United International University
School of Science and Engineering
Final Examination Trimester: Fall-2022
Course Title: Calculus and Linear Algebra
Course Code: Math 183/ Math 2183 Marks: 40
Time: **2 hours**

Answer all questions.

1. a) Solve the following system by the elementary row operation.

$$2x - 2y + 4z + 6q = 0$$
$$6x + 2y - 4z + 2q = 4$$
$$-2x + 4y - 2z - 2q = -2$$

[4]

b) Solve the homogeneous system of linear equation.

$$3x - 6y + 3z - 3w = 0$$
$$-3y + 2z + w = 0$$
$$6x - 2y + z - w = 0$$

[3]

c) Find the reduced row echelon form the matrix.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 4 & 1 \\ 3 & 2 & 0 \end{bmatrix}$$

[3]

2. a) Find the Eigenvalues and corresponding Eigenvector of matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$. Also draw the Eigen-space in xy −plane. [4+1]

b) i) Find the inverse of $A = \begin{bmatrix} -1 & 1 & 0 \\ 2 & -1 & 1 \\ 1 & 3 & 1 \end{bmatrix}$. [4]

ii) If $X = A^{-1}A$, what is X? [1]

3. Consider a Matrix $A = \begin{bmatrix} 1 & 5 \\ 2 & -3 \end{bmatrix}$

i) Find $p(A)$ for $p(x) = x^2 - 2x + 3$. [3]

ii) Verify that $(A^T)^{-1} = (A^{-1})^T$. [2]

iii) Find x, Such that $\mathrm{tr}(A) = x^2 + 2x$. [2]

iv) Find A^{-3}. [2]

v) Find AB, where $B = \begin{bmatrix} 1 & 5 \\ 1 & 9 \\ 7 & 4 \end{bmatrix}$. [1]

4. a) Solve $y'' - 8y' + 16y = e^{-4x} + \sin 4x - 2$. [6]

b) Solve the initial value problem

$$y'' - 6y' + 9y = 0 \; ; \; y(0) = -2, \quad y'(0) = 1$$ [4]