

ANALYSIS OF THE BRAIN SYNCHRONIZATION

Problem. How does the brain synchronization algorithm work?

Let $X_{T \times V}$ and $Y_{T \times V}$ be the data sets. In general $V \gg T$. Let the rank of X is r .

Let us assume that both X and Y have identical spatial correlation structure. Therefore, $X^T X = Y^T Y$. Let us consider SVD decomposition of X and Y .

$X = U_X \Sigma_X V_X^T$ and $Y = U_Y \Sigma_Y V_Y^T$. We can choose U , Σ and V matrices to be real.

$$X^T X = Y^T Y = V_X \Sigma_X^2 V_X^T = V_Y \Sigma_Y^2 V_Y^T.$$

Since this decomposition is unique if there are no repeated singular values, we can say $\Sigma_X = \Sigma_Y$ and $V_X = V_Y$. Even in case of repeated singular values we can choose $V_X = V_Y$. So removing subscripts, we have

$$X = U_X \Sigma V^T \text{ and } Y = U_Y \Sigma V^T.$$

What is the cross covariance matrix?

$$XY^T = U_X \Sigma V^T V \Sigma U_Y^T = U_X \Sigma^2 U_Y^T.$$

The optimal rotation given by BrainSync is $O = U_X U_Y^T$.

$$X = OY = U_X U_Y^T U_Y \Sigma V = U_X \Sigma V.$$

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We have proven: X and Y have same spatial correlation $\implies \Sigma_X = \Sigma_Y$ and $V_X = V_Y$. The proof in other direction is easy by substitution. Therefore we have proven the following claim.

Claim. X and Y have same spatial correlation $\iff \Sigma_X = \Sigma_Y$ and $V_X = V_Y$. By orthogonal transformation, we are changing temporal basis so that they match with the basis of target brain under the exact identity $XX^T = YY^T$.

Remark. Similar analysis can be done by ICA. All the information about XX^T is in Σ_X and V_X . So for brain network analysis in the resting state, we should just keep those two. For the task data spatial basis and singular values represent brain networks. Does the temporal basis represent driving processes such as thoughts or motor actions? It would be interesting to check correlation of spatial basis and block design process in task data. The basis that has highest correlation can be kept and rest could be nulled out. This will give us brain data that is driven by the task and remove the one that is driven by physiological processes.