Graph Search

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1 Introduction

2 Branch and Bound for Multi-stop Ordering

2.1 Search Tree Formulation

2.2 Integer Program Formulation

The linear program is formulated in the following method:
Notation:
c_{ij} = cost of path from vertex i to vertex j
Variables: x_{ij} , for all $i \in [0, n)$ and $j \in [0, n)$, such that $i \neq j$ (where the depth of the search tree is n). The integer program is formulated such that we must have $x_{ij} \in \{0, 1\}$
Objective function is to minimize : $\sum c_{ij} x_{ij}$
The constraints are:

$$\sum x_{ij} = n \tag{1}$$

for each $k \in [0, n)$

$$\sum x_{kj} = 1 \quad \forall j \in [0, n), j \neq k \tag{2}$$

for each $k \in [1, n)$

$$\sum x_{ik} = 1 \quad \forall i \in [0, n), i \neq k \tag{3}$$

$$x_{ij} + x_{ji} \leq 1 \tag{4}$$

In this formulation:
(1) ensures that exactly n edges in the search tree are picked.
(2) ensures that each vertex picked only visits one other vertex in the search tree
(3) ensures that each vertex is only visited once
(4) ensures that if we pick edge (i,j), we do not then pick edge (j,i) (ensuring that we do not go backwards in the search tree.
Together, these ensure that the output will be a valid path down the search

tree which is a valid hamiltonian path in the graph; this therefore will give us an ordering of vertices to visit. Since the integer program minimizes the function,$\sum c_{ij}x_{ij}$, which represents the total cost of the hamiltoninan path, the output will be the optimal one.

2.3 Branch and Bound

We then use the branch and bound algorithm, with a linear programming approximation of the formulation in 2.2 as the heuristic.