2 Buckingham-Pi

2.1 Dimensional analysis

Dimensional analysis is the analysis of a relationship between physical quantities by considering their units of measure. For example, it will be meaningless to construct an equation in which the dimension of the terms in the equation is of the form

$$L + M = T, \tag{2.1}$$

where L is measured in meters, M in kilograms and T in seconds. Simply put, if an equation models a physical process, then, all the terms in it that are separated by +, - or = must have the same physical dimension. If they did not, we would be saying something ridiculous like:

apples + eggs = light bulbs + whisky,

Even though, it is only reasonable to compare apples with apples.

This is, perhaps, the most basic of the many consistency checks that youshould build into your mathematics.

An equation like (2.1) will be called dimensionally inconsistent, or dimensionally non-homogeneous. Therefore, an equation that has the correct and same dimension termwise, will be called dimensionally homogeneous (or dimensionally consistent).

Dimensional analysis is a method for reducing the number (and complexity) of measureable quantities which affect a given physical process by using a compacting technique. That is, if a phenomena depends upon n dimensional variables, dimensional analysis will reduce the problem to k dimensionless variables, where $k < n$.

The standard notation for the dimensions of a quantity is a square bracket [.] around the quantity. All the scientific units that we will be using can be written in terms of the 7 primary dimensions or fundamental dimensions which are

- Mass m: $[m] = M$ measured in kilograms
- Time t: $[t] = T$ measured in seconds
- Length l: $[l] = L$ measured in meters

- Electric current i: $[i] = I$ measured in ampere
- Temperature θ: $[\theta] = \Theta$ measured in kelvin (or $^\circ C$)
- amount of substance with dimension [mol], measured in mole
- luminoscity with dimension [cd], measured in candela

All well-posed models must have consistent units.

2.2 Examples

2.2.1 Newton's second law of motion: $F = ma$

Check that the dimensions of Newton's 2nd law are consistent:

$$[F] = Newtons = \frac{ML}{T^2}$$
$$[m] = M$$
$$[a] = \frac{L}{T^2}$$

$$[ma] = [F] = \frac{ML}{T^2}$$

2.2.2 The Logistic model

Let the dimension for population numbers be such that $[P] = N$, where N means an amount or quantity . Consider the Logistic equation:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right). \tag{2.2}$$

where r is the per capita growth rate of the population measured in s^{-1}, K is the carrying capacity and P is the population size.

LHS:

$$\left[\frac{\mathrm{d}P}{\mathrm{d}t}\right] = \frac{N}{T}$$

RHS: Since $\left[\dfrac{P}{K}\right] = 1$

$$\left[rP\left(1 - \frac{P}{K}\right)\right] = [rP] = N[r] = \frac{N}{T}$$

2.2.3 Integrals and derivatives

Integrals and derivatives are defined by limits. For example,

$$\left[\frac{dP}{dt}\right] = \left[\lim_{\delta t \to 0} \frac{\delta P}{\delta t}\right] = \lim_{\delta t \to 0}\left[\frac{\delta P}{\delta t}\right] = \lim_{\delta t \to 0} \frac{N}{T} = \frac{N}{T}. \tag{2.3}$$

Suppose that $v(t)$ is a velocity. Then $[v] = \dfrac{L}{T}$. What are the units of

$$\int_a^b v(t)dt? \tag{2.4}$$

$$\left[\int_a^b v(t)dt\right] = \left[\lim_{n \to \infty} \sum_{i=1}^{n} v(t_i)\Delta t_i\right] = \frac{L}{T} \times T = L$$

We can find the units of certain parameters in equations by matching the dimensions. Consider the linear partial differential equation governing the diffusion of a substance, for which $u(x,t)$, is the concentration of the substance in units of amount of a substance per unit volume .

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}. \tag{2.5}$$

Given that $[u] = N/L^3$, $[t] = T$ and $[x] = L$, what are the dimensions of α?

LHS:

$$\left[\frac{\mathrm{d}u}{\mathrm{d}t}\right] = \frac{N}{L^3} * \frac{1}{T} = \frac{N}{L^3 T}$$

RHS:

$$\left[\alpha^2 \frac{\partial^2 u}{\partial x^2}\right] = [\alpha^2]\frac{N}{L^3} * \frac{1}{L^2} = [\alpha^2]\frac{N}{L^5}.$$

Comparing both sides gives

$$\frac{N}{L^3T} = [\alpha^2]\frac{N}{L^5}$$

for which

$$[\alpha^2] = \frac{L^2}{T}$$

and

$$[\alpha] = \frac{L}{\sqrt{T}}$$

What are the units of β in the following expression:

1. $\exp[\beta t]$
2. $\sin(\beta t)$
3. $\cos(\beta t)$

solution

The argument in each of the three function must be dimensionless. In particular, the cos and sin function are defined in terms of angle θ which is dimensionless since $\theta = \frac{s}{r}$ where s is some arc length and r , a radius.

Also, these three functions are written as sums of different powers by taking Taylors expansion about the origin, e.g.,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

A dimensional argument will mean comparing length with area with volume, etc.

Thus,

$$[\beta t] = [\beta]T = 1$$

$$[\beta] = \frac{1}{T}$$

2.3 The Principle of Dimensional Homogeneity

If an equation truly expresses a proper relationship between dimensional variables in a physical process, it will be dimensionally homogeneous. This means that each of its additive terms will have same dimensions.

2.4 Buckingham-Pi theorem

Let $W_1, W_2, W_3, \ldots, W_n$ be n dimensional variables that are physically relevant in a given problem and that are inter-related by a dimensionally homogeneous equation. This dimensionally homogeneous equation has the form

$$F(W_1, W_2, W_3, \ldots, W_n) = 0, \quad \text{or equivalently} \quad W_1 = f(W_2, \ldots, W_n). \tag{2.6}$$

If k is the number of fundamental dimensions rquired to describe the n variables, then there will be $j = (n - k)$ dimensionless and independent quantities or **Pi groups** $\Pi_1, \Pi_2, ..., \Pi_{n-k}$. The functional relationship can thus be reduced to the much more compact form

$$H(\Pi_1, \Pi_2, ..., \Pi_n) = 0, \tag{2.7}$$

or equivalently,

$$\Pi_1 = h(\Pi_2, ..., \Pi_n). \tag{2.8}$$

Note that the set of dimensionless quantities $\Pi_1, \Pi_2, ..., \Pi_{n-k}$ is not unique. They are, however, independent and form a complete set.

It is important to differentiate between the following:

Dimensional variables: These are quantities which actually vary during a given experiment, and thee quantities are often plotted against each other to show the data.

Dimensional constants: These are quantities that may vary between experiments, biut they are often held constant during a particular experiment. Examples are density ρ, gravitational acceleration g, pressure P, viscosity μ, etc.

Pure constants: These quantities have no dimensions, and arise from mathematical manipulations. For example, π and e are pure constants.

Note that dimensionally homogeneous means that the units balance on either side of the equation.

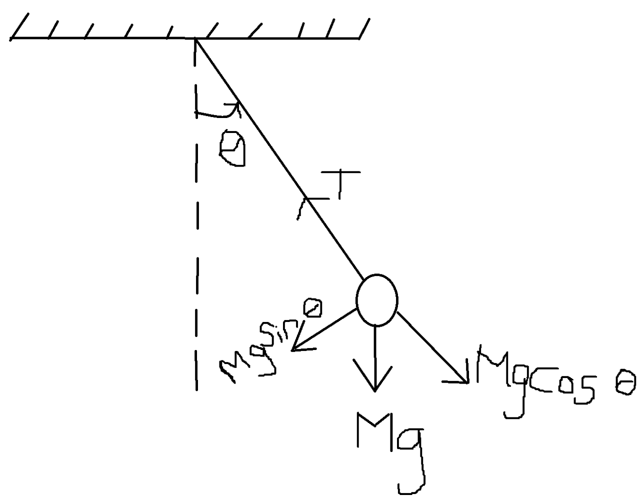
When building a model, initial understanding can be obtained from dimensional analysis and Buckingham-Pi theory.

2.4.1 The simple pendulum

Dimensional analysis is a powerful method for analysing problems without much knowledge of:

- The underlying physics
- Differential equations

We will now illustrate this by considering the period of a pendulum.



Measurable quantities (variables and parameters):

- M - mass of the pendulum.
- l - length of the pendulum

- $x(t)$ - arc length from the vertical or equilibrium configuration
- $\theta(t)$ - angle of deflection from the vertical in radians

The problem is to determine the period of the pendulum.

Method 1: Differential equations

We formulate and solve the ordinary differential equation (ODE) that models the dynamics.

By Newton's second law of motion

$$M\frac{d^2x}{dt^2} = -Mg\sin\theta(t). \tag{2.9}$$

But

$$x(t) = l\theta(t), \tag{2.10}$$

which gives

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0. \tag{2.11}$$

Now consider small amplitude oscillations so that θ is small. Then

$$\sin\theta = \theta + O(\theta^3). \tag{2.12}$$

Neglecting terms of order θ^3 leads to

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0. \tag{2.13}$$

The general solution is of the form

$$\theta(t) = A\cos(\omega t + \alpha), \tag{2.14}$$

where

$$\omega = \left(\frac{g}{l}\right)^{1/2}, \tag{2.15}$$

and A and α are constants. Note that the solution does not depend on the mass of the pendulum M.

Exercise 1

Suppose that you are given the initial conditions

$$\theta(0) = \theta_0, \qquad \frac{d\theta}{dt}(0) = 0. \tag{2.16}$$

Find A and α in (2.14).

Now, let T_0 be the period of the simple pendulum. This means that

$$\theta(t) = \theta(t + T_0). \tag{2.17}$$

That is,

$$A\cos(\omega t + \alpha) = A\cos(\omega t + \alpha + \omega T_0)$$

Thus,

$$\omega T_0 = 2\pi$$

Thus

$$T_0 = 2\pi/\omega = 2\pi\left(\frac{l}{g}\right)^{1/2}, \tag{2.18}$$

where T_0 is the period of oscillation of the pendulum, when terms of order θ^3 are neglected.

Method 2: Dimensional analysis

In this method, an alternative derivation of the period T_0 is provided.

Physical quantities in which T_0 could depend:

- l - length of the pendulum
- g - acceleration due to gravity
- m - mass of the pendulum
- θ - angle of deflection

Assume that there exists a function f such that

$$T_0 = f(l, g, m, \theta). \tag{2.19}$$

Denote the dimensions of a physical quantity z by square brackets $[z]$. So

$$[z] = dimensions\ of\ z. \tag{2.20}$$

Fundamental dimensions:

- M - Mass

- L - length
- T - time

M, L, T are the fundamental dimensions of quantities in mechanics. In this problem we will only need the mechanical fundamental dimensions. There are other fundamental dimensions such as Temperature etc.

We will now express the quantities in terms of the fundamental dimensions. The variable θ is dimensionless.

- $[T_0] = T$
- $[l] = L$
- $[g] = L/T^2$
- $[m] = M$
- $[\theta] = M^0 L^0 T^0 = 1$

We consider a product of powers of l, g, m, θ such that we can form dimensionless products. Therefore we write

$$[T_0^a \theta^b l^c m^d g^e] = M^0 L^0 T^0. \tag{2.21}$$

We want to find values for a-e so that this product is dimensionless. Substituting in the dimensions leads to

$$T^a (M^0 L^0 T^0)^b L^c M^d (LT^{-2})^e = M^0 L^0 T^0. \tag{2.22}$$

Equating exponents gives

$$d = 0, \qquad c + e = 0, \qquad a - 2e = 0. \tag{2.23}$$

The solution to this system is

$$a = 2e, \quad c = -e, \quad d = 0, \tag{2.24}$$

with b arbitrary. There are infinitely many solutions which can be represented by

$$\begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} = e \begin{pmatrix} 2 \\ 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{2.25}$$

A basis is given by the two vectors $(2\ 0\ \ -1\ 0\ 1)^T$ and $(0\ 1\ 0\ 0\ 0)^T$. One product is thus (by choosing the basis elements)

$$\Pi_1 = T_0^2 l^{-1} g, \tag{2.26}$$

and the other is

$$\Pi_2 = \theta. \tag{2.27}$$

These Π_i form a complete set of dimensionless products as any other dimensionless product can be formed by taking products $\Pi_1^e \Pi_2^b$ for any b and e values.

The Buckingham-Pi theorem gives

$$H(T_0^2 l^{-1} g, \theta) = 0, \tag{2.28}$$

which implies that

$$T_0 = \sqrt{\frac{l}{g}} h(\theta), \tag{2.29}$$

where h is an arbitrary function of θ. One requires more knowledge to find h. We can either conduct experiments to determine h or use a differential equation. From using the differential equation, we see that h is a constant. We can also show that h is a constant by using the following argument: Since there is no damping, $h(\theta)$ is an even function of θ. Expanding in a Taylor series gives

$$h(\theta) = h(0) + \theta h'(0) + \frac{\theta^2}{2} h''(0) + O(\theta^3). \tag{2.30}$$

Now, since h is an even function of θ, $h'(0) = 0$ and $h'''(0) = 0$ and so on. Therefore,

$$h(\theta) = h(0) + \frac{\theta^2}{2} h''(0) + O(\theta^4). \tag{2.31}$$

Thus, correct to first order in θ

$$T_0 = h(0)\sqrt{\frac{l}{g}}. \tag{2.32}$$

Comparison:

Differential equation: $T_0 = 2\pi\sqrt{\dfrac{l}{g}}$

Dimensional analysis: $T_0 = h(0)\sqrt{\dfrac{l}{g}}$.

From this we can conclude that dimensional analysis gives correctly the functional form of the solution in terms of the physical quantities. It does not, however, solve for constant factors. These can be obtained experimentally.

Note that for this example the number of variables and parameters is 5 and the number of equations is 3. If there were more dimensions we would get more equations. Now 5-3 is 2 which is the number of dimensionless products.

2.5 General procedure

Measurable quantities:

A quantity u is to be determined in terms of n measurable quantities (variables and parameters) $W_1, W_2, ..., W_n$ with dimensions $[u]$ and $[W_i]$.

We assume that there exists an unknown function f such that

$$u = f(W_1, W_2, ..., W_n). \tag{2.33}$$

We analyse this function using fundamental dimensions $(L_1, L_2, ..., L_m)$ where $m \leq n$.

Fundamental dimensions:

We postulate that all descriptive quantities in mathematical models have dimensions that are products of powers of fundamental dimensions, $L_1, L_2, ..., L_m$.

In the SI system there are seven fundamental dimensions:

- $L_1 = M$ (mass)
- $L_2 = L$ (length)
- $L_3 = T$ (time)
- $L_4 = A$ (electric current)
- $L_5 = k$ (Temperature)
- $L_6 = mol$ (amount of substance)
- $L_7 = cd$ (luminoscity)

Note that these are different to units. For example the SI units of mass is kg, length is metre and time is second.

Let z be some quantity. The dimensions of z, denoted by $[z]$, is a product of powers of the fundamental dimensions

$$[z] = L_1^{\alpha_1} L_2^{\alpha_2} ... L_m^{\alpha_m}, \tag{2.34}$$

for some real numbers α_i.

The dimension vector of z is the column vector

$$\alpha = (\alpha_1 \alpha_2 ... \alpha_m)^t. \tag{2.35}$$

A quantity z is dimensionless provided

$$[z] = L_1^0 L_2^0 ... L_m^0. \tag{2.36}$$

Exercise 2

Find the dimensions and SI units for the following:

- velocity
- density
- force
- pressure
- energy
- power
- heat flux
- specific heat
- kinematic viscosity

2.6 Further examples

2.6.1 Energy released by the first nuclear bomb 1945

In 1947 a picture of the explosion was published. The energy released by the explosion was strictly classified. G I Taylor determined the energy released during the explosion using only the radius of the expanding blast wave at time t from the published picture, and dimensional analysis. The energy released was still classified but his calculations proved to be quite accurate.

A nuclear explosion is approximated by the release of a large amount of energy E from a point. This results in an expanding fireball.

Let $r =$ the radius of the expanding fireball. We want to find a relationship between r and the other relevant parameters. These other relevant parameter are:

- $t-$ time elapsed after the explosion takes place
- $E-$ energy released by the explosion
- ρ_0- initial or ambient air density
- P_0- initial or ambient air pressure

Thus, we write

$$r = f(t, E, \rho_0, P_0)$$

assuming f exists.

Express measurable quantities in terms of Fundamental dimensions:

- $[r] = L \qquad [t] = T \qquad [E] = \dfrac{ML^2}{T^2}$
- $[\rho_0] = \dfrac{M}{L^3} \qquad [P_0] = \dfrac{M}{T^2 L}$

Consider the products of powers of r, t, E, ρ_0 and P_0 that is dimensionless:

$$[(r)^a (t)^b (E)^c (\rho_0)^d (P_0)^e] = 1$$

$$L^a T^b (ML^2T^{-2})^c (ML^{-3})^d (MT^{-2}L^{-1})^e = 1$$

This gives the three equations

$$L: \quad a + 2c - 3d - e = 0, \tag{2.37}$$

$$M: \quad c + d + e = 0, \tag{2.38}$$

$$T: \quad b - 2c - 2e = 0. \tag{2.39}$$

We have 3 equations and 5 unknowns. There are , therefore, 5-3 = 2 dimensionless products Let's solve in terms of c and d:

$$e = -c - d, \tag{2.40}$$

$$b = 2c + 2e = 2c - 2c - 2d = -2d, \tag{2.41}$$

$$a = -2c + 3d + e = -2c + 3d - c - d = -3c + 2d. \tag{2.42}$$

Thus we can write

$$\begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} = c \begin{pmatrix} -3 \\ 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} + d \begin{pmatrix} 2 \\ -2 \\ 0 \\ 1 \\ -1 \end{pmatrix}. \tag{2.43}$$

Using these basis vectors we get

$$\Pi_1 = r^{-3} E P_0^{-1}, \qquad \Pi_2 = r^2 t^{-2} \rho_0 P_0^{-1}. \tag{2.44}$$

But we want to solve explicitly for r so we form a new dimensionless product:

$$\Pi_1 = \Pi_1 \Pi^{\frac{3}{2}} = E t^{-3} \rho_0^{\frac{3}{2}} P_0^{\frac{-5}{2}}$$

and

$$\Pi_2 = \Pi_1^{-1} \Pi^2 = r^5 E^{-1} t^{-2} \rho_0$$

Buckingham-Pi theorem gives

$$H(\Pi_1, \Pi_2) = 0, \qquad \text{or} \quad \Pi_2 = G(\Pi_1)$$

$$r = \left(\frac{E t^2}{\rho_0} \right)^{\frac{1}{5}} F \left(P_0^{\frac{5}{2}} \rho_0^{\frac{3}{2}} \frac{E}{t^3} \right)$$

2.6.2 Heat conduction

Consider a one-dimensional heat conduction in an infinite bar. The bar is heated by a point source of heat. Let

$$u = \text{temperature at any point on the bar} \tag{2.45}$$

We assume that

$$u = f(x, t, \rho, c, k, Q), \tag{2.46}$$

where

- $x-$ distance along the bar from the point source of heat
- $t-$ elapsed time after the initial heating
- $\rho-$ mass density of the bar
- $c-$ the specific heat of the bar
- $\kappa-$ thermal conductivity of the bar.
- $Q-$ strength of the heat source measured in energy per length squared.

Fundamental dimensions:

- $L_1 = M$
- $L_2 = L$
- $L_3 = T$
- $L_4 = K$

Dimensions of the quantities:

- $[u] = K \qquad [t] = T \qquad [x] = L \qquad [Q] = \dfrac{M}{T^2}$
- $[\rho] = \dfrac{M}{L^3} \qquad [\kappa] = \dfrac{ML}{T^3K} \qquad [c] = \dfrac{L^2}{KT^2}$

There are 4 fundamental dimensions for this problem, and 7 measurable quantities. we therefore have $7 - 4 = 3$ dimensionless products. To obtain them let

$$[u^a x^b t^c \rho^d c^e \kappa^f Q^g] = 1. \tag{2.47}$$

This gives

$$L: \quad b - 3d + 2e + f = 0, \tag{2.48}$$

$$M: \quad d + f + g = 0, \tag{2.49}$$

$$T: \quad c - 2e - 3f - 2g = 0, \tag{2.50}$$

$$K: \quad a - e - f = 0. \tag{2.51}$$

Writing these equations in terms of e, f and d gives

$$a = e + f, \quad g = -d - f \quad c = 2e + 3f + 2g = 2e - 2d + f, \quad b = 3d - 2e - f. \tag{2.52}$$

Therefore we get

$$\begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \end{pmatrix} = e \begin{pmatrix} 1 \\ -2 \\ 2 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + f \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} + d \begin{pmatrix} 0 \\ 3 \\ -2 \\ 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}. \tag{2.53}$$

From these basis vectors we get

$$\Pi_1 = ux^{-2}t^2c, \quad \Pi_2 = ux^{-1}tkQ^{-1}, \quad \Pi_3 = x^3t^{-2}\rho Q^{-1}. \tag{2.54}$$

We need only one of the dimensional product Π_i to depend on u. We now eliminate u in one of the Π's and complete the problem.

Take the product $\Pi_1\Pi_2^{-1}$ (or $\Pi_1^{-1}\Pi_2$) to obtain a new dimensionless product which we call $\bar{\Pi}_2$

$$\bar{\Pi}_2 = \Pi_1\Pi_2^{-1} = x^{-1}tck^{-1}Q$$

Thus,

$$\Pi_1 = H(\bar{\Pi}_2, \Pi_3)$$

That is,

$$u = \frac{x^2}{t^2c}H(x^{-1}tck^{-1}Q, x^3t^{-2}\rho Q^{-1})$$

Exercise 3

2.6.3 The damped pendulum

For the damped pendulum, assume that the drag force F is proportional to v and use dimensional analysis to find the period.
Hint: let the force F be given by

$$\underline{F} = -\alpha \underline{v}, \tag{2.55}$$

where $[\alpha] = M/T$. Now look for products of the form

$$[T_0^a \theta^b l^c m^d g^e \alpha^f] = M^0 L^0 T^0. \tag{2.56}$$

Many of you probably got the answer

$$\Pi_1 = T_0^2 l^{-1} g, \quad \Pi_2 = \theta, \quad \Pi_3 = T_0^{-1} m \alpha^{-1}. \tag{2.57}$$

Using (Π_1, Π_2, Π_3) we obtain

$$\Pi_1 = F(\Pi_2, \Pi_3), \tag{2.58}$$

which gives

$$T_0^2 l^{-1} g = F(\theta, T_0^{-1} m \alpha^{-1}). \tag{2.59}$$

Solving for T_0 leads to

$$T_0 = \sqrt{\frac{l}{g}} G(\theta, T_0^{-1} m \alpha^{-1}). \tag{2.60}$$

Recall that the period for the simple pendulum is given by

$$T_0 = \sqrt{\frac{l}{g}} F(\theta). \tag{2.61}$$

Clearly, these two expressions show the difference between the period of the simple and the damped pendulum. However, the expression for the damped pendulum does not solve explicitly for T_0.

If I ask you to solve explicitly for T_0, then multiply

$$\Pi_1 \Pi_3^2 = l^{-1} g m^2 \alpha^{-2} = \Pi_3^*. \tag{2.62}$$

Now using the set (Π_1, Π_2, Π_3^*) we get

$$T_0^2 l^{-1} g = F(\theta, l^{-1} g m^2 \alpha^{-2}), \tag{2.63}$$

which gives

$$T_0 = \sqrt{\frac{l}{g}}G(\theta, l^{-1}gm^2\alpha^{-2}). \tag{2.64}$$

In this case, we have an explicit expression for T_0 and we can compare the result to the simple pendulum.

Suppose we use the set (Π_3, Π_2, Π_3^*). Then we get

$$T_0 = \frac{m}{\alpha}F(\theta, l^{-1}gm^2\alpha^{-2}). \tag{2.65}$$

In this case, we have an explicit expression for T_0 but we cannot compare it easily to the simple pendulum.

Which case is best?

2.6.4 Terminal velocity of a falling raindrop (students to complete)

Determine the terminal velocity of a raindrop failing from a motionless cloud using Buckingham Pi theory. Let

- $v-$ terminal velocity of the raindrop
- $r-$ radius of raindrop
- $\rho-$ density of air
- $\mu-$ viscosity of air
- $g-$ gravity

You are given that

$$[\mu] = ML^{-1}T^{-1}, \tag{2.66}$$

where:
M represents the dimensions of mass.
T represents the dimensions of time.
L represents the dimensions of length.

Use products of the form

$$v^a r^b \rho^c \mu^d g^e. \tag{2.67}$$

2.6.5 Wind force on a moving van (students to complete)

Find the wind force F of a van moving down the freeway.
What affects the force F of the wind on a van?

Thus we have the relationship

Consider dimensionless products of the form

Equating coefficients gives

which yields an infinitude of solutions

The dimensionless product is given by

and using Buckingham-Pi we get
