CSE409

18/11/2023

Computer Graphics

Rasterization → zoom করলে ফেটে যায়, no lighting, > 50% হলে color that pixel, otherwise not

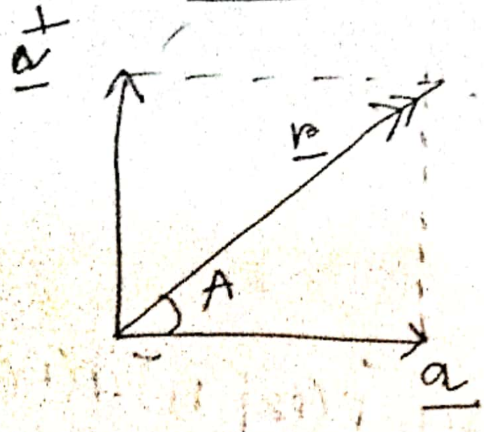
Ray casting

↳ প্রতিটি pixel-এর জন্য একটি light ray from our perspective, যদি object-এর hit করে → color।
Reflected ray-গুলো যদি track করা হয়, একে বলে Ray Tracing.

Book → Schaum**

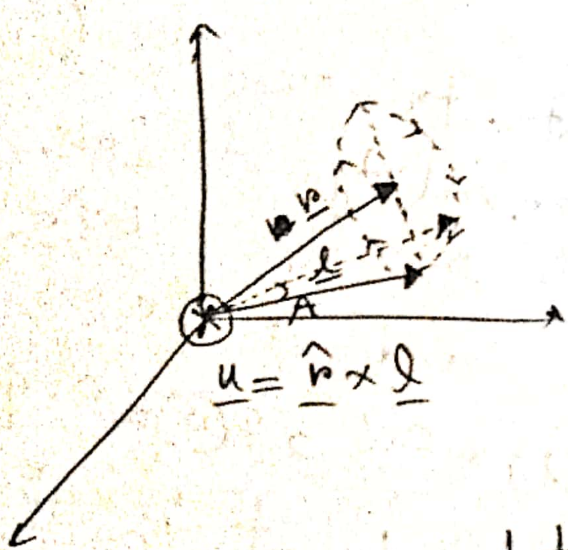
CSE409 — 21/11/2013

Vector Tools for Computer Graphics



$$\underline{r} = (\cos A).\underline{a} + (\sin A).\underline{a}^{\perp}$$

Rotation in 3d:



new $\underline{l}$

$$\underline{z} = \underline{l}\cos A + \underline{u}\sin A$$
$$= \underline{l}\cos A + (\hat{\underline{r}} \times \underline{l})\sin A$$

$\hat{\underline{r}} \rightarrow$ যার সাপেক্ষে rotate

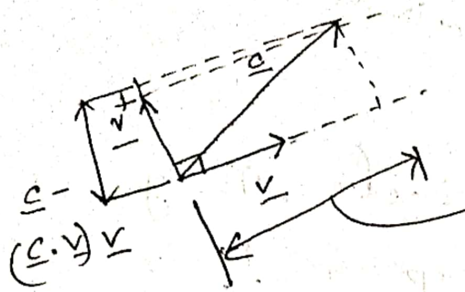
$\underline{l} \rightarrow$ যাকে rotate

$A \rightarrow$ angle of rotation

$\underline{l}$, rotates around $\underline{r}$ in a circular plane.

Orthogonal Projection:

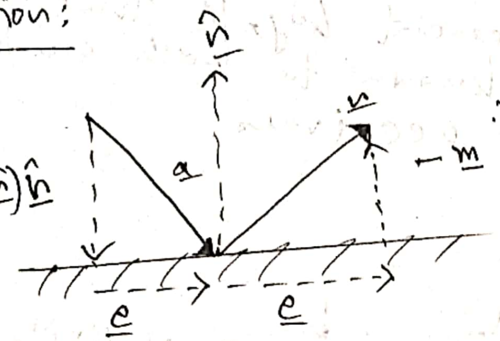
v is unit vector



$(\underline{c}\cdot\underline{v})\,\underline{v}$

$\underline{c} - (\underline{c}\cdot\underline{v})\,\underline{v}$

Reflection:



$\underline{m} = (\underline{a}\cdot\hat{\underline{n}})\hat{\underline{n}}$

$\underline{e} = \underline{a} - (\underline{a}\cdot\hat{\underline{n}})\hat{\underline{n}}$

$\overline{r} = \underline{e} - \underline{m}$

$\therefore \boxed{\overline{r} = \underline{a} - 2(\underline{a}\cdot\hat{\underline{n}})\hat{\underline{n}}}$

22/11/2023

CSE 305

\# Modern cryptography → randomness.

\# Not appropriate randomness in substitute cipher. Cryptanalysis → Most freq letter in plaintext corresponds to most " " ciphertext.

Sym Key : One-time pad

Key → n bit string. n bit দিয়ে 2^n টি distinguished number generate করা যায়। random ভাবে Key select করা হয়।

MAC এবং Hashing-এ used, not in cryptography since its not IND-CPA secure.

Next class:

MAC-then-encrypt → Decrypt করার আগে বোঝা যায় না if tempered. Costly.

vs.

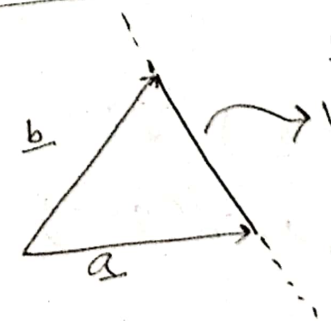
Encrypt-then-MAC it → More robust to mistakes.

28/11/2023

CSE309

Linear combination of vectors:

(1)



$\underline{v} = c_0\underline{a} + c_1\underline{b}$, $\boxed{c_0 + c_1 = 1}$ → Affine combination

v always lies on this line

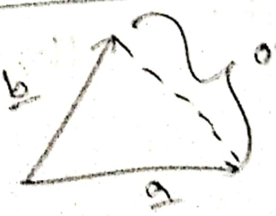
Proof. $\underline{v} = c_0\underline{a} + (1-c_0)\underline{b}$

$= c_0(\underline{a} - \underline{b}) + \underline{b}$

$= \underline{b} + c_0(\underline{a} - \underline{b})$;

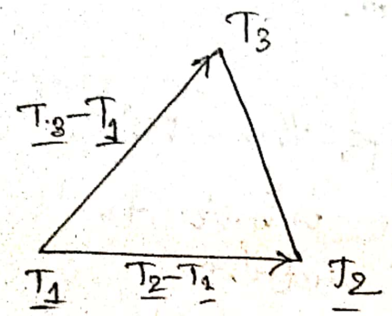
this is the equation of a straight line where $\underline{a}$ and $\underline{b}$ are given points.

(2) Convex combination: $c_0 + c_1 = 1$, $c_0 \geq 0$, $c_1 \geq 0$



only here lies v

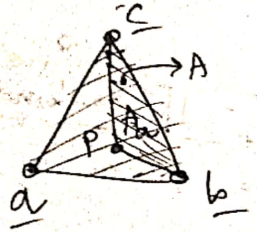
Ray-Triangle Intersection



$\alpha(\underline{T_2}-\underline{T_1})+\beta(\underline{T_3}-\underline{T_1})$; these points lie on the triangle for α, β.

Barycentric definitions

$P(\alpha,\beta,\gamma) = \alpha\,\underline{a} + \beta\,\underline{b} + \gamma\,\underline{c}$; $\boxed{\alpha+\beta+\gamma=1}$



only include points inside $\Leftarrow$ $\boxed{0<\alpha<1,\ 0<\beta<1,\ 0<\gamma<1}$

$$P(\beta,\gamma) = (1-\beta-\gamma)\,\underline{a} + \beta\,\underline{b} + \gamma\,\underline{c}$$
$$= \underline{a} + \beta(\underline{b}-\underline{a}) + \gamma(\underline{c}-\underline{a})$$

$$\boxed{\alpha = \frac{A_a}{A},\ \beta = \frac{A_b}{A},\ \gamma = \frac{A_c}{A}}$$

For intersection,

$$P(t) = P(\beta,\gamma)$$
$$\Rightarrow \underline{P_0} + t\,\underline{v} = \underline{a} + \beta(\underline{b}-\underline{a}) + \gamma(\underline{c}-\underline{a})$$

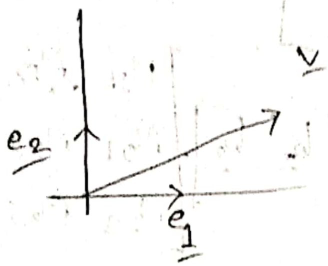
$\hookrightarrow$ 3 unknown (α,β,t); 3 eqn (x,y,z)

Intersect if $\beta + \gamma \leq 1$ and $\beta > 0$ and $\gamma > 0$

Modeling Transformations

5) Vector space: 2D or 3D, sum of all vectors.

Basis:



$\underline{v} = c_1 \underline{e_1} + c_2 \underline{e_2}$

infinite # basis for a vector space.

$$\underline{v} = c_0 \underline{a} + c_1 \underline{b} = [\underline{a} \;\; \underline{b}] \underbrace{\begin{bmatrix} c_0 \\ c_1 \end{bmatrix}}_{\text{coordinate}}.$$

Linear Translation

(1) $\mathcal{L}(\underline{u} + \underline{v}) = \mathcal{L}(\underline{u}) + \mathcal{L}(\underline{v})$

(2) $\mathcal{L}(a\underline{v}) = a . \mathcal{L}(\underline{v})$

$$\mathcal{L}(\underline{v}) = \mathcal{L}\left(\sum_i c_i \underline{b_i}\right) = \sum_i c_i . \mathcal{L}(\underline{b_i})$$

$\Downarrow$

Transformation of a vector is just only transformation of basis

$L(\underline{b_i})$ is also a new vector that exists in the vector space defined by the $\underline{b_i}$ vectors..

<u>Thus, for 3D,</u>

$$L(\underline{b_1}) = \begin{bmatrix} \underline{b_1} & \underline{b_2} & \underline{b_3} \end{bmatrix} \begin{bmatrix} M_{11} \\ M_{21} \\ M_{31} \end{bmatrix}$$

$$\therefore \begin{bmatrix} L(\underline{b_1}) & L(\underline{b_2}) & L(\underline{b_3}) \end{bmatrix} = \begin{bmatrix} \underline{b_1} & \underline{b_2} & \underline{b_3} \end{bmatrix} \cdot \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

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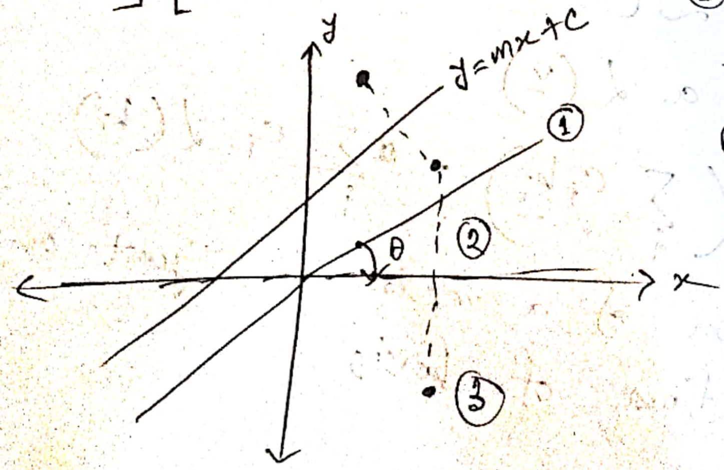
Composition of Transformation 02/12/2023

$$\underset{③}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{②}{\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{①}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}}$$



$\theta = \tan^{-1} m$

Withdrawal $\Rightarrow$ "Inner flow" থেকে withdraw করা হচ্ছে

Injection $\Rightarrow$ " -তে যুক্ত হচ্ছে!

CSE409 05/12/2023

Av: aligning vector $\bar{v}$ with $\hat{k}$

$$A_{v,k} = R_{-\phi,j} * R_{\theta,i}$$

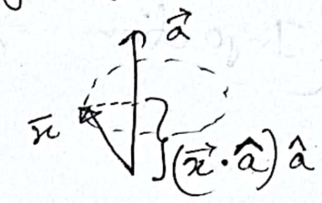
$$= \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\lambda}{|\vec{v}|} & \frac{-ab}{\lambda|\vec{v}|} & \frac{-ac}{\lambda|\vec{v}|} & 0 \\ 0 & \frac{c}{\lambda} & -\frac{b}{\lambda} & 0 \\ \frac{a}{|\vec{v}|} & \frac{b}{|\vec{v}|} & \frac{c}{|\vec{v}|} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A^{-1}_{v,\hat{k}} :=$ align $\hat{k}$ with $\bar{v}$ $= A^{T}_{\bar{v},\hat{k}}$

\# $A^{-1}_{\bar{v},\hat{k}} \cdot R_{\theta,\hat{k}} \cdot A_{\bar{a},\hat{k}}$ $\Rightarrow$ rotate $\bar{v}$ with respect to $\bar{a}$.

Rotate $\vec{x}$ with respect to $\vec{a}$ by angle θ:



$$\vec{x} = \vec{x}_{\parallel} + \vec{x}_{\perp} \quad \text{——— (i)}$$

$$\vec{w} = \vec{a} \times \vec{x}$$
$$= \vec{a} \times (\vec{x}_{\parallel} + \vec{x}_{\perp})$$
$$= \cancel{(\vec{a} \times \vec{x}_{\parallel})}^{\,0} + (\vec{a} \times \vec{x}_{\perp}) \quad [\vec{x}_{\parallel} \text{ is parallel with } \vec{a}]$$
$$= \vec{a} \times \vec{x}_{\perp} \quad \text{——— (ii)}$$

$$R(\vec{x}) = R(\vec{x}_{\parallel}) + R(\vec{x}_{\perp}) \quad [\text{Linear transformation}]$$
$$= \vec{x}_{\parallel} + R(\vec{x}_{\perp}) \quad [\text{since } \vec{x}_{\parallel} \parallel \vec{a}, \text{ rotating it w.r.t. } \vec{a} \text{ doesn't change it}]$$

$$= \vec{x}_{\parallel} + \vec{x}_{\perp} \cos\theta + \vec{w} \sin\theta$$
$$= (\vec{x} \cdot \hat{a})\hat{a} + \left\{\vec{x} - (\vec{x} \cdot \hat{a})\hat{a}\right\} \cos\theta + \vec{w} \sin\theta$$
$$= (\vec{x} \cdot a)\hat{a} + \vec{x}\cos\theta - (\vec{x} \cdot \hat{a})\hat{a}\cos\theta + (\vec{a} \times \vec{x}_{\perp}) \sin\theta$$

$$\boxed{R(\vec{x}) = \vec{x}\cos\theta + (1 - \cos\theta)(\vec{x} \cdot \hat{a})\hat{a} + (\hat{a} \times \vec{x}_{\perp}) \sin\theta}$$

#

$$R(k,\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cos\theta + (1-\cos\theta) \begin{bmatrix} a_x^2 & a_x a_y & a_x a_z \\ a_x a_y & a_y^2 & a_z a_y \\ a_x a_z & a_y a_z & a_z^2 \end{bmatrix} + \sin\theta \cdot \begin{bmatrix} 0 & -a_z & a_y \\ -a_z & 0 & a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$(\vec{x}\cdot\vec{a})\,\vec{a} = \left(\begin{bmatrix} a_x & a_y & a_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) \begin{bmatrix} a_x & a_y & a_z \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \cdot \left(\begin{bmatrix} a_x & a_y & a_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right)}$$

$$= \cancel{\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}} \cdot \begin{bmatrix} a_x \cdot x + a_y \cdot y + a_z \cdot z \end{bmatrix} \begin{bmatrix} a_x & a_y & a_z \end{bmatrix}$$

$$= \begin{bmatrix} a_x^2 x + a_x a_y y + a_x a_z z & a_x a_y x + a_y^2 y + a_z a_y z & a_x a_z x + a_y a_z y + a_z^2 z \end{bmatrix}$$

$$= \begin{bmatrix} a_x^2 & a_x a_y & a_x a_z \\ a_x a_y & a_y^2 & a_z a_y \\ a_x a_z & a_y a_z & a_z^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\vec{a} \times \vec{x} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ x & y & z \end{bmatrix} = \begin{bmatrix} -y a_z + z a_y \\ -z a_x + x a_z \\ -x a_y + y a_x \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -a_z & a_y \\ -a_z & 0 & a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$