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# HW3

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PARALLEL PROGRAMMING

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1a.

Computation:

$$A \cdot N^2 / P$$

Communication:

$$KN$$

Comm-to-Comp Ratio

$$K \cdot P / N$$

Efficiency:

$$1 / (1 + (K \cdot P / N))$$

Isoefficiency:

$$\text{Let } W = N^2$$

We know that  $N = \sqrt{W}$

$$P / \sqrt{W} = k; W = (P/k)^2$$

We know that the allgather on a hypercube has a depth of  $\log(p)$

Communication Cost:

We start with  $N/P$  data and that amount will increase with every level of the gather.

This gives us a sum of  $(\text{Alpha} + \text{Beta} \cdot N/P \cdot 2^i)$  for when  $i = 0$  to when  $i = \log(p) - 1$

b.

Computation:

$$A \cdot N / P$$

Communication:

$$KN$$

Comm-to-Comp Ratio

$$K \cdot P / A$$

Efficiency:

$$1 / (1 + (K \cdot P / A))$$

Isoefficiency:

Problem size is  $W$

2. a.

$$\begin{aligned} \text{i) Diameter} &= (x-1) + (y-1) + (z-1) \\ &= 29 + 39 + 49 \\ &= 117 \end{aligned}$$

Degree = [3, 6] (range of 3 to 6, inclusive)

The diameter of this given mesh is 117 and the degree for each node is between 3 and 6.

ii) If we had a 2D mesh of 30x50, the bisection width of this would be 30. If we have a 3D mesh of 30x40x50 we just have the 40 layers of the 30x50 stacked one top of one another. Because of this we have a bisection width of  $30 * 40 = 1200$ . In order to find the bisection bandwidth of the mesh we will need to multiply the bisection by the channel bandwidth. We are given a channel bandwidth as 4 GB/x

$$\begin{aligned}\text{bisection bandwidth} &= \text{channel bandwidth} * \text{bisection width} \\ &= 4(\text{GB/s}) * 1200 = 4800(\text{GB/s})\end{aligned}$$

So we have a resulting bisection bandwidth of the network will be 4800 GB/s

iii) Since we need to minimize bandwidth usage we need to take advantage of all the 2GB/s channel bandwidths. Because of this we need to cut parallels to the x axis apart from the z axis.

$$\begin{aligned}\text{Bisection width} &= 40 * 50 \\ &= 2000\end{aligned}$$

This give us 2 GB/s

$$\begin{aligned}\text{Bisection bandwidth} &= \text{channel bandwidth} * \text{bisection width} \\ &= 2 * 2000 \\ &= 4000 \text{ GB/s}\end{aligned}$$

b.

Assumptions:

1. The original 2D grid of processes are i, j, and k where  $i = [0,199]$ ,  $j = [0, 299]$  and  $k = [0,0]$
2. The 3D grid of the processes defined above are labeled with  $i'$ ,  $j'$ ,  $k'$  where  $i' = [0, 29]$ ,  $j' = [0, 39]$ ,  $k' = [0, 49]$
3. The total # of nodes in all of the cases are the same:
  - a.  $(200 * 300 \Rightarrow 60000 \Rightarrow 30 * 40 * 50)$  is true

There are various mappings that can be used in such a situation.

The simplest mapping would take advantage of this:

$$30 * 10 = 300$$

$$40 * 50 / 10 = 200$$

From this we get the following mapping

$$f(i', j', k') = (j/10, i/5, k/4)$$

From this mapping from the y-axis from the original grid to the x-axis of the 3D mesh.

This uses the x axis then from the original grid. This finally creates the y and z axes of the 3D meshes.