United International University
School of Science and Engineering
Final Examination Trimester: Fall-2023
Course Title: Calculus and Linear Algebra
Course Code: MATH 2183 Marks: 40 Time: 2 Hour

Answer all the questions.

1. a) Consider following system of linear equations

$$-x + 2y - z = -1$$
$$2x - y - 2z = 2$$
$$-x - 3y - 2z = 0$$

i) Solve the system of linear equations by Using Gaussian elimination method. [4]
ii) Using Cramer's rule find the values of x and z. [3]

b) Solve the homogeneous system of linear equations [3]

$$4x + y - z - 4w + 3p = 0$$
$$-x - y + 2z - w + 5p = 0$$

2. a) Solve the following second order ordinary differential equations [5]

i) $\frac{d^2y}{dx^2} + 18\frac{dy}{dx} + 81y = 0\,; \quad y(0) = 1,\ y'(0) = 0.$
ii) $y'' + y' + 3y = 0$

b) Solve $y'' - y' + 5y = 8e^{-5x} - 5 + e^{-4x}sinx.$ [5]

3. Consider the matrices given below for the followings:

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad u = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \quad v = [0 \quad -1 \quad 0 \quad 3]$$

a) Find A^{-1} by using matrix inversion algorithm and calculate $B = A^{-1}C$ to find the unknown values of x, y, z for matrix B. [5]

b) Evaluate $u.v$. [2]

c) Find the Matrix $2A + \textbf{Adjoint matrix of}(A) + 3$. [3]

4. a) Solve $(2x + y)dx + (x + 3y^2)dy = 0$ [5]

b) Find the Eigenvalues and corresponding Eigenvector of Matrix $A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$. Also draw the Eigen space in xy −plane. [5]