Compiler Optimizations with

Datalog and Equality Saturation

Anjali Pal

```
def foo(x):
    return x + 1
print foo(1)
```

```
def foo(x):
  return x + 1
print foo(1)
```

print 2

```
if x < 2:
 y = x + x
 r = y * 2
else:
 r = x + x
return r
```

```
if x < 2:
 y = x + x
 r = y * 2
else:
  r = x + x
return r
```

```
if x = 4:
y = 5 * x
elif x = 5:
 y = 4 * x
else:
 y = 20
return y
```

```
if x = 4:
y = 5 * x
elif x = 5:
y = 4 * x
else:
 y = 20
return y
```

return 20

```
if x > 0:
else:
z = y \geqslant 0
return z
```

```
if x > 0:
y = x
else:
z = y \geqslant 0
return z
```

return true

```
if x > 0:
else:
if y < 10:
 z = 5
else:
z = -5
return z
```

```
if x > 0:
y = 2
else:
 y = 3
if y < 10:
z = 5
else:
z = -5
return z
```

return 5

egglog: a fixpoint reasoning system that combines *Datalog* and *Equality Saturation*

Equality Saturation

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Equality Saturation

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Patalog EqSat egglog eggcc

Datalog is a <u>declarative programming language</u> consisting of **facts** and **rules**

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Datalog Fact

Parent("Alice", "Bob")

log EqSat egglog eggcc

Datalog Fact

Parent("Alice", "Bob")

Alice is a parent of Bob

```
Ancestor(X, Y) :-
Parent(X, Y)
```

If X is a parent of Y, then X is an ancestor of Y

og EgSat egglog eggcc

```
Ancestor(X, Z) :-
Parent(X, Y),
Ancestor (Y, Z)
```

```
Ancestor(X, Z):-
Parent(X, Y),
Ancestor (Y, Z)
```

If X is a parent of Y and Y is an ancestor of Z, then X is an ancestor of Z

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```
Program
Ancestor(X, Y):- Parent(X, Y)
Ancestor(X, Z):-
   Parent(X, Y),
   Ancestor(Y, Z)
Parent("Alice", "Bob")
Parent("Bob, "Charlie")
```

Program Ancestor(X, Y):- Parent(X, Y) Ancestor(X, Z):-Parent(X, Y),Ancestor(Y, Z) Parent("Alice", "Bob") Parent("Bob, "Charlie")

What does the program mean?

How do we evaluate it?

Program

```
Ancestor(X, Y):- Parent(X, Y)
Ancestor(X, Z):-
   Parent(X, Y),
   Ancestor(Y, Z)
Parent("Alice", "Bob")
Parent("Bob, "Charlie")
```

Parent

Ancestor

Program Ancestor(X, Y):- Parent(X, Y) Ancestor(X, Z):-Parent(X, Y), Ancestor(Y, Z) Parent("Alice", "Bob") Parent("Bob, "Charlie")

Parent	
Alice	Bob

Ancestor

Program Ancestor(X, Y):- Parent(X, Y) Ancestor(X, Z):-Parent(X, Y),Ancestor(Y, Z) Parent("Alice", "Bob") Parent("Bob, "Charlie")

Parent	
Alice	Bob
Bob	Charlie

Ancestor

```
Program
Ancestor(X, Y):- Parent(X, Y)
Ancestor(X, Z):-
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Parent("Alice", "Bob")
Parent("Bob, "Charlie")
```

Parent	
Alice	Bob
Bob	Charlie

Ancestor	
Alice	Bob

Program Ancestor(X, Y):- Parent(X, Y) Ancestor(X, Z):-Parent(X, Y), Ancestor(Y, Z) Parent("Alice", "Bob") Parent("Bob, "Charlie")

Parent	
Alice	Bob
Bob	Charlie

Ancestor	
Alice	Bob
Bob	Charlie

```
Program
Ancestor(X, Y):- Parent(X, Y)
Ancestor(X, Z):-
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```

Parent	
Alice	Bob
Bob	Charlie

Ancestor	
Alice	Bob
Bob	Charlie
Alice	Charlie

Program Ancestor(X, Y):- Parent(X, Y) Ancestor(X, Z):-Parent(X, Y), Ancestor(Y, Z) Parent("Alice", "Bob") Parent("Bob, "Charlie")

Parent	
Alice	Bob
Roh	Charlie
Apply rules until there are no more facts to add "fixpoint"	
VETOC	מטט
Bob	Charlie
Alice	Charlie

Datalog: Summary

- Program consists of facts and rules
- Rules can match on complex queries relating multiple facts
- Evaluate program by iteratively applying rules until fixpoint

Equality Saturation

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$$(a * 2) / 2 \implies a$$

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$$(a * 2) / 2 \implies a$$

Rewrite Rules

$$(x * y) / z \Longrightarrow x * (y / z)$$
 $x * 2 \Longrightarrow x << 1$
 $x / x \Longrightarrow 1$ $x * y \Longrightarrow y * x$
 $x * 1 \Longrightarrow x$ $x * y \Longrightarrow x * 1$

(a * 2) / 2
$$\implies$$
 a



$$(a * 2) / 2 \Longrightarrow a * (2 / 2) \Longrightarrow a * 1 \Longrightarrow a$$

Rewrite Rules

(a * 2) / 2
$$\implies$$
 a



$$(a * 2) / 2 \Longrightarrow (a << 1) / 2$$

Rewrite Rules

$$(x * y) / z \Longrightarrow x * (y / z)$$
 $x * 2 \Longrightarrow x << 1$
 $x / x \Longrightarrow 1$ $x * y \Longrightarrow y * x$
 $x * 1 \Longrightarrow x$ $x \Longrightarrow x * 1$

(a * 2) / 2 \implies a

$$(a * 2) / 2 \Longrightarrow (2 * a) / 2 \Longrightarrow (a * 2) / 2 \Longrightarrow \dots$$

Rewrite Rules

$$(x * y) / z \Longrightarrow x * (y / z)$$
 $x * 2 \Longrightarrow x \ll 1$
 $x / x \Longrightarrow 1$ $x * y \Longrightarrow y * x$
 $x * 1 \Longrightarrow x$ $x \Longrightarrow x * 1$

$(a * 2) / 2 \implies a$



$$a \Longrightarrow a * 1 \Longrightarrow (a * 1) * 1 \Longrightarrow \dots$$

Rewrite Rules

$$(x * y) / z \Longrightarrow x * (y / z)$$
 $x * 2 \Longrightarrow x \ll 1$
 $x / x \Longrightarrow 1$ $x * y \Longrightarrow y * x$
 $x * 1 \Longrightarrow x$ $x \Longrightarrow x * 1$

$$(a * 2) / 2 \implies a$$

USEFUL

$$(x * y) / z \Longrightarrow x * (y / z)$$

 $x / x \Longrightarrow 1$
 $x * 1 \Longrightarrow x$

NOT SO USEFUL

$$x * 2 \Longrightarrow x << 1$$

$$x * y \Longrightarrow y * x$$

$$x \Longrightarrow x * 1$$

(a * 2) / 2
$$\implies$$
 a

But <u>critical</u> for other inputs!

USEFUL

$$(x * y) / z \Longrightarrow x * (y / z)$$

 $x / x \Longrightarrow 1$
 $x * 1 \Longrightarrow x$

NOT SO USEFUL

$$x * 2 \Longrightarrow x << 1$$

$$x * y \Longrightarrow y * x$$

$$x \Longrightarrow x * 1$$

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$(a * 2) / 2 \implies a$

Which rewrite? When?

USEFUL

$$(x * y) / z \Longrightarrow x * (y / z)$$

 $x / x \Longrightarrow 1$
 $x * 1 \Longrightarrow x$

NOT SO USEFUL

$$x * 2 \Longrightarrow x << 1$$

$$x * y \Longrightarrow y * x$$

$$x \Longrightarrow x * 1$$

$$(a * 2) / 2 \implies a$$

All of them at once!

USEFUL

$$(x * y) / z \Longrightarrow x * (y / z)$$

 $x / x \Longrightarrow 1$
 $x * 1 \Longrightarrow x$

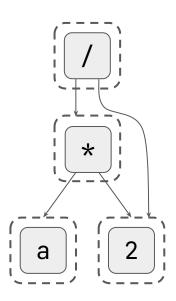
NOT SO USEFUL

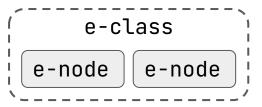
$$x * 2 \Longrightarrow x \ll 1$$

$$x * y \Longrightarrow y * x$$

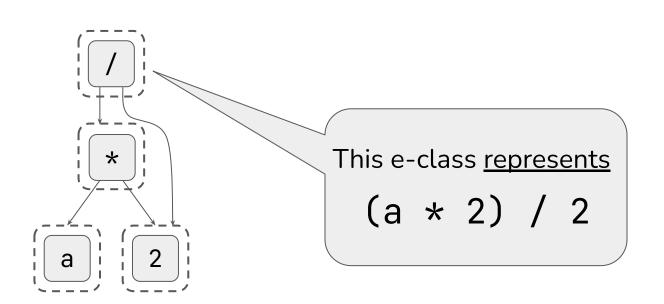
$$x \Longrightarrow x * 1$$

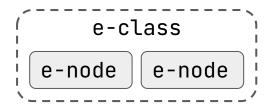
Equivalence Graphs (e-graphs)



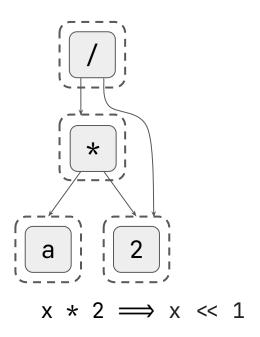


Equivalence Graphs (e-graphs)

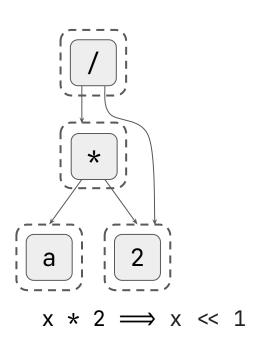




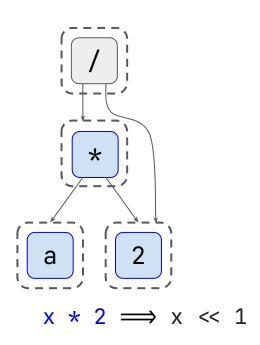
eggcc



eggcc



Find a term that looks like the left, Add a term that looks like the right, And mark them equivalent

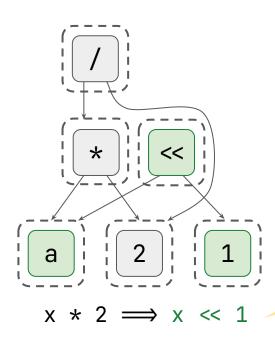


Find a term that looks like the left, Add a term that looks like the right, And mark them equivalent

50

eggcc

eggcc

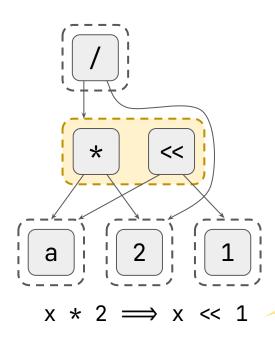


Find a term that looks like the left,

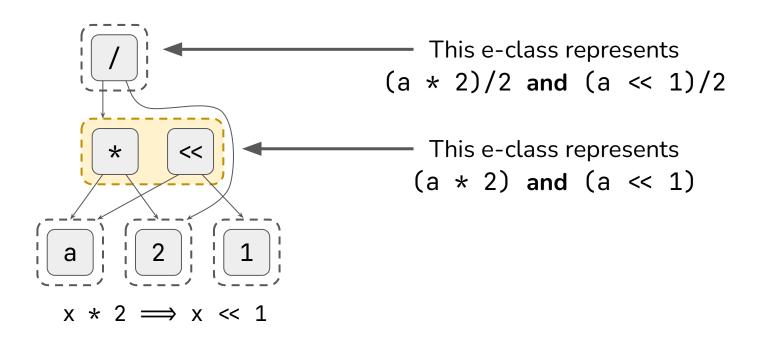
Add a term that looks like the right,

And mark them equivalent

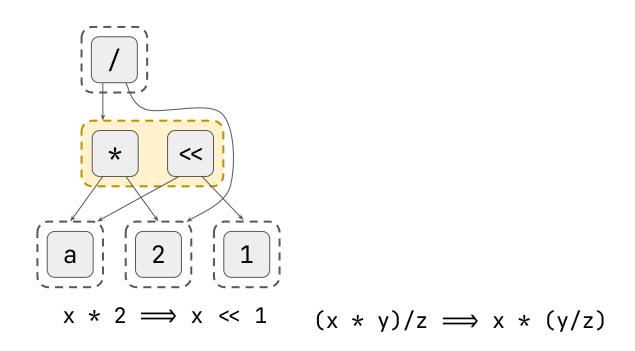
eggcc

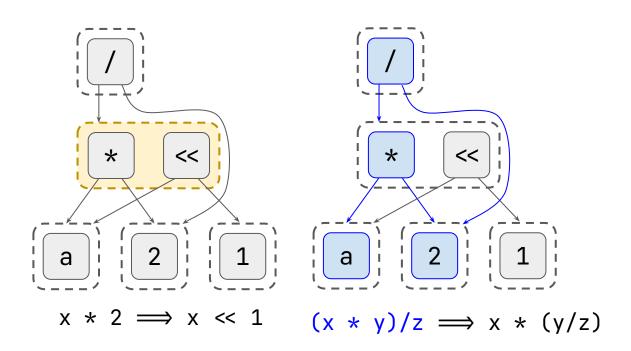


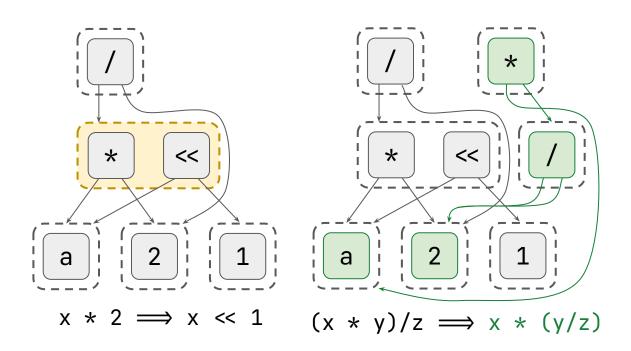
Find a term that looks like the left, Add a term that looks like the right, And mark them equivalent

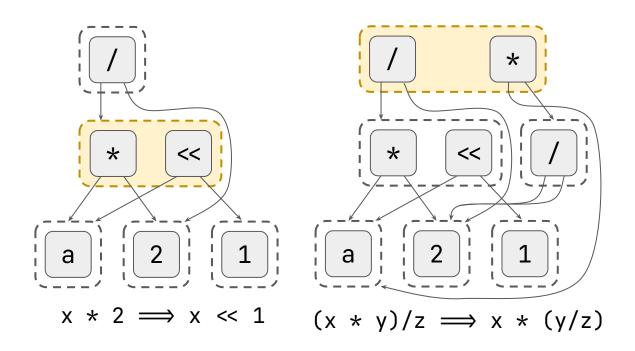


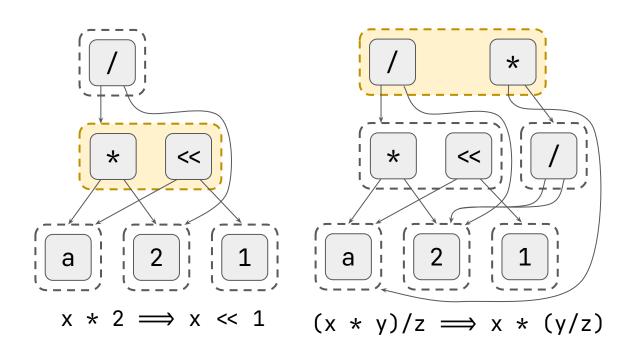
eggcc



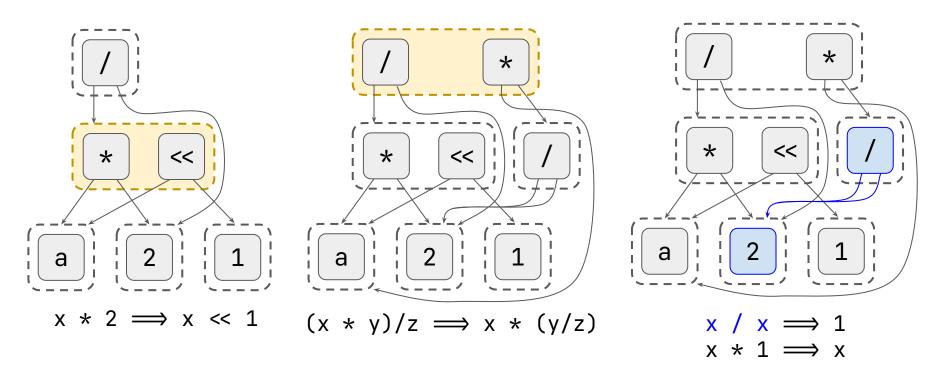


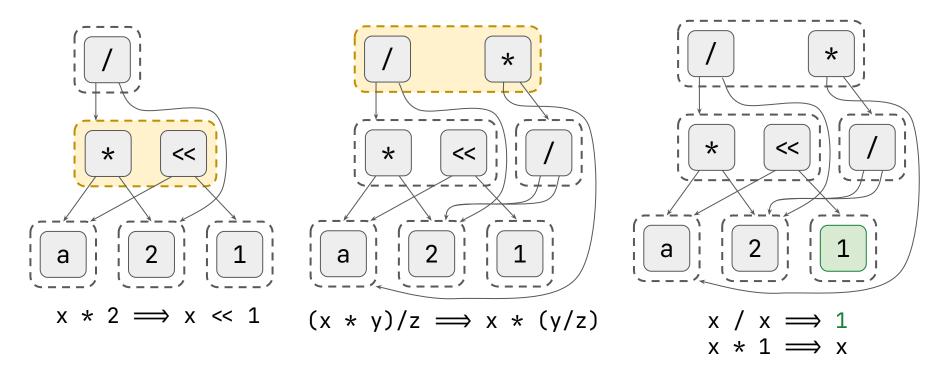


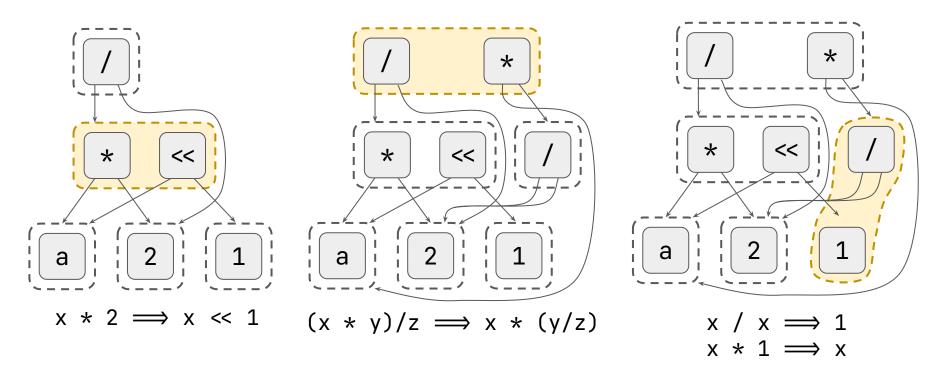


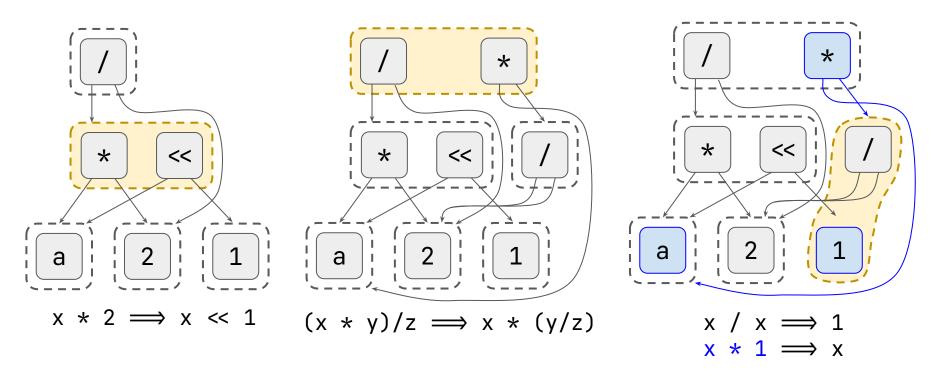


$$\begin{array}{c} x / x \Longrightarrow 1 \\ x * 1 \Longrightarrow x \end{array}$$



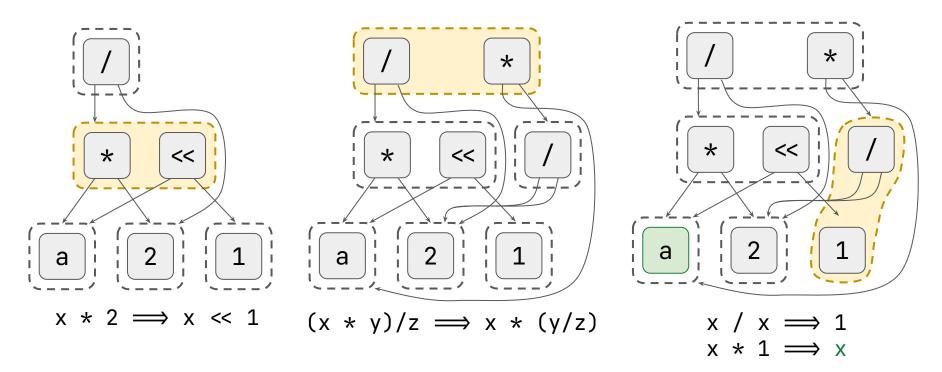


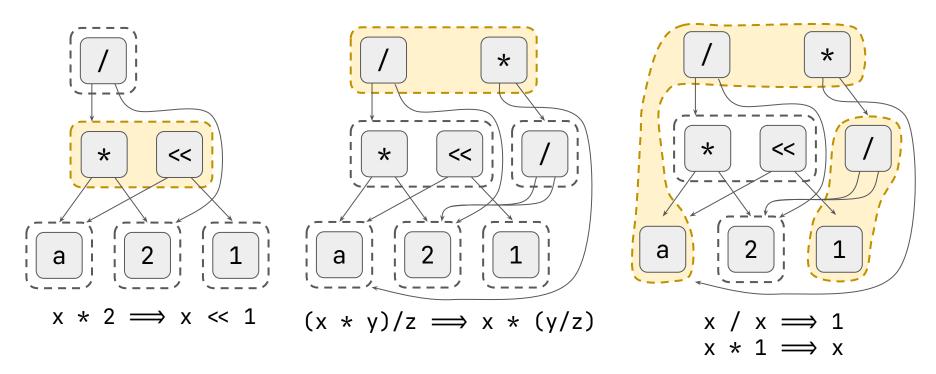


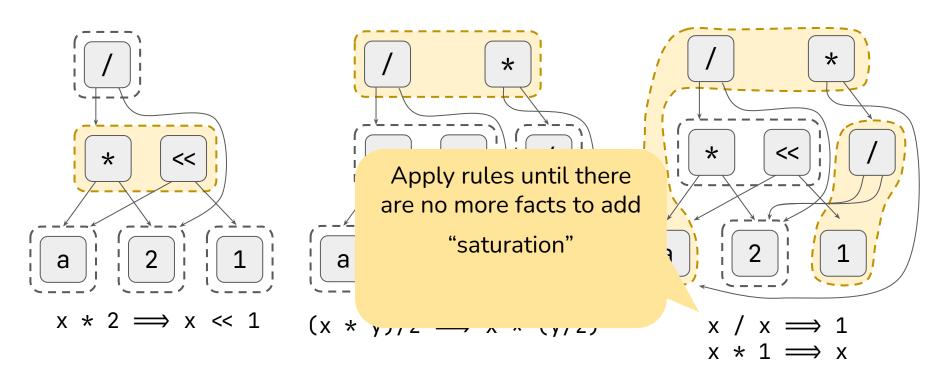


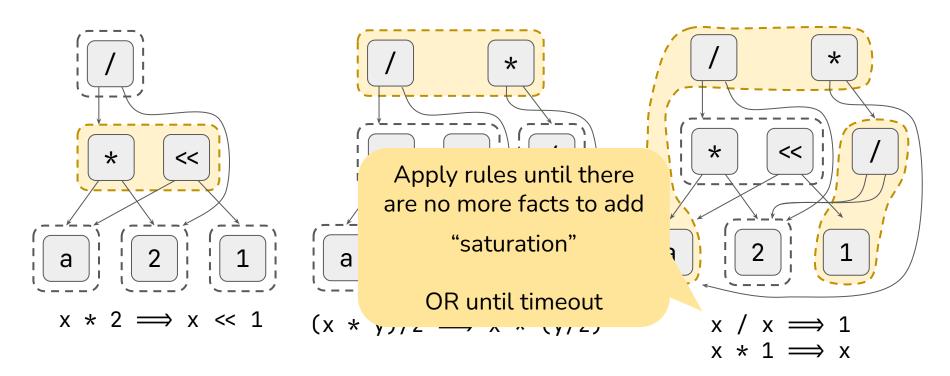
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eggcc



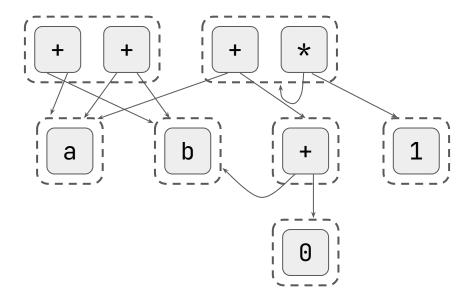


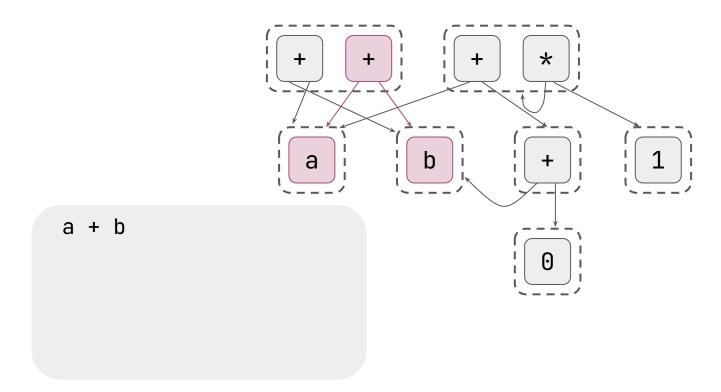




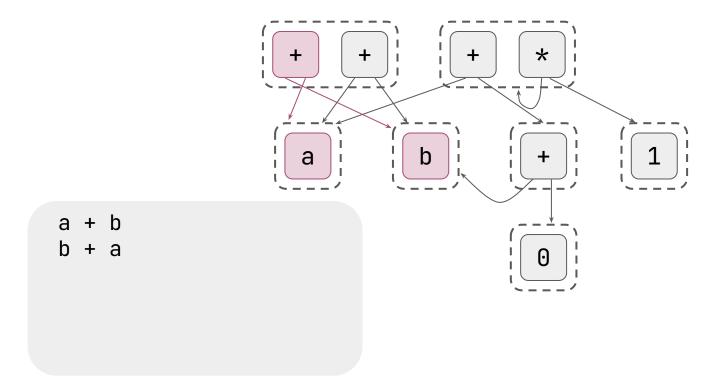
Congruence

$$a = b \text{ implies } f(a) = f(b)$$



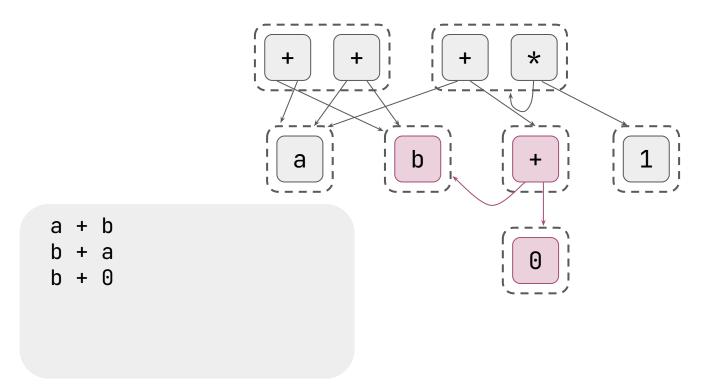


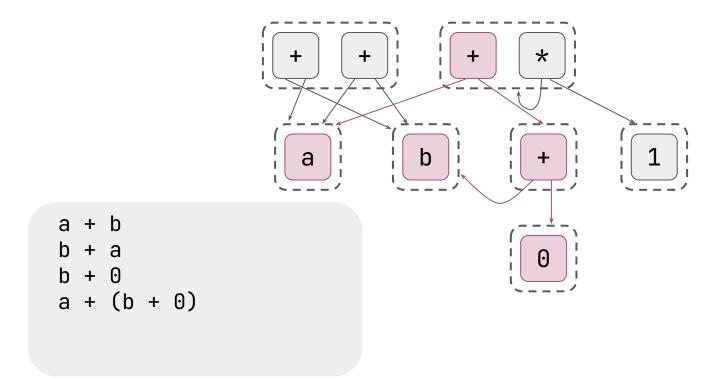
eggcc



70

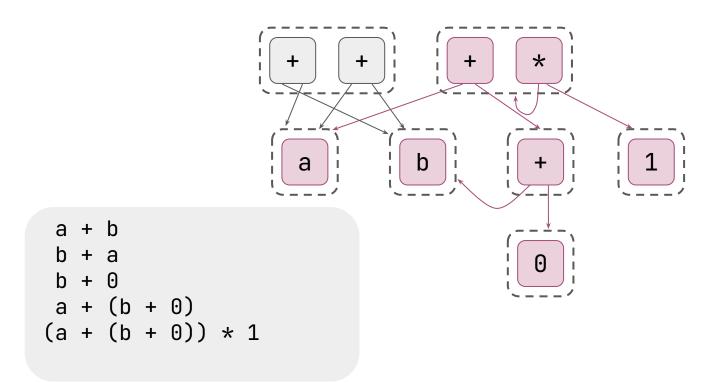
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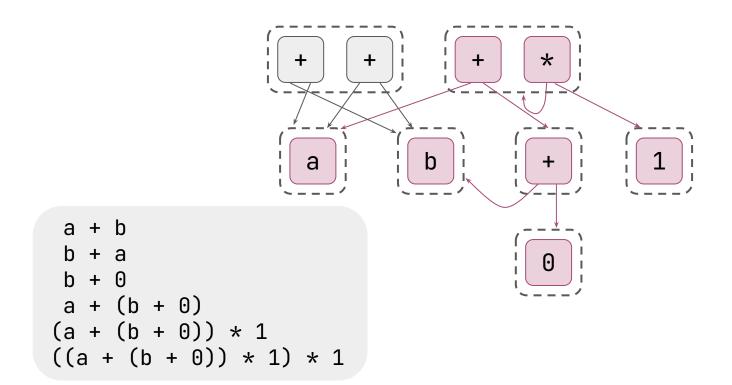
72

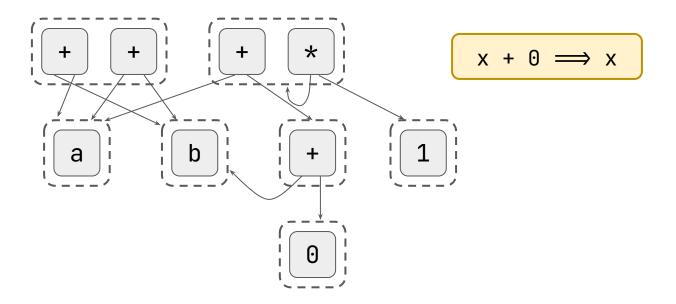
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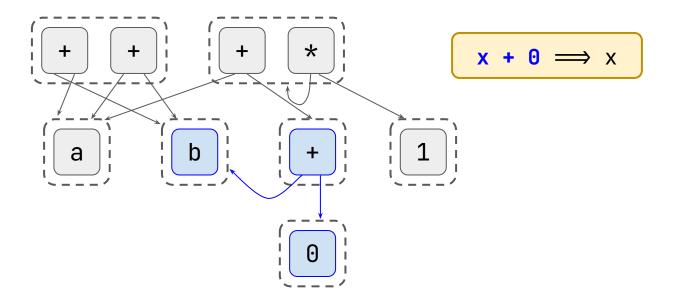


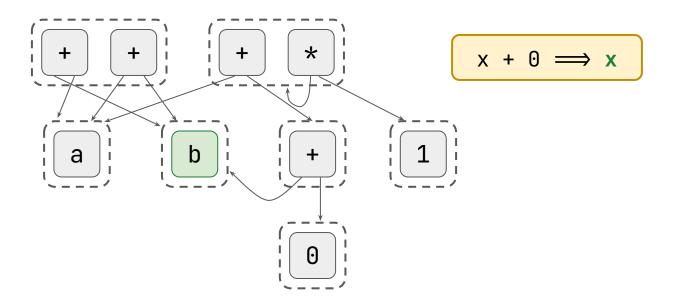
73

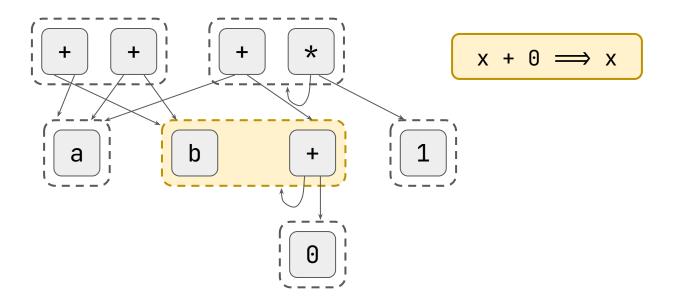
eggcc





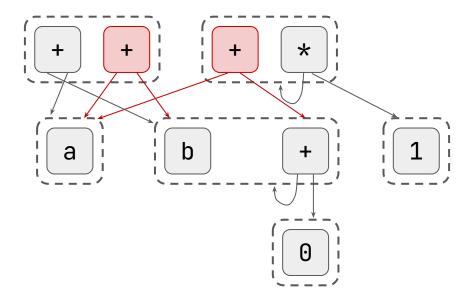






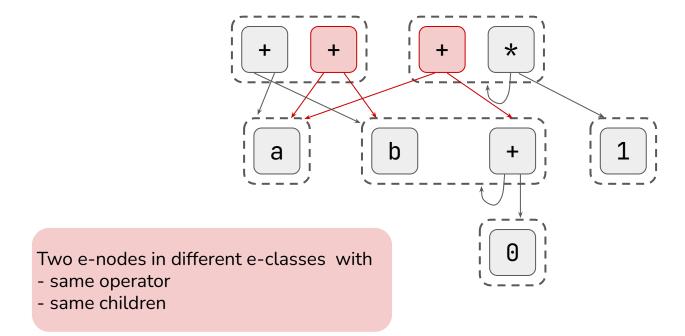
78

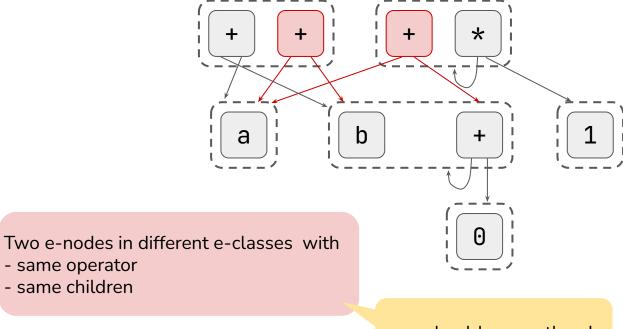
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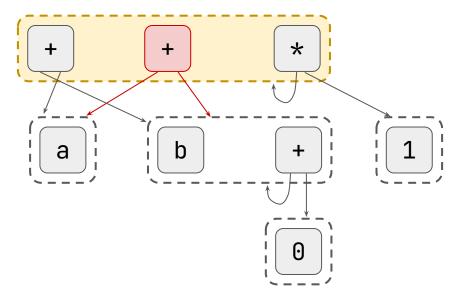




we should merge them!

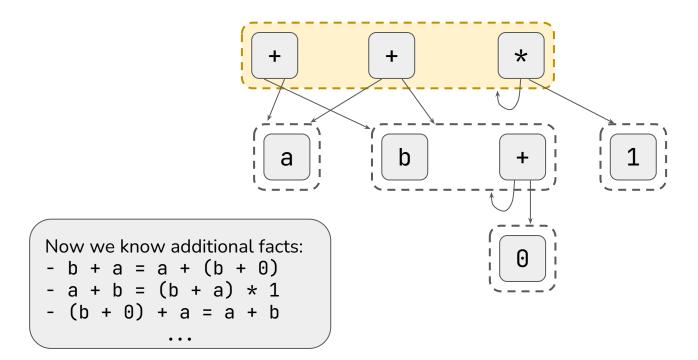
81

Datalog EqSat egglog eggcc



82

eggcc



e-graphs: Summary

- E-graphs compactly represent many equivalent terms
- Rewrite rules have the form lhs => rhs
- Equality saturation grows an e-graph by applying rewrite rules
 - Until saturation or timeout
- E-graphs maintain congruence

egglog is a fixpoint reasoning system that unifies Datalog and Equality Saturation

Match on one or more terms in the database

Match on one or more terms in the database

Add new terms and/or mark terms equivalent

```
Ancestor(X, Y):- Parent(X, Y)
Ancestor(X, Z):-
   Parent(X, Y),
   Ancestor(Y, Z)
Parent("Alice", "Bob")
Parent("Bob, "Charlie")
```

Patalog EqSat egglog eggcc

```
(relation parent
  (String String))
(relation ancestor
  (String String))
(parent "Alice" "Bob")
(parent "Bob" "Charlie")
(rule ((parent x y))
      ((ancestor x y)))
(rule ((parent x y)
      (ancestor y z))
      ((ancestor x z))
(run); run the rules
```

```
Ancestor(X, Y):- Parent(X, Y)
Ancestor(X, Z):-
   Parent(X, Y),
   Ancestor(Y, Z)
Parent("Alice", "Bob")
Parent("Bob, "Charlie")
```

```
(relation parent
  (String String))
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  (String String))
(parent "Alice" "Bob")
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(rule ((parent x y))
      ((ancestor x y)))
(rule ((parent x y)
      (ancestor y z))
      ((ancestor x z))
(run); run the rules
```

Parent Id

Ancestor Id

```
(relation parent
  (String String))
(relation ancestor
  (String String))
(parent "Alice" "Bob")
(parent "Bob" "Charlie")
(rule ((parent x y))
      ((ancestor x y)))
(rule ((parent x y)
      (ancestor y z))
      ((ancestor x z))
(run); run the rules
```

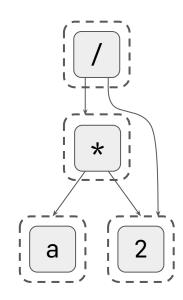
Parent		ld
Alice	Bob	1
Bob	Charlie	2

Ancestor	ld
----------	----

```
(relation parent
  (String String))
(relation ancestor
  (String String))
(parent "Alice" "Bob")
(parent "Bob" "Charlie")
(rule ((parent x y))
      ((ancestor x y)))
(rule ((parent x y)
      (ancestor y z))
      ((ancestor x z))
(run); run the rules
```

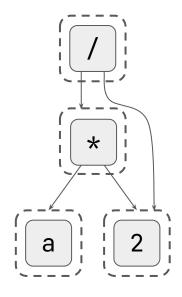
Parent		ld
Alice	Bob	1
Bob	Charlie	2

Ancestor		ld
Alice	Bob	3
Bob	Charlie	4
Alice	Charlie	5



Datalog EqSat egglog eggcc

```
; Declare a datatype
(datatype Expr
  (Var String)
  (Num i64)
  (Mul Expr Expr)
  (Div Expr Expr))
; Add a term
(Div (Mul (Var "a") (Num 2))
     (Num 2))
```



```
; Declare a datatype
(datatype Expr
 (Var String)
  (Num i64)
 (Mul Expr Expr)
  (Div Expr Expr))
; Add a term
(Div (Mul (Var "a") (Num 2))
     (Num 2))
```

Div		ld
id 3	id 1	4
Mul		ld
id 2	id 1	3
Var		ld
Var "a"		ld 2

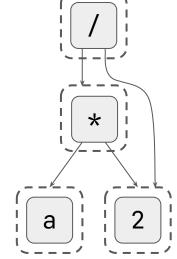
Datalog EqSat egglog eggcc

```
; Declare a datatype
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  (Var String)
 (Num i64)
  (Mul Expr Expr)
  (Div Expr Expr))
; Add a term
(Div (Mul (Var "a") (Num 2))
     (Num 2))
```

```
(x * y) / z \Longrightarrow x * (y / z)
 x / x
 x * 1
```

```
; Declare rules
(rule
  ((= e (Div (Mul x y) z)))
  ((union e (Mul x (Div y z)))))
(rule
  ((= e (Div x x)))
  ((union e (Num 1))))
(rule
  ((= e (Mul x (Num 1))))
  ((union e x)))
(run); run the rules
```

```
(x * y) / z \Longrightarrow x * (y / z)
x / x \Longrightarrow 1
x * 1 \Longrightarrow x
```

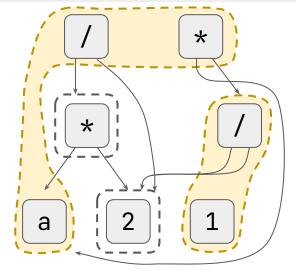


```
; Declare rules
(rule
  ((= e (Div (Mul x y) z)))
 ((union e (Mul x (Div y z)))))
(rule
 ((= e (Div x x)))
  ((union e (Num 1))))
(rule
 ((= e (Mul x (Num 1))))
  ((union e x)))
(run); run the rules
```

```
(x * y) / z \Longrightarrow x * (y / z)

x / x \Longrightarrow 1

x * 1 \Longrightarrow x
```



egglog subsumes Datalog and Equality Saturation

Rules can match on multiple facts, like in Datalog

egglog subsumes Datalog and Equality Saturation

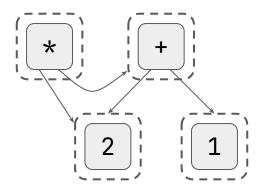
Rules can match on multiple facts, like in Datalog

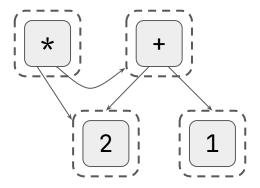
egglog subsumes Datalog and Equality Saturation

Rules can mark terms equivalent, like in Equality Saturation

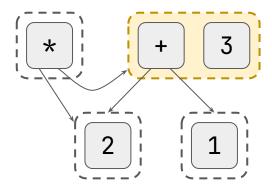
Now let's build a compiler!

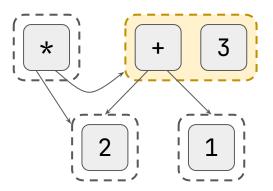
```
x = (1 + 2 + 3) * (5 + 6 + 7)
print x
print 108
```



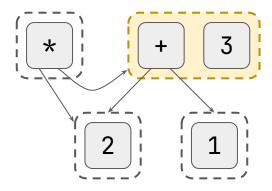


```
(rule ((= e (Add (Num x) (Num y))))
      ((union e (Num (+ x y)))))
```

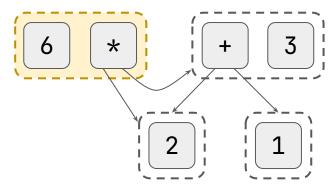




Optimization: Constant Folding



Optimization: Constant Folding



```
x = \dots // \text{ between } 2-4
y = ... // between 3-5
print (x + y < 100)
print true
```

```
(datatype Expr ...)

(datatype Interval
   (IntI i64 i64)
   (BoolI bool bool))
(function ival (Expr) Interval)
```

```
(datatype Expr ...)
(datatype Interval
  (IntI i64 i64)
  (BoolI bool bool))
(function ival (Expr) Interval)
                                      ival
(let one (Num 1))
(set (ival one) (IntI 1 1))
                                       (Num 1)
                                                  (IntI 1 1)
```

```
(datatype Expr ...)

(datatype Interval
   (IntI i64 i64)
   (BoolI bool bool))
(function ival (Expr) Interval)

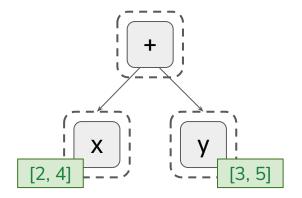
(let one (Num 1))
    ival
```

```
(let one (Num 1))
(set (ival one) (IntI 1 1))
(let t (Bool true))
(set (ival t) (BoolI true true))
```

ival	
(Num 1)	(IntI 1 1)
(Bool true)	(BoolI true true)

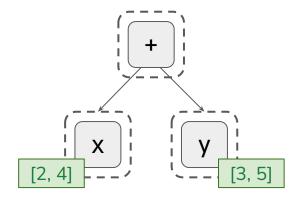
```
(rule
  ((= e (Add x y))
    (= (IntI lo-x hi-x) (ival x))
    (= (IntI lo-y hi-y) (ival y)))
  ((set (ival e) (IntI (+ lo-x lo-y) (+ hi-x hi-y)))))
```

```
x = ... // between 2-4
y = ... // between 3-5
print (x + y < 100)</pre>
```



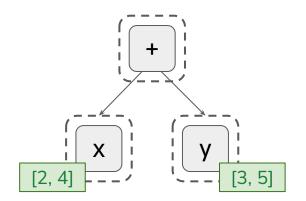
```
(rule
  ((= e (Add x y))
        (= (IntI lo-x hi-x) (ival x))
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        ((set (ival e) (IntI (+ lo-x lo-y) (+ hi-x hi-y)))))
```

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x = ... // between 2-4
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```



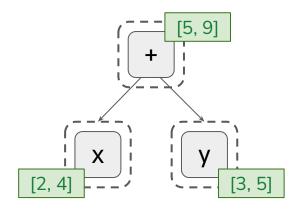
```
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  ((= e (Add x y))
    (= (IntI lo-x hi-x) (ival x))
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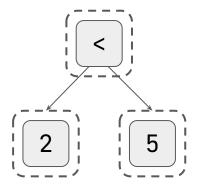


```
(rule
  ((= e (Add x y))
    (= (IntI lo-x hi-x) (ival x))
    (= (IntI lo-y hi-y) (ival y)))
  ((set (ival e) (IntI (+ lo-x lo-y) (+ hi-x hi-y)))))
```

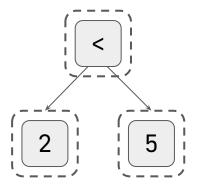
```
x = ... // between 2-4
y = ... // between 3-5
print (x + y < 100)</pre>
```



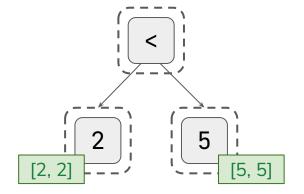
```
(rule
  ((= e (Num x)))
  ((set (ival e) (IntI x x))))
```



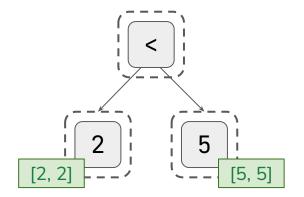
```
(rule
  ((= e (Num x)))
  ((set (ival e) (IntI x x))))
```



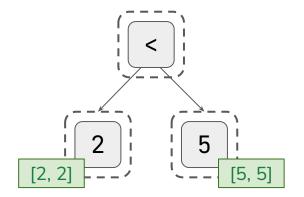
```
(rule
  ((= e (Num x)))
  ((set (ival e) (IntI x x))))
```



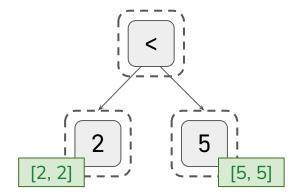
```
(rule
  ((= e (LessThan x y))
    (= (IntI lo-x hi-x) (ival x))
    (= (IntI lo-y hi-y) (ival y)))
  ((set (ival e) (BoolI (< hi-x lo-y) (< lo-x hi-y)))))</pre>
```



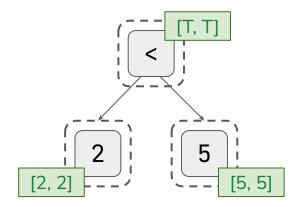
```
(rule
  ((= e (LessThan x y))
    (= (IntI lo-x hi-x) (ival x))
    (= (IntI lo-y hi-y) (ival y)))
  ((set (ival e) (BoolI (< hi-x lo-y) (< lo-x hi-y)))))</pre>
```



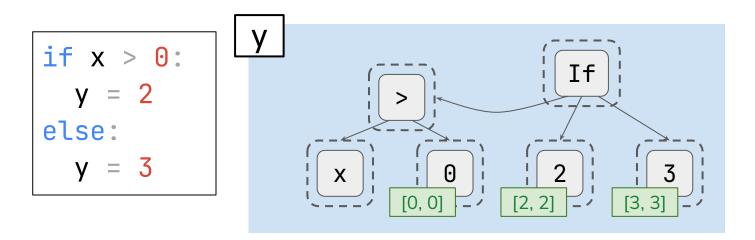
```
(rule
  ((= e (LessThan x y))
    (= (IntI lo-x hi-x) (ival x))
    (= (IntI lo-y hi-y) (ival y)))
  ((set (ival e) (BoolI (< hi-x lo-y) (< lo-x hi-y)))))</pre>
```

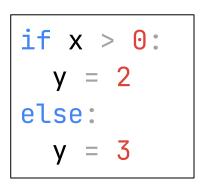


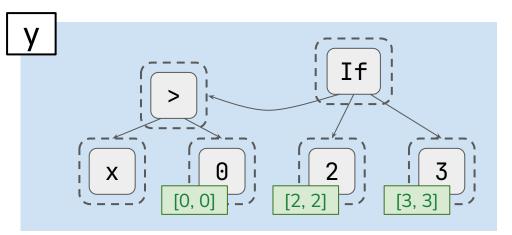
```
(rule
  ((= e (LessThan x y))
    (= (IntI lo-x hi-x) (ival x))
    (= (IntI lo-y hi-y) (ival y)))
  ((set (ival e) (BoolI (< hi-x lo-y) (< lo-x hi-y)))))</pre>
```

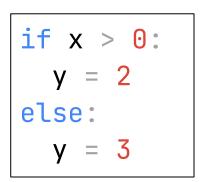


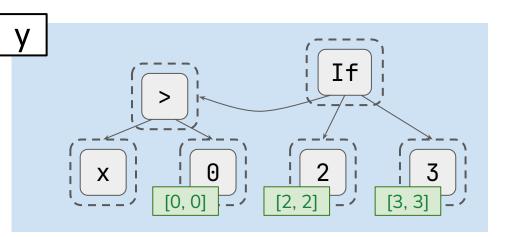
```
if x > 0:
else:
if y < 10:
 z = 5
else:
return z
```



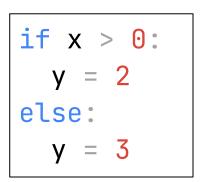


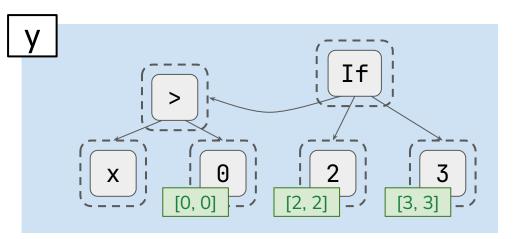


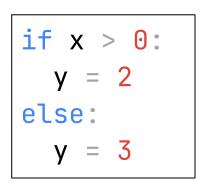


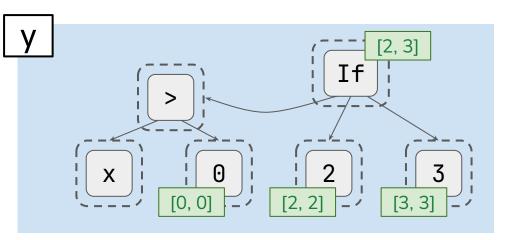


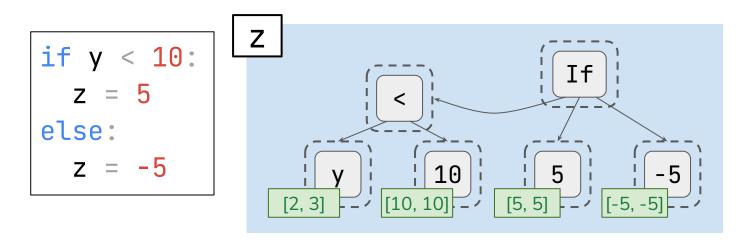
```
(rule ((= e (If pred then else))
       (= then-ival (ival then))
       (= else-ival (ival else)))
  ((set (ival e) (interval-union then-ival else-ival))))
```





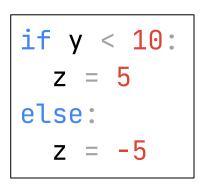


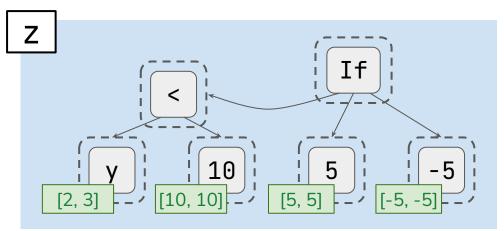


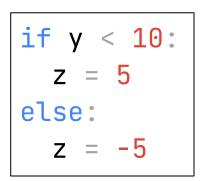


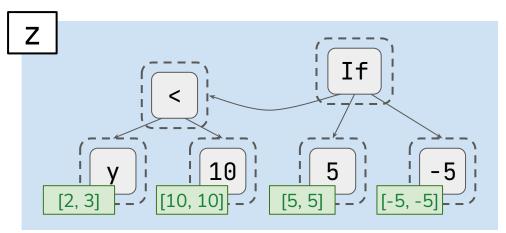
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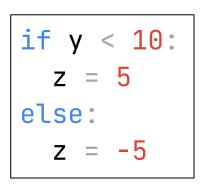
eggcc

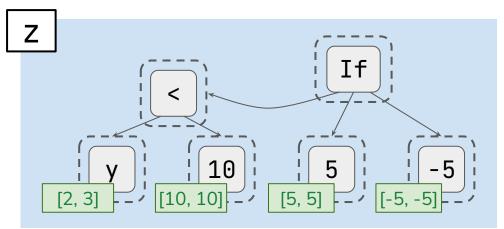


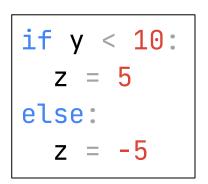


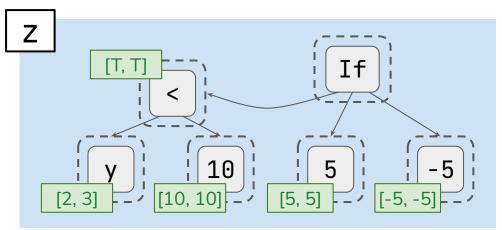




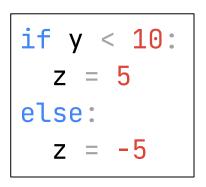


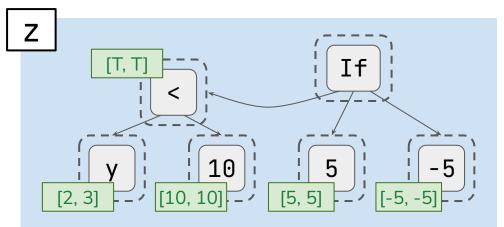




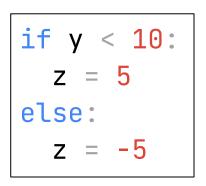


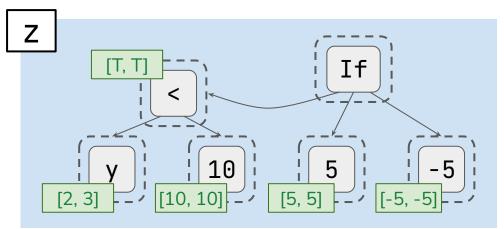
```
(rule
  ((= e (If pred then else))
   (= (BoolI true true) (ival pred)))
  ((union e then)))
```



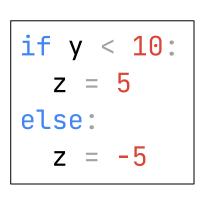


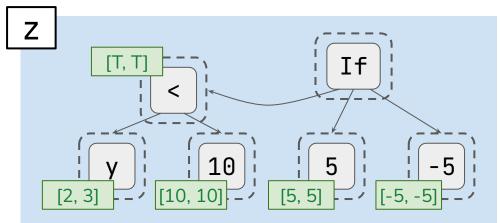
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    (= (BoolI true true) (ival pred)))
    ((union e then)))
```



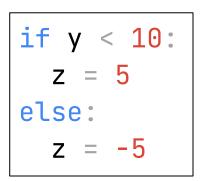


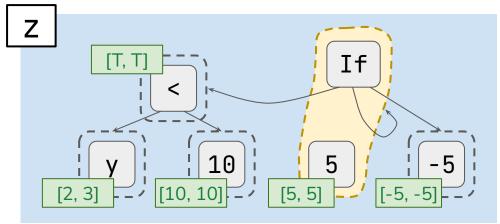
```
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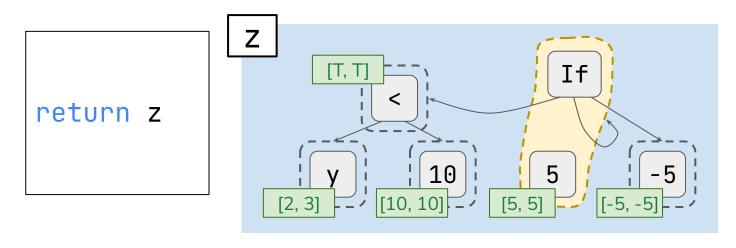




```
(rule
  ((= e (If pred then else))
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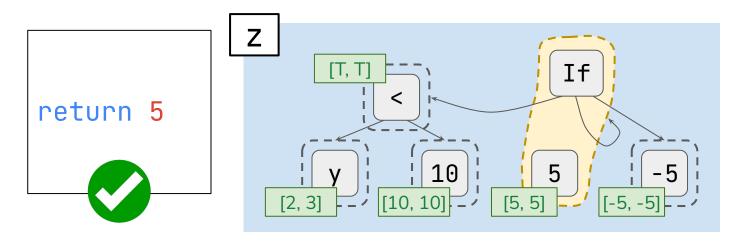






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eggcc



```
if x > 0:
y = 2
else:
y = 3
if y < 10:
z = 5
else:
z = -5
return z
```

return 5

What about imperative code?

```
def foo():
                       def foo():
  print "hi"
                         print "hi"
  return 3
                         return 3
foo() + foo()
                       2 * foo()
       Are these programs equivalent?
```

eggcc: An optimizing compiler built with Egglog

- Building a compiler is hard
 - Optimizations don't compose well
 - Incremental analysis is hard
 - Phase ordering gets in the way
- Building a compiler using egglog is better
 - Optimizations are written as declarative rewrite rules
 - Leverage composable analyses from Datalog with fast equational reasoning from EqSat
- There are still challenges
 - Encoding control flow and effectful programs is difficult
 - Tension between sharing and context
 - Extracting the optimal terms from the e-graph after optimization is hard

All of this work is still in progress!

I'd love to talk about it in more detail if you're interested!

Get in touch: anjalip@uw.edu





eggcc team



















