United International University
Department of Computer Science and Engineering

CSE 2213/CSI 219: Discrete Mathematics
Final Examination : Fall 2023
Total Marks: 40 Time: 2 hours

Answer all the 5 questions. Numbers to the right of the questions denote their marks.
Any examinee found adopting unfair means will be expelled from
the trimester / program as per UIU disciplinary rules.

1. Prove the following using mathematical induction, whenever n is a non-negative integer. [4]

$$3 + 3 \cdot 5 + 3 \cdot 5^2 + \cdots + 3 \cdot 5^n = \frac{3 \cdot (5^{n+1} - 1)}{4}$$

2. (a) **Seven (7)** women and **nine (9)** men are on the faculty in the Mathematics department at a school. How many ways are there to select a committee of **six (6)** members of the department if at least **two (2)** women and at least **one (1)** man must be on the committee? [3]

   (b) How many **4-digit** numbers are possible that are **divisible by 3** can be formed using the digits **3, 5, 6, 1 and 7** such that no digit is repeated? (Hint: A number is divisible by 3 if the sum of the digits is divisible by 3.) [4]

   (c) How many cards must be chosen from a standard deck of 52 cards to guarantee that there are **at least two (2)** cards of each of **two (2) different suits**? (Note: There are a total of 4 suits of cards) [2]

3. (a) What type of directed graph is the one given in Figure 1? If you change the directed graph in Figure 1 to an undirected graph, what kind of undirected graph would it be? Would there be any difference between the undirected graph from Figure 1 and the graph in Figure 2? State your reasoning. [3]

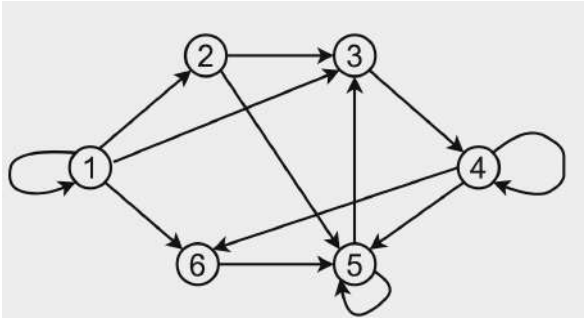


Figure 1: Graph for Question 3(a), 3(b)

   (b) Find the in-degree and out-degree for each of the vertices of the graph provided in Figure 1. Thus, show whether the Handshaking theorem is preserved in the graph of Figure 1 or not. [3]

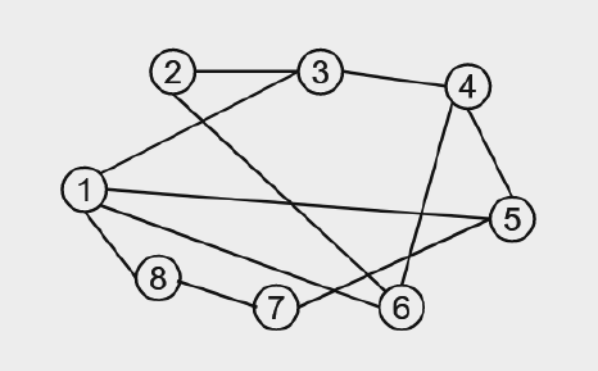


Figure 2: Graph for Question 3(a), 3(c)

   (c) Find out if the graph in Figure 2 is bipartite or not, using two coloring algorithms. If bipartite, show the graph in bipartite form. [3]

4. (a) Given below is the adjacency matrix of a graph. Draw the graph. [3]

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(b) Is the graph in Figure 3 strongly connected? If not, find out strongly connected components of this graph.[3]

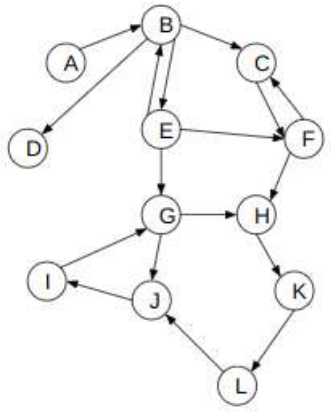


Figure 3: Graph for Question 4(b)

(c) Determine whether the graphs in Figure 4 are isomorphic or not. [3]

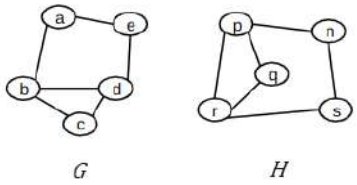


Figure 4: Graphs for Question 4(c)

5. (a) Consider vertex ***"a"*** as the **root** of the tree in Figure 5 and answer the following questions. [1 + 1 = 2]

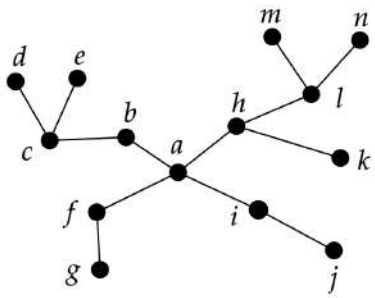


Figure 5: Tree for Question 5(a)

i. What is the maximum level of the tree?
ii. How many internal vertices are there in the tree?

(b) Consider the following list of numbers: [2 + 2 = 4]

7, 3, 10, 1, 5, 8, 11

Now answer the following questions.

i. Construct a binary search tree, inserting the given numbers one by one [draw only the final tree].

ii. Provide the preorder and postorder traversals of the tree you constructed [you can write only the final results].

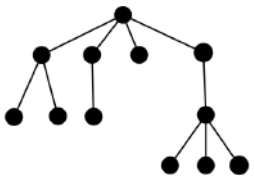
(c) State, with short justification, whether the following statements are "True" or "False". [1 × 3 = 3]

i. Figure 6(a) represents a *tree*.

ii. The tree in Figure 6(b) is a *3-ary* tree.

iii. A full *m-ary* tree is always a balanced tree.

(a)



(b)

Figure 6: Graphs for Question 5(c)