbb{R}^n$ is a vector not in M. Then $\mathbf{x}^* \in M$ is the closest vector to $\mathbf{y}$ if and only if $\mathbf{y} - \mathbf{x}^* \in M^\perp$. (Here $M^\perp$ denotes the orthogonal complement of the subspace M.) *You can (and should) use other theorems proved in class to prove this theorem.*

Solution:

See Corollary 5.1.2 on page 161.

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