UNIVERSITY OF MARYLAND

Geodesics For Face Recognition

Anne Jorstad   Advisor: David Jacobs

Mentor: Alain Trouvé, Centre de Mathématiques et de Leurs Applications, Ecole Normale Supérieure de Cachan, France

UMIACS University of Maryland Institute for Advanced Computer Studies

Thesis Problem

Develop a metric that assigns a similarity cost to faces in the presence of changes in expression and lighting based on dense correspondences.

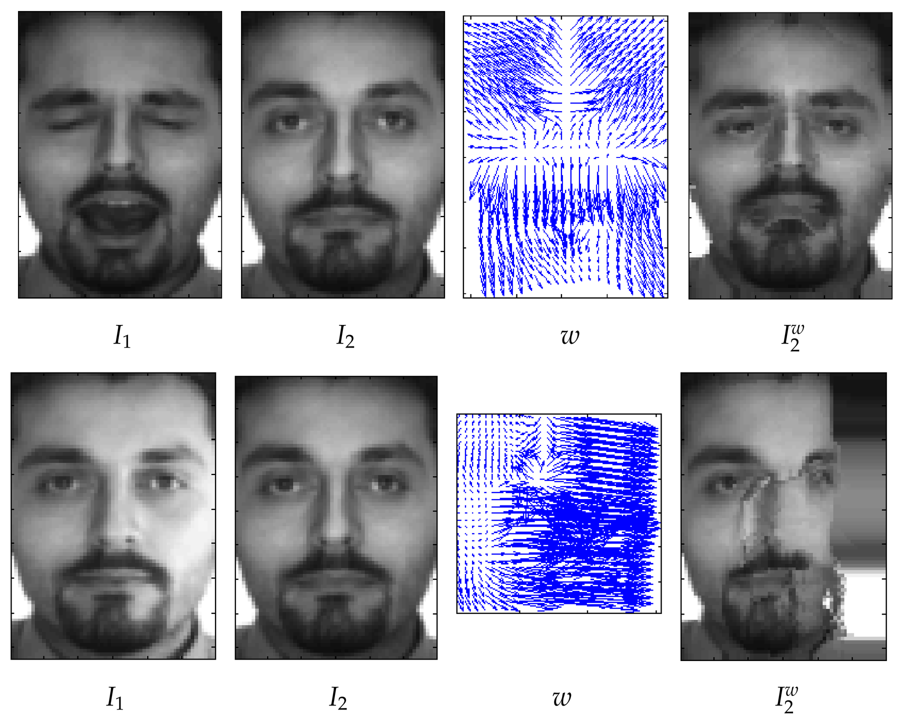
Stage 1: Use the metric in a new framework for face recognition.

Stage 2: Update correspondences through a series of morphings considering the metric on the manifold of faces.

Dense Correspondences for Image Deformation

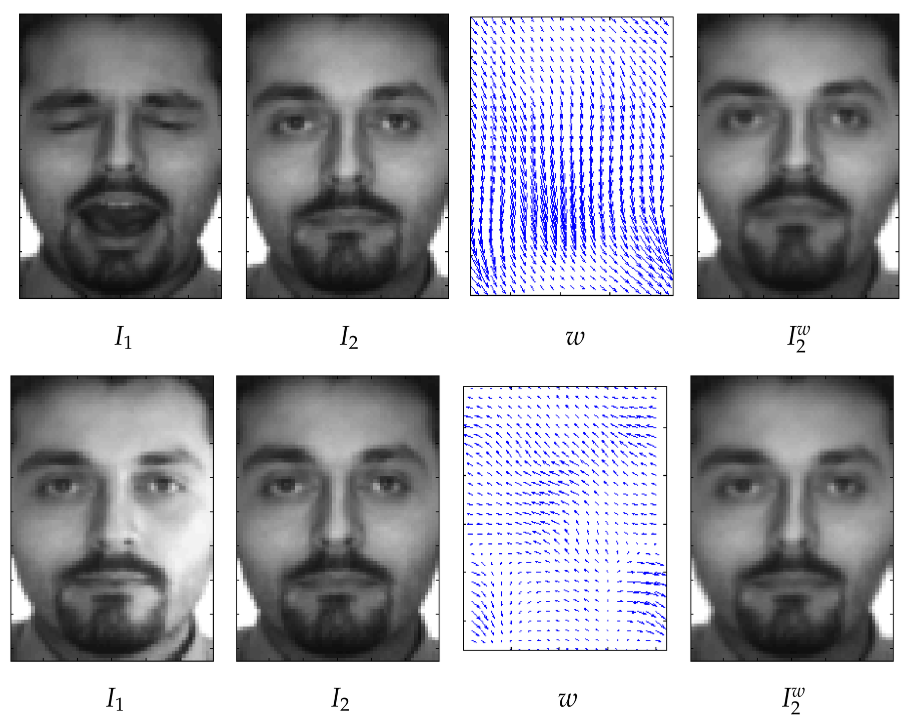
Goal: Compute the lowest cost deformation and intensity changes relating two face images, and use these for recognition.

Poor results are achieved when the traditional Black and Anandan flow is used:



w: the flow from I_1 to I_2
I_2^w: I_2 warped backwards along w to match I_1

Results from our proposed method are more stable:



Expression change: The top lip has been correctly matched while keeping the overall flow smooth.

Lighting change: In spite of significant change in lighting there has been no deformation, and the flow is small.

A Deformation and Lighting Insensitive Metric

Goal: Define a metric on face image flows that is insensitive to changes in expression and lighting.

$$E_{\text{DLI}}(w) = (1-\lambda)E_b(w) + \lambda E_r(w), \qquad \text{where } w \text{ is the flow from } I_1 \text{ to } I_2.$$

Photometric Term:

$$E_b(w) = \frac{1}{2}\sum_{ij}\frac{\|\nabla(I_2^w - I_1)\|^2}{\|\nabla I_1\|^2 + \epsilon^2} = \frac{1}{2}\sum_{ij}(E^x_{b_{ij}})^2 + (E^y_{b_{ij}})^2$$

- Invariant to multiplication by a scalar and addition by a constant.
- Insensitive to changes caused by the effects of lighting variation in 3D scenes.
  (ex: Moving a light can greatly change the gradient at the edge of a polyhedron, as the two faces are exposed to light differently.)

Regularization Term:

$$E_r(w) = \frac{1}{2}\langle K^{-1}w,\, w\rangle_G = \frac{1}{2}\sum_{ij}(E^x_{r_{ij}})^2 + (E^y_{r_{ij}})^2$$

- K : a symmetric positive definite matrix (eg a gaussian)
- G : a generalized inner product defined on $M \times N \times 2$ structures (dimensions of the flow)

Calculating Flows

Find the optimal flow between images:

- Minimize $E_{\text{DLI}}(w)$ with modified Gradient Descent.
- Sobolev Gradient: $\nabla_K E = K\nabla E$, smoother, superior convergence rates[1].
- Optimize over dual variable α:

$$w_n = K\alpha_n \qquad \text{(a convolution)}$$
$$\alpha_{n+1} = \alpha_n - \Delta t \cdot \nabla E(w_n)$$

Learning for Improved Results

Given pixel matching costs for known image pairs,

- Learn typical Gaussian distributions through 4D cost vectors $[E^x_{b_{ij}}\ E^y_{b_{ij}}\ E^x_{r_{ij}}\ E^y_{r_{ij}}]$ (assume pixel independence).
- Train separate models for same-person and different-person image pairs.
- The final similarity measure: $S(I_1, I_2) = \dfrac{P_{\text{same}}(\vec{E}(w))}{P_{\text{diff}}(\vec{E}(w))}$

Flow Experiments

Identity of unknown face determined by gallery image resulting in lowest matching cost.

Images from the AR Face Database[2] with variations in expression and lighting.

Variation	Accuracy	Variation	Accuracy
Smile	97.6%	Left light	98.8%
Frown	91.6%	Right light	99.6%
Scream	79.6%	Both lights	98.4%

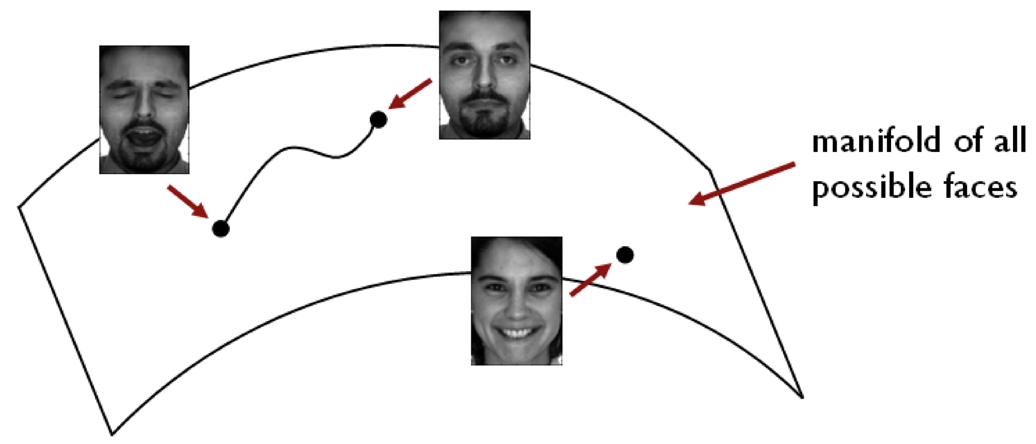
Cost Function	Expression	Lighting	Overall
Direct	82.0%	96.0%	89.0%
After Learning	89.6%	98.9%	94.3%
Smile gallery			
After Learning	86.8%	91.2%	89.0%
Borders removed			
After Learning	85.1%	96.4%	90.7%

Geodesics on the Face Manifold

Goal: Slowly introduce changes in face images over several steps for increased robustness.

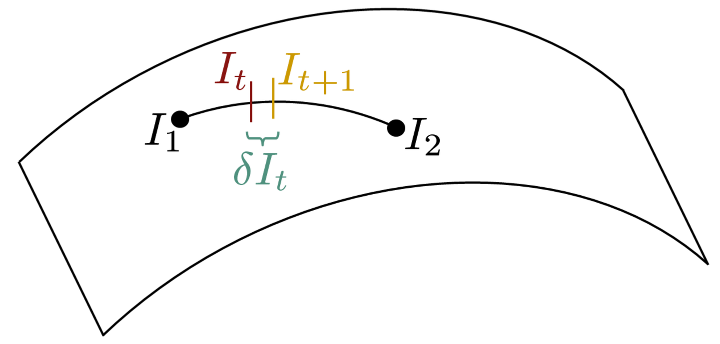
Consider face images as points on a high-dimensional manifold:

- Curves on manifold describe face transformations through time.
- Define similarity as length of geodesic (shortest path) connecting two faces.



Require a metric to give local structure to the manifold:

$$E_{\text{DLI}} = \sum_{t=1}^{T}\sum_{i=1,j=1}^{M,N}(1-\lambda)E_b + \lambda E_r$$



- Geodesic parameterized by time t.
- $\delta I_t = I_{t+1} - I_t$ replaces $I_2^w - I_1$ in E_b.

Geodesic Optimization

Optimize over $M \times N \times T$ variables: too big!

Gradient Descent stepsize choice: line search schemes regularly fail long before a minimum is reached.

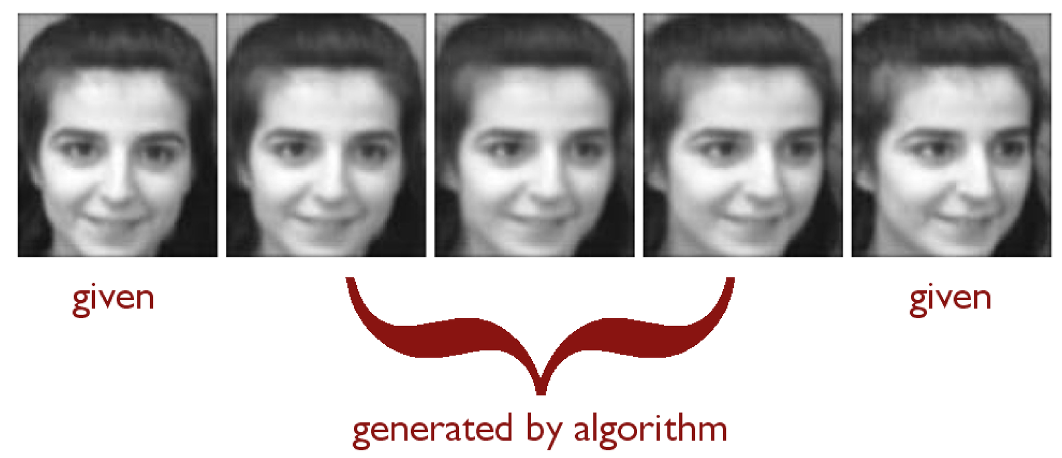
1st order approximation often poor, but 2nd order methods too computationally intensive; considering Preconditioned Conjugate Gradient.

Use lower dimensional representation of image path: Splines or wavelets.

- Reduces redundancy and numerical error.
- Gradient calculation more complicated.

Preliminary Results

From a pose change study[3] (note: pose has 3 degrees of freedom, expression change has infinite degrees of freedom):



Calculating geodesics on the manifold of faces should provide a robust method for face recognition across changes in expression and lighting.

[1] J. W. Neuberger. Sobolev Gradients and Differential Equations, 2nd Edition. Springer, 2010.

[2] A. Martinez and R. Benavente. The AR Face Database. CVC Technical Report #24, 1998.

[3] A. Trouvé and L. Younes. Metamorphoses Through Lie Group Action. Foundations of Computational Mathematics, 2005.