# Draft Project Proposal: The Particle Method, Local Volatility and Local Correlation

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## 1 Objective

Recently there has been significant interest in implementing calibration in Monte Carlo, via the particle method (Guyon & Henry-Labordere 2011),(Guyon 2013). There are several reasons that this is of interest:

- Calibration based on forward induction in partial differential equations (PDEs) is restricted to low dimensional, Markovian models. If we wish to add features such as stochastic volatility, stochastic interest rates etc. the dimensionality quickly increases. This is relevant both for accurately modelling derivative products and for XVA calculations, where it may become necessary to jointly model multiple market risk factors in addition to a stochastic credit factor.
- The particle method offers a relatively generic approach to calibration, making it easier and quicker to implement alternative model specifications (in theory). Usually calibration is a fairly difficult and bespoke problem, especially as the number of factors increases.
- Regulators and internal model validation groups are much more focused on benchmarking models and being able to make quantitative (not just qualitative) statements about their limitations eg. the effect of not modelling specific factors. This is hard to do without being able to efficiently implement a higher factor model including the additional factors.

Whilst the idea of the particle method is easy to state - essentially just a kernel regression implementation of forward induction in Monte Carlo - very little has been published on some of the implementational choices such as optimal choice of:

- Bandwidth/number of buckets.
- Kernel function.
- Time discretisation.
- Number of Monte Carlo paths.

Additionally of particular interest to us, since the method is based on Monte-Carlo, is how accurately volatility risk can be calculated.

Although the real power of the method is in its application to higher dimensional problems, we suggest starting with a low dimensional problem with a well-known solution in order to explore these effects. Depending on how successful part 1 of the project is, we then propose a second, more challenging phase, exploring a higher dimensional problem which is a genuine challenge facing the industry.

At each stage of both part 1 and part 2, the accuracy can be checked against the Black-Scholes formula, and we have a simple alternative model benchmark for comparison. Both pricing and calibration can be implemented through Monte-Carlo, which does not need to be heavily optimised or sophisticated in its implementation, since the focus of the project is on calibration accuracy, not numerical efficiency. BAML has some experience of using the particle method, so can offer guidance, but we have not explored the method exhaustively so we would be very interested in a systematic study.

So that the project team do not have to waste time worrying about building volatility surfaces according to complicated industry conventions, and to eliminate interpolation discrepancies, BAML will supply clean test data: interpolated implied volatility values by strike and maturity to the required degree of granularity, and interpolated FX forward rates by maturity.

## 2 Part 1: Deterministic Local Volatility

There is a well known way of calibrating deterministic local volatility directly to an implied volatility surface (Dupire 1994).

Part 1 of the project consists of implementing an alternative, particle method calibration of the deterministic local volatility, along with Monte-Carlo pricing. The pricing needs to be implemented sufficiently accurately that the dominant source of error is calibration. This can be done by using an arbitrarily large number of paths and timesteps in pricing, since the goal of the project is not to optimise the efficiency of pricing but to focus on calibration. Study and analyse:

- Accuracy of calibration of PV of vanilla options with all maturities by comparing pricing in deterministic local volatility model against value computed using Black-Scholes formula directly using implied volatility surface. Accuracy should be quoted in implied volatility terms.
- A useful benchmark would be to compare both local volatility surface and PV of vanilla options based on a conventional calibration using the Dupire formula.
- Benchmark the (bucketed) volatility risk of vanilla options against the analytic expression for Vega i.e. bump input implied volatility surface up, recalibrate local volatility surface and reprice. Also look at risk to deformations of the skew and convexity of the volatility surface. Are there any issues of numerical noise, especially for out of the money options?

## 3 Part 2: Deterministic Local Correlation

If we calibrate deterministic local volatility to two currency pairs that form two sides of a currency triangle eg. USDJPY and EURUSD and then correlate them together using a flat correlation, we will not accurately match the smile of the cross pair, in this case EURJPY. In order to match the cross smile, we need to introduce a local correlation - this is a relatively new area and there is not much literature, but it is discussed in (Austing 2014). The approach that is outlined there involves assuming that the local volatility locally satisfies the triangle rule. However, as pointed out in (Guyon 2013), this is just one of an infinite family of local correlation functions that could be consistent with the observed cross smile. It also suffers certain drawbacks - for example it may imply correlations greater than 1. Locally capping and flooring the correlation at $\pm 1$ introduces a bias which cannot be controlled. The positive-definiteness problem potentially gets worse when we add additional stochastic factors, for example a stochastic volatility factor, or increase the number of currencies.

Implementing calibration of the local correlation in the particle method would enable us to easily experiment with different correlation specifications and control the accuracy of calibration.

As before, we have a straightforward test of accuracy: valuation of vanilla options on the cross pair, and a benchmark approach for comparison: the local volatility triangle calibration.

# References


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