United International University

School of Science and Engineering
Final Examination   Trimester: Fall-2023
Course Title: Fundamental Calculus
Course Code: Math 1151   Marks: 40   Time: 2 Hours

Answer <u>all</u> the questions. Answer all parts of a question together.

1. Consider the function $f(x) = x^2 - 2x$; $x_0 = -1$ and $x_1 = 2$.
   (a) **Find** the instantaneous rate of change of $f(x)$ with respect to x at an arbitrary value of x_0. **[2]**
   (b) Use part **(a)** to **find** the slope of the tangent lines for the value of x_0. **[1]**
   (c) **Find** the average rate of change of function in the interval $[x_0, x_1]$. **[1]**
   (d) **Find** the equation of the tangent line to the function $f(x)$ at x_0. **[2]**
   (e) **Find** the equation of the secant line to the function $f(x)$ on the interval $[x_0, x_1]$. **[2]**
   (f) **Draw** the graph of $f(x)$ together with the tangent lines and secant lines. **[2]**

2. (a)   **Roughly sketch** the derivative graph of the following given function.   **[2]**
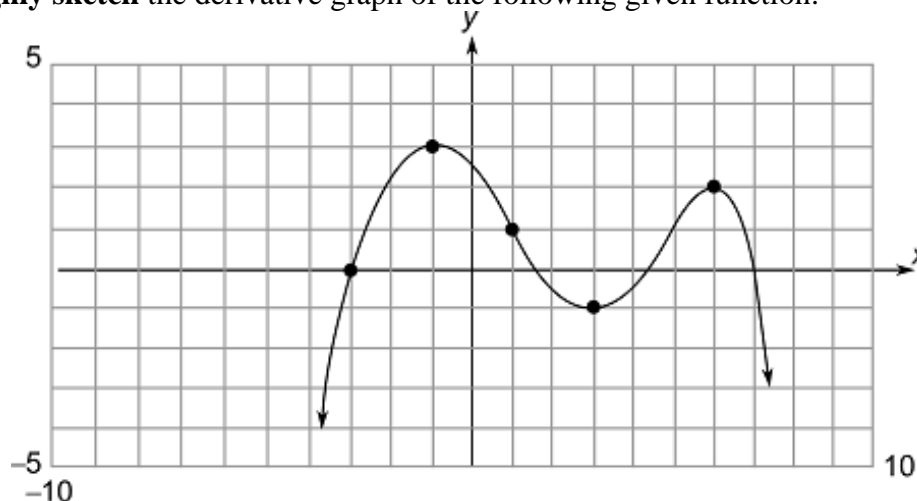
   (b)   Consider the function   **[5]**
$$f(x) = \begin{cases} 4 - x^2, & x > -1 \\ 2x + 5, & x \le -1 \end{cases}$$
      (i)   **Sketch** the graph of $f(x)$.
      (ii)   **Determine** whether the function $f(x)$ is continuous and differentiable at $x = -1$.

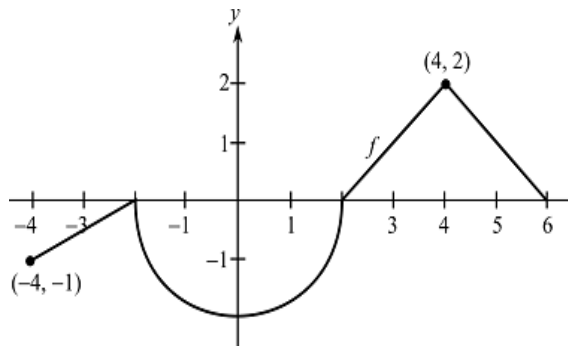   (c)   **Find** $\frac{dy}{dx}$, where $y = \cot^3 \sqrt{2x - 3\sin x}$.   **[3]**

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3. **(a)** Use appropriate formula of geometry to **evaluate** the following integrals: **[5]**

      (i) $\int_{-3}^{3}\{(x+4)-\sqrt{9-x^2})\}dx$ **(ii)** $\int_{-2}^{3}(5-|x+1|)dx$

  (b) The graph of $f(x)$ is shown. **Evaluate** the following definite integrals.

      (i) $\int_{-2}^{6}f(x)dx$ **[3]**

      (ii) $\int_{0}^{4}|f(x)|dx$ **[2]**

4. **(a)** Evaluate the following integrals: **[6]**

      (i) $\int \frac{t^3-t^2\cos t-5t+1}{t^2}dt$

      (ii) $\int \frac{x}{1+x^4}dx$

      (iii) $\int x\sin 2x\,dx$

  (b) **Find** and **Sketch** the area of the region enclosed by the parabola $y=x^2$ and the line $y=x+2$. **[4]**

BEST OF LUCK!!!