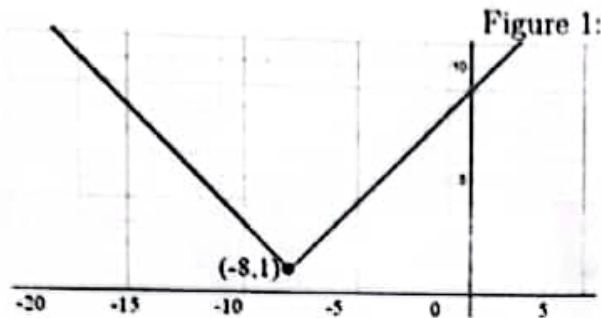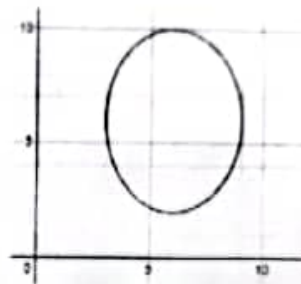United International University
School of Science and Engineering
Midterm Exam, Spring 2023
Fundamental Calculus (MATH 1151)
Total Marks : 30 Time : 1 hour 45 minutes
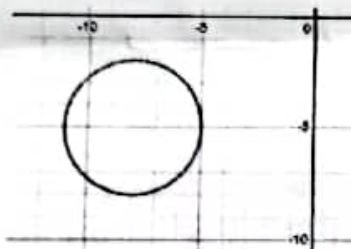
Solve all questions.

1. (a) Determine whether the following curve in(Figure 1) is a function of x. If it is, sketch the domain and range of the functions. [4]
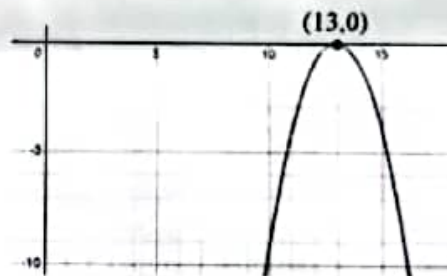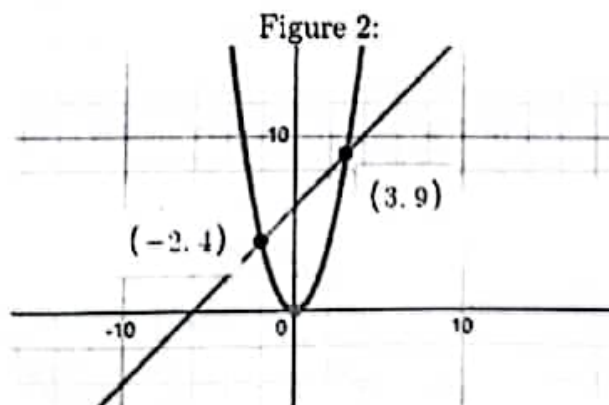
Figure 1:

(a)

(b)

(c)

(d)

(b) The graph (Figure 2) of function f in and g (the straight line) are given. [3]

Figure 2:

i. State the values of $f(0)$ and $g(3)$.

ii. On what interval(s) is $g(x) \leq f(x)$?

iii. On what interval(s) is f increasing and decreasing?

(c) Sketch the graph of the following function. Write the range of the function in an interval notation.

$$f(x) = \begin{cases} \sqrt{9 - x^2} & \text{if } -3 \le x \le 0 \\ x + 3 & \text{if } 0 < x \le 4 \end{cases}$$

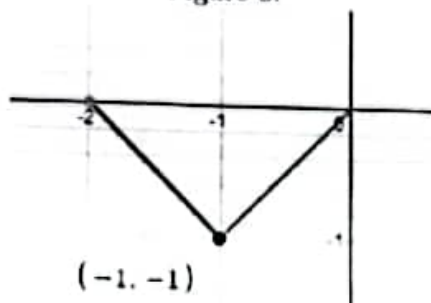2. Draw the graph of the following functions. [6]

    (a) $y = -e^x - 1$

    (b) $y = -|x - 2| - 1$

    (c) $y = -\ln(x - 1)$

    (d) $y = 2\cos 2x$

3. Use the given graph (Figure 3) of $y = g(x)$ to sketch the following functions: [4]

Figure 3:

$(-1, -1)$

    (a) $y = g(x - 2)$

    (b) $y = -|g(x)|$

    (c) $y = g(\frac{x}{3})$

    (d) $y = g(3x)$

4. Determine whether the following functions are one to one, or many to one. Find the inverse of each function (if possible). Draw the graph of each function and its inverse in the same diagram. Also, state the domain and range of the inverse function(s). [4]
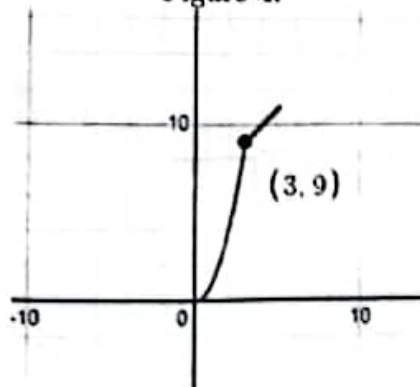
    (a) $f(x) = \ln(x - 3), \ x \ge 3$

    (b) $f(x) = -x^2 + 4, \ -2 \le x \le 0$

5. (a) The graph (Figure 4) of the function is defined for $x \ge 0$. Complete the graph for $x < 0$ to make it (a) an even function & (b) an odd function. [2]
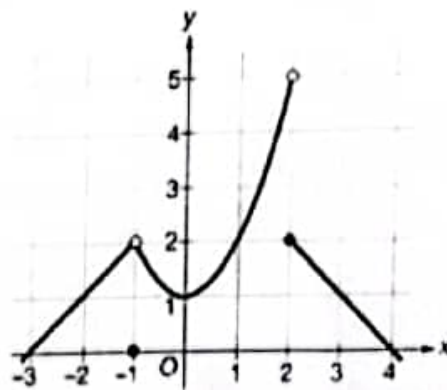
Figure 4:

$(3, 9)$

(b) Determine (mathematically) whether the following functions be even, odd, or neither. [1]

    i. $f(x) = \sqrt{x-1} - 2$

    ii. $g(x) = -\frac{1}{x^2}$

6. Consider following graph of a function $y = f(x)$. [3]

Figure 5:

Graph of f

From the figure determine the following limits:

(a) $\lim\limits_{x \to -1^+} f(x)$

(b) $\lim\limits_{x \to -1^-} f(x)$

(c) check the continuity at $x = 1$