United International University

School of Science and Engineering

Final Examination Trimester: Summer-2024

Course Title: Calculus and Linear Algebra

Course Code: MATH 2183 Marks: 50 Time: 2 Hours

Answer all the questions

1. a) Given that,

$$x + 2y - z = 1$$
$$-x + y - z = 0$$
$$3x + y - z = 2$$

i) Write the above system of linear equations in the form $\boldsymbol{AX = B}$, where A, X and B are matrices. [1]

ii) Find the inverse of $\boldsymbol{A}$ by using matrix inversion algorithm and hence solve the above system of linear equations. [7]

iii) Find the unknown variables $\boldsymbol{z}$ by using Cramer's rule. [2]

b) Find the infinite solution of the following system of linear equations [5]

$$2x_1 + 2x_2 + x_3 - x_4 + 2x_5 = 0$$
$$-x_1 - x_2 + 3x_3 - x_4 - 4x_5 = 0$$
$$x_1 + 2x_2 - 5x_3 - x_4 + 2x_5 = 0$$

2. **a)** Solve $\boldsymbol{y'' + y' = e^{-2x} + cos\,2x - ln2 + 4^x + e^{-x}sin5x}$. [8]

b) Solve the following second order ordinary differential equation [7]

i) $\boldsymbol{y'' - y' + 5y = 0}$

ii) $\boldsymbol{2y'' - 11y' + 12y = 0 \qquad y(0) = 5 \qquad y'(0) = 15}$

3. **a)** Find eigenvalues and eigenvectors of the Matrix $\boldsymbol{A = \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix}}$. Also sketch the eigenspace in $\boldsymbol{xy}$ −coordinates. [8]

b) [6]

Given $\boldsymbol{A = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}}$, $\boldsymbol{B = [1 \quad -2 \quad 1 \quad 3]}$

i) Find $\boldsymbol{\det(A)}$ and $\boldsymbol{det(B)}$

ii) Evaluate $\boldsymbol{AB + 5}$.

iii) Find $\boldsymbol{x}$, Such that $\boldsymbol{tr(AB) = 2x^2 - 3}$

c) Solve $\boldsymbol{(2xy + y^2 + 1)dx + (x^2 + 2xy + 1)dy = 0}$ [6]