



**United International University**  
**School of Science and Engineering**  
 Final Examination Trimester: Fall 2022  
 Course Title: Coordinate Geometry and Vector Analysis  
 Course Code: Math 2201 Marks: 40  
 Total Time: 2 hours

Answer all questions.

✓ 1. a) Consider,  $F(x, y) = 2e^{2x} \sin y \mathbf{i} + e^{2x} \cos y \mathbf{j}$  [6]

i) Show that  $F$  is a conservative vector field on the entire  $xy$ -plane.

ii) Find the potential function  $\phi(x, y)$ .

iii) Find  $\int_{(0,0)}^{(1, \frac{\pi}{2})} F \cdot d\mathbf{r}$  using  $\phi(x, y)$

b) Using Green's theorem find the value of  $\oint_C F \cdot d\mathbf{r}$  Where [4]

$F(x, y) = (e^{3x} - y^2)\mathbf{i} + (y^3 + 2x^2)\mathbf{j}$  and  $C$  is the closed circle  $x^2 + y^2 = 4$

✓ 2. a) Evaluate  $\int_C (x-1)dx + (2y-x)dy$  along the rectangle with vertices [5]  
 $(0, 0), (0, 2), (2, 2)$  and  $(2, 0)$

b) Evaluate the surface integral  $\iint_{\sigma} x^2 z^2 ds$ ;  $\sigma$  is the part of the cone [5]

$z = \sqrt{x^2 + y^2}$  that lies between the planes  $z = 1$  and  $z = 3$ .

✓ 3. a) Find the flux of the vector field  $F(x, y, z) = 2x\mathbf{i} - y\mathbf{j} + 2z\mathbf{k}$  across  $\sigma$ , [5]  
 where  $\sigma$  is the portion of the surface  $z = 5 - x^2 - y^2$  that lies above the  $xy$ -plane and suppose that  $\sigma$  is oriented up.

b) Use the Divergence Theorem to find the outward flux of the vector field [5]  
 $F(x, y, z) = 2x^3\mathbf{i} + 2y^3\mathbf{j} + 2z^3\mathbf{k}$  across the surface of the region that is enclosed by  $z = \sqrt{9 - x^2 - y^2}$  and the plane  $z = 0$ .

✓ 4. a) Use cylindrical coordinate systems to evaluate: [5]

$$\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{9-2(x^2+y^2)} (x^2 + y^2) dz dy dx$$

Or

Using double integral to find the area enclosed by the equations

$$-x - 2y = 2, x - y = 1 \text{ and } y = 0.$$

b) Use triple integral to find the volume of the solid  $G$  enclosed by the sphere [5]  
 $x^2 + y^2 + z^2 = 9$  and the cone  $z = \sqrt{x^2 + y^2}$ .



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Final Examination Trimester: Summer: 2022

Course Title: Coordinate Geometry and Vector Analysis

Course Code: MAT 2109 / Math 201 Marks: 40 Time: 2 Hours.

Formula:

1.

$$\int_C f(x, y) dx + g(x, y) dy = \iint_R \left( \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA$$

2.

$$\int_{(x_0, y_0)}^{(x_1, y_1)} \mathbf{F} \cdot d\mathbf{r} = \int_{(x_0, y_0)}^{(x_1, y_1)} \nabla \phi \cdot d\mathbf{r} = \phi(x_1, y_1) - \phi(x_0, y_0)$$

3.

$$\iint_{\sigma} f(x, y, z) dS = \iint_R f(g(y, z), y, z) \sqrt{\left(\frac{\partial x}{\partial y}\right)^2 + \left(\frac{\partial x}{\partial z}\right)^2 + 1} dA$$

$$\iint_{\sigma} f(x, y, z) dS = \iint_R f(x, y, g(x, y)) \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1} dA$$

$$\iint_{\sigma} f(x, y, z) dS = \iint_R f(x, g(x, z), z) \sqrt{\left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2 + 1} dA$$

4.

$$\iint_{\sigma} \mathbf{F} \cdot \mathbf{n} dS = \iint_R \mathbf{F} \cdot \left( -\frac{\partial z}{\partial x} \mathbf{i} - \frac{\partial z}{\partial y} \mathbf{j} + \mathbf{k} \right) dA$$

$\sigma$  of the form  $z = g(x, y)$   
and oriented up

$$\iint_{\sigma} \mathbf{F} \cdot \mathbf{n} dS = \iint_R \mathbf{F} \cdot \left( \frac{\partial z}{\partial x} \mathbf{i} + \frac{\partial z}{\partial y} \mathbf{j} - \mathbf{k} \right) dA$$

$\sigma$  of the form  $z = g(x, y)$   
and oriented down

5.

$$\iint_{\sigma} \mathbf{F} \cdot \mathbf{n} dS = \iiint_G \operatorname{div} \mathbf{F} dV$$

6.

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_{\sigma} (\operatorname{curl} \mathbf{F}) \cdot \mathbf{n} dS$$