United International University
Department of Computer Science and Engineering

CSE 2213/CSI 219: Discrete Mathematics

Mid-term Examination : Fall 2021

Total Marks: 30 Time: 1 hour 45 minutes

Answer all the 5 questions. Numbers to the right of the questions denote their marks.

1. (a) Find the inverse, converse and contrapositive of the following sentence: [0.5×3=1.5]

   "People feel stressed when they have a lot on their plate."

   (b) Prepare the truth table for the following compound proposition: [2.5]

   $$(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$$

   (c) Using propositional laws, prove that $(p \to q) \to r$ and $(\neg r \to p) \land (q \to r)$ are logically equivalent. [2]

2. (a) Consider the following predicates:

   $T(x)$: x is a teacher of CSE.
   $L(x)$: x is a Lecturer.
   $D(x)$: x teaches Discrete Mathematics.
   $S(x, y)$: The substitute teacher of x is y.

   Represent the following statements using the above predicates, quantifiers and logical connectives. The domain of all variables consists of all people of the world. [1×3=3]

   i. Some teachers of CSE are Lecturers.
   ii. All teachers of CSE teach Discrete Mathematics.
   iii. The substitute of some Discrete Mathematics teachers of CSE are some lecturers.

   (b) State and explain the truth values of each of the following expression, where the domain of all variables is all real numbers. [1×3=3]

   i. $\forall x \exists y (x^2 = y)$
   ii. $\exists x \forall y (xy = 0)$
   iii. $\forall x \forall y \exists z (z = \frac{x+y}{2})$

3. (a) Suppose $A \subset B$. Determine whether the following statements are true or false (with reasoning): [1.5×2=3]

   i. $B' \subset A'$
   ii. $B - A = \emptyset$

   (b) Suppose you have two sets $A = \{1, 2\}$ and $B = \{a, b\}$.

   i. Determine $A \times B$. [1]
   ii. Find the power set $P(A \times B)$. [1]
   iii. Show that $|P(A \times B)| = 2^{|A||B|}$. [1]

4. (a) Find $f \circ g$ and $g \circ f$, where $f(x) = \frac{1}{x} - \frac{2}{x+1}$ and $g(x) = \frac{x-1}{x+2}$ are functions from R to R. [2]

   (b) Determine if the following functions are invertible with necessary explanation: [2×2=4]

   i. $f : Z^+ \to R, f(x) = \frac{x-1}{x+1}$
   ii. $f : R - \{1\} \to R, f(x) = \frac{1}{x-1}$

5. (a) Prove the following statement using a direct proof: [3]

   "If n is a multiple of 3, then $2n + 3$ is a multiple of 3."

   (b) Prove the following statement using a proof by contradiction: [3]

   "The product of a non-zero rational number and an irrational number is irrational."

United International University (UIU)

Dept. of Computer Science & Engineering (CSE)

Mid Exam. : : Trimester: Spring 2020

Course Code: CSE 2213, Course Title: DISCRETE MATHEMATICS

Total Marks: **30** Duration: 1 hour 45 min

Answer all the questions. Figures are in the right-hand margin indicate full marks.

Question 1.		
a)	Find f o g and g o f, where $f(x) = x^3$ and $g(x) = (x^2 + 1)/(x^2 + 2)$ are functions from R to R.	[1 + 1 = 2]
b)	Determine if the following functions are invertible. i) $f: R - \{1/3\} \rightarrow \boldsymbol{R}, \boldsymbol{f(x) = (2x + 7)/(3x - 1)}$ ii) $f: R \rightarrow R, f(x) = x^3 + 1$	[2 x 2 = 4]
Question 2.		
a)	Draw the Venn Diagram of the following sets. i) $(B' \cup A') \cap C$ ii) $((B - C) \cap (A - B)) \cup C$	[1.5×2=3]
b)	Suppose you have a set S = {a, {b, c}, ∅} i Find the power set P(S). ii Find the cardinality of the set P(P(S)). iii Determine S × S.	[3×1=3]
Question 3:		
a)	Prove $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$using logical equivalence laws.	[2]
b)	Construct a truth table for the following compound proposition: $(x \vee (y \leftrightarrow z)) \oplus (\neg x \rightarrow z)$	[2.5]
c)	Write down the converse, contrapositive, and inverse of the following proposition: "He will pass the exam if he studies hard."	[1.5]
Question 4:		
a)	Let $P(x)$ be the statement "x is a football player", $Q(x)$ be the statement "x is physically strong", and $R(x)$ be the statement "x is athletic". Express the following sentences in terms of $P(x), Q(x), R(x)$, quantifiers and logical connectives: (i) There is a football player who is athletic but not physically strong. (ii) Every football player is physically strong or athletic but not both.	[1.5 x 2 = 3]
b)	With brief explanation, determine the truth values of the following propositions. Here, the domain of each variable consists of all real numbers. (i) $\forall x \exists y (y^2 = x)$ (ii) $\exists y \forall x (x^2 + y^2 = x^2)$	[1.5 x 2 = 3]
Question 5:		
a)	Prove the following by using the principle of mathematical induction $\frac{1}{(1 \cdot 2)} + \frac{1}{(2.3)} + \frac{1}{(3.4)} + \ldots.. + \frac{1}{\{n(n+1)\}} = \frac{n}{(n+1)}$ where $n \in Z^+$	[3]
b)	Show that, if xy is even, then x is even or y is even. Here, x and y are integers.	[1.5]
c)	Using Proof by Contraposition, prove that, if n is an integer and 7n + 4 is even, then n is also even.	[1.5]

United International University (UIU)

Dept. of Computer Science & Engineering (CSE)

Mid Exam. : : Trimester: Summer 2020

Course Code: CSE 2213, Course Title: DISCRETE MATHEMATICS

Total Marks: **20** Duration: 1 hour

Answer all the questions. Figures are in the right-hand margin indicate full marks.
"Any examinee found adopting unfair means will be expelled from the trimester / program as per UIU disciplinary rules."

Question 1.		
a)	i) Find the power set $P(S)$ for the set $S = \{0, \{\emptyset\}, \emptyset\}$. ii) Draw the Venn Diagram of the following set. $(A \cap C) \cup (A \cap B)$	[1.5×2=3]
b)	For each of the following "functions" f, determine whether they are Bijection. i) $f: Z \to Z^+, f(x) = \lvert x \rvert + 1$ ii) $f: Z \to Z^+, f(a) = \frac{a^3+1}{a^2+1}$	[2×1=2]
Question 2:		
a)	Prove the following using truth table: (i) $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$	[3]
b)	Given propositions A: X is a good person B: X respects everyone C: X lacks manner Translate the logical expression into a English sentence. $(A \leftrightarrow B) \vee (C \to \neg A)$	[2]
Question 3:		
a)	$P(x)$: x is attentive. $Q(x)$: x does a good result in the examination. Write down the following sentences using the above predicates, appropriate quantifiers and logical connectives: (i) All attentive students do good result in the examination. (ii) Some students do not do a good result though they are attentive.	[1 x 2 = 2]
b)	With brief explanation, determine the truth values of the following propositions. Here, the domain of each variable consists of all real numbers. (i) $\forall x \exists y (x = y^2)$ (ii) $\exists y \forall x (x = y^2)$ (iii) $\neg \forall x (1 - x = x + 1)$	[1 x 3 = 3]
Question 4:		
a)	Prove the following by using the principle of mathematical induction, $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{2}$ whenever n is a nonnegative integer.	[3]
b)	Using proof by contraposition, prove the following: "For all integer n, if $n^2 + 5$ is even, then n is odd."	[2]

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Answer all the questions. Figures are in the right-hand margin indicate full marks.
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Question 1.		
a)	Given that A and B are two sets such that: $A \cap \overline{B} = \{10, 22, 31, 76\}$ $B = \{50, 64, 97, 84\}$ (i) Find out A U B. Order the elements of your set in ascending order. (ii) Given that $A \cap B = \{50, 64, 97\}$, find out |A| (iii) Given that Set C is a single-element set containing the letter 'a', Find out P((A ∩ B) X C)	[1+0.5+1=2.5]
b)	(i) Consider the following function: $f: Z \rightarrow R, f(x)=x^3$ What type of function is this? Explain if this function can have an inverse. (ii) Now consider a different function, g: $g: A \rightarrow B, g(x)=x+1$ $where, A = \{a \in Z^+ \mid a \text{ is even and } a \leq 10\}$ $B = \{b \in Z^+ \mid b \text{ is odd and } b \leq 12\}$ a. State the elements of the domain set, the codomain set and the image set of the function g. b. Find the composition function, $f \circ g$	[1+1+0.5=2.5]
Question 2:		
a)	Write down whether each of the following statements is true or false. Explain the reason of your answer. Domain consists of real numbers. i. $\forall x \forall y (xy < 0 \rightarrow \exists z(z^{xy} > 0))$ ii. $\exists x \forall y (x^y y^x = 1)$ iii. $\forall x \forall y \exists z ((yz)^x = 1)$	[3 x 1 = 3]
b)	Look at the following predicates: $P(x)$: x owns a car. $Q(x)$: x is rich. $R(x, y)$: x drives y's car. Represent the following sentences using the above predicates, appropriate quantifiers and logical connectives. Domain consists of all people. i. There is a rich man who owns a car. ii. A poor man does not own a car. iii. Not all rich man drive their own cars. iv. A man who owns a car is not poor.	[4 x 0.5 = 2]
Question 3:		
a)	Prove that $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology using a sequence of logical equivalences law	[2]
b)	Translate the following sentences into a logical expression. i. I come to class only if there is going to be a CT.	[1 x 3 = 3]

	ii. For you to get an A in this course, it is necessary and sufficient that you do well in this mid-term exam. iii. Your guarantee is good whenever you bought your laptop less than 90 days ago or you didn't damage it physically.	
Question 4:		
a)	Prove the following by using the principle of mathematical induction, n is a positive integer. $1^3 + 2^3 + \ldots\ldots + n^3 = (n(n + 1)/2)^2$	[3]
b)	Using direct proof technique, prove that if x even and y odd, then xy is even".	[2]