United International **University**
School of Science and Engineering
Final Examination Trimester: Spring 2023
Course Title: Coordinate Geometry and Vector Analysis
Course Code: Math 2201 Marks: 40
Total Time: 2 hours

Answer all questions.

1. a) Consider, $\boldsymbol{F(x,y) = e^x siny\, i + e^x cosy\, j}$ [5]
   i) Show that $\boldsymbol{F}$ is a conservative vector field on the entire $\boldsymbol{xy}$ −plane.
   ii) Find the potential function $\boldsymbol{\phi(x,y)}$.Find $\int_{(0,0)}^{\left(1,\frac{\pi}{2}\right)} \boldsymbol{F.dr}$ using **(ii).**

   b) Using Green's theorem find the value of $\oint_c \boldsymbol{F.dr}$
   Where $\boldsymbol{F(x,y) = (2e^{-5x} - y^2)i + (y^3 + 2x^2)j}$ and C is the closed circle with parametric equations $\boldsymbol{x = 4cost, and\ y = 4sint.}$ [5]

2. a) Evaluate $\int_c \boldsymbol{(2x - y)dx - (y - x)dy}$ along the rectangle with vertices $(0,\ 0), (0,3), (3,3)$ and $(3,0)$. [5]
   b) Evaluate the surface integral $\iint_\sigma \boldsymbol{(x + y)\, ds}$; σ is the part of the plane $\boldsymbol{x + y + z = 4}$ that lies in the first octant. [5]

3. a) Find the flux of the vector field $\boldsymbol{F(x,y,z) = 2xi - yj + 2zk}$ across σ, where σ is the portion of the surface $\boldsymbol{z = 9 - x^2 - y^2}$ that lies above the xy − plane and suppose that σ is oriented up. [5]
   b) Using double integral to find the area enclosed by the equations $\boldsymbol{-x + y = -2, x + y = -2}$ and $\boldsymbol{y = 0}$. [5]

4. a) Use cylindrical coordinate systems to evaluate: [5]

$$\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{0}^{16-4x^2-4y^2} 5xdzdxdy$$

   b) Find the volume of the sphere by using spherical coordinate system where the radius of sphere is 2. [5]

Or,

Using triple integral find the volume of the solid bounded by the $\boldsymbol{x^2 + y^2 = 2}$, $\boldsymbol{xy - plane}$ and $\boldsymbol{z = 3}$.