



# United International University

## Department of Computer Science and Engineering

CSE 2213/CSI 219: Discrete Mathematics

Final Examination — Summer 2024

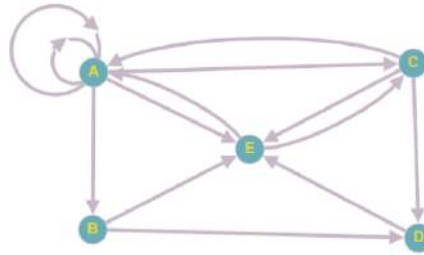
Total Marks: 50 Time: 2 hours

Any examinee found adopting unfair means will be expelled from the trimester / program as per UIU disciplinary rules.

1. Prove the following using Mathematical Induction, whenever  $n$  is a positive integer. [5]

$$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \cdots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

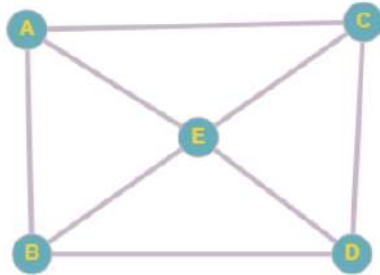
2. (a) Does the following graph follow the Handshaking Theorem? Mathematically justify your answer. [2]



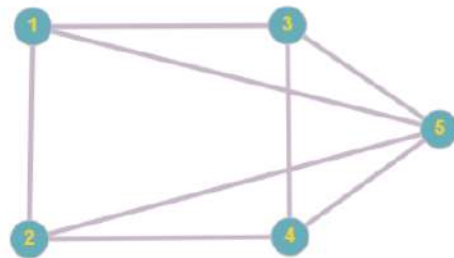
- (b) Draw the graph from the following incidence matrix. [2]

	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	$e_6$	$e_7$	$e_8$
$a$	1	1	0	0	0	0	1	0
$b$	1	0	1	0	0	1	0	1
$c$	0	1	0	0	0	1	0	0
$d$	0	0	0	1	1	0	0	0
$e$	0	0	1	1	0	0	1	1

- (c) Determine whether the following pair of graphs are isomorphic. [3]

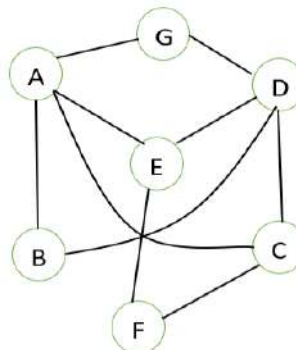


Graph  $G$



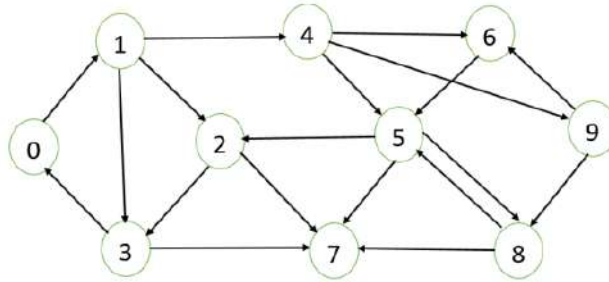
Graph  $H$

3. (a) Determine whether the graph in the following figure is bipartite or not, using the 2-coloring algorithm. If the graph is bipartite, redraw the graph in bipartite form. [3]



**Question 3 contd.**

- (b) Is the following graph strongly connected or weakly connected? If it is weakly connected, please provide the reasoning and identify the strongly connected components. [4]

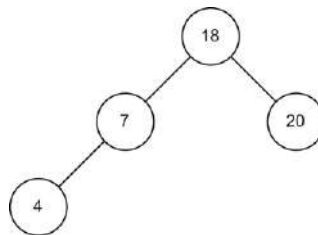


- (c) The University of Ethior has **12 social clubs** and **60 students** who are members of these clubs. Each club holds regular meetings and organize events, and every student participates in **exactly  $x$  clubs**. Each club awards a unique participation certificate to all of its participating members after the event. Now, to facilitate better interaction among the students, the university is planning to host a social gathering event where all the clubs will participate, and each student must interact with every other student at least once.

Based on the above information, answer the following questions:

- If the sum of the total number of certificates awarded by the clubs and the total number of certificates received by their members is 480, find  $x$ . [2.5]
- Find out the minimum number of unique interactions among the students in this event. [2.5]

4. (a) The following figure contains the beginning of an unfinished binary search tree.

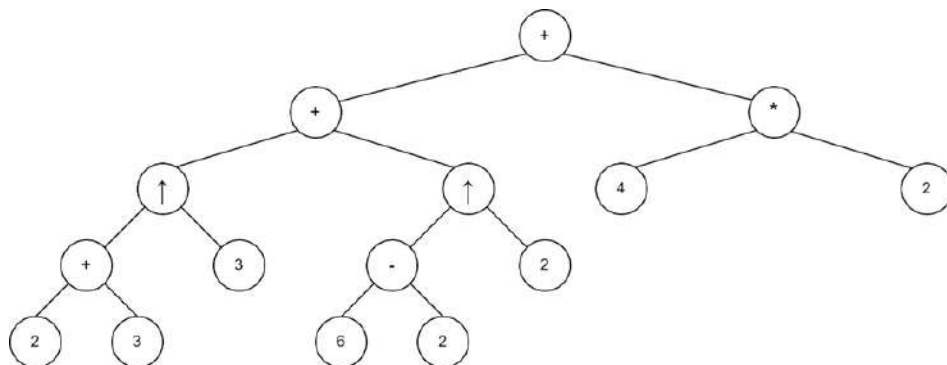


- Complete the mentioned binary search tree by inserting the following values. [1]

26, 27, 8, 16, 24, 14

- Which tree traversal technique will you use to sort the values of the complete binary search tree in ascending order? Show the traversal. [1 + 2 = 3]

- (b) Consider the following binary expression tree.



- Determine the prefix notation from the mentioned binary expression tree. [1]
- Calculate the result of the prefix notation. [3]

**Question 4 contd.**

- (c) In the small town of Rocky Beach, the three investigators — Kishore, Musa and Robin are investigating a case. Though teenager, their intelligence is way over some of the finest detectives in the US Police. To solve the case, they have to find out the whereabouts of a golden Rolls Royce, stolen from a rent-a-car company. To know this, they have devised a mindblowing technique they call “Bhoot Theke Bhoot”, where each of them calls three of his friends and tells them to either inform if they have seen the car, or forward the same request to three of their friends.

If there are a total of 4,728 teenagers (including Kishore, Musa and Robin) in the town, and each detective covers exactly the same number of friends, then answer the following questions —

- i. How many friends chose to forward the message? [3]
  - ii. How many total phone calls had to be made? [2]
5. (a) How many strings of lowercase letters (no repetition) are there of length four or five? [2]
- (b) What is the minimum number of students required to guarantee that at least 100 students are admitted from the same district in Bangladesh, given that there are 64 districts? [3]
- (c) How many ways can the letters of the word “COMPUTATION” be rearranged, if — [1.5 × 2 = 3]
- i. All the vowels are together?
  - ii. All the T's are together?
6. (a) You have a string and some beads of three different shapes, as follow: [3]
- You have **5 round beads**, 3 of which are red and 2 of which are blue.
  - You have **6 diamond shaped beads**, each of the colors violet, blue, green, yellow, orange and red.
  - you have **2 triangular beads**, one of black color and another one of white color.



You are arranging the beads in a string as per the design given in the above figure for making a decoration piece. Now based on the beads of different shapes and colors and the design which you are following, in how many ways can you prepare the decoration piece? Show the necessary steps and calculations in favor of your answer.

- (b) During the vacation, you arrive at a beautiful 5-storey villa with 4 apartments on each floor. Soon after, 43 friends and family members join you. To ensure everyone has enough space and the apartments don't feel overcrowded, you need to distribute everyone (including yourself) across the villa. What is the maximum number of people who will need to stay in one apartment to minimize crowding? Show your reasoning and calculations. [2]