CS472 Module 10 Part A - What is computational complexity?

Athens State University

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Algorithm Analysis vs. Computational Complexity

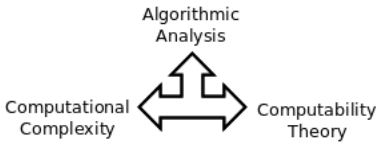
Computational complexity branch of computer science that focuses on classifying computational problems according to their inherent difficulty, and relating those classes to each other

- Study of all possible algorithms that solve a given problem
- Determine a lower bound on efficiency of all algorithms for a given problem
- Problem analysis as opposed to algorithm analysis

How to measure complexity?

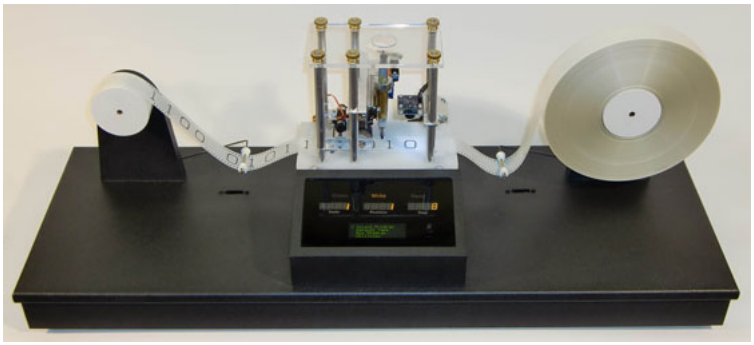
- A problem is difficult if its solution requires significant resources whatever the algorithm used

Algorithm Analysis vs. Computational Complexity



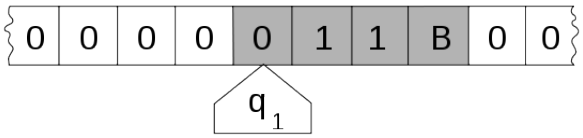
- **Algorithm analysis**: determine amount of resources required by a particular algorithm to solve problem
- **Complexity theory**: What algorithms for solving a problem can be found for a problem given a restricted set of resources?
- **Computability theory**: What can of problems can, in principle be solved using algorithms?

Some results from computability theory: Turing Machines



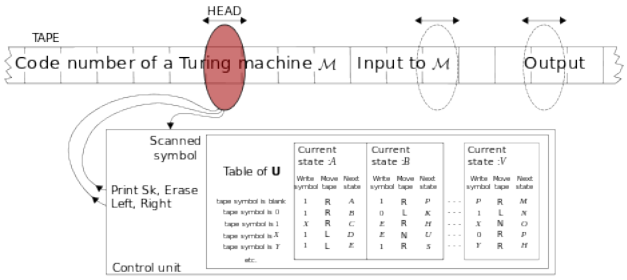
A **Turing Machine** is hypothetical device that manipulates symbols on strip of table according to a table of rules. A Turing machine can be adapted to simulate the logic of any computer algorithm.

Computability theory: Operation of a Turing Machine



There is a mathematical notation that can be used to describe the structure and behavior of a Turing machine. The details of this notation may be of interest but is not required for this discussion.

Computability theory: A Universal Turing Machine



A **Universal Turing Machine** (UTM) is a Turing machine that can simulate an arbitrary Turing machine running on any arbitrary input.

There are results in the theory of computer science that says a UTM can calculate any recursive function, decide any recursive language, and accept any recursively enumerable language. In particular, the Church-Turing thesis says the problems solvable by a UTM are exactly those problems solvable by an algorithm for any reasonable definition of the term.

Computability theory and Computational Complexity

- There will be some results from the theory of computation that we will take as assumptions in this discussion:
  - The *Church-Turing thesis* states that if a problem can be solved by an algorithm, then there exists a *Turing machine* that can solve the problem.
    - It can be shown that modern digital computers are, in fact, a form of a Turing machine.
  - Every model of computation identified to date can be converted to a Turing machine (or vice versa) without providing any extra computational power.

Some terms

Computational problem A (possibly infinite) collection of *instances* together with a *solution* for each instance

Problem instance A string composed of items from an alphabet. Typically, we work with *bitstrings* taken from the binary alphabet $0, 1^*$

- Objects other bitstrings must be suitably encoded; for example, integers can be represented in binary notation, graphs by encoding their adjacency matrix or adjacency list

Decision problem vs. function problem

Decision problem Any arbitrary yes-or-no question on an infinite set of inputs

- Example: "Given two numbers x and y, does x evenly divide y?"
- A decision problem that can be solved by an algorithm is called a *decidable* problem

Function problem A computational problem where a single output is expected for every input, but the output is more complex than that of a decision problem

- Example: The Traveling Salesman Problem

Function problems are just decision problems

- It is possible, perhaps with a representation change, to recast a function problem into an integer
- For example, the multiplication of two integers can be expressed as a set of triples (a, b, c) such that $a * b = c$. Deciding if a given triple is a member of this set corresponds to solving the problem of multiplying two integers

Measuring the size of an instance

- To reason about what it means to solve a problem using a given amount of time and space, we need:
  - A mathematical model of computation such as the deterministic Turing machine
  - Some measure of the number of steps such a machine will take in order for the machine to halt and output the answer to decision problem of interest.
- Other measures of complexity exist but in general we can operate with the asymptotic behavior of input size

Measuring the size of an instance

- Algorithm analysis uses the rate-of-growth functions to reason about the performance of algorithms
- For algorithm analysis, we look at upper bound on time complexity of algorithms
- For computational complexity, we look towards the lower bound on the amount of time an algorithm requires to solve an instance