United International University (UIU)

Dept. of Computer Science and Engineering (CSE)

Mid Exam Year: 2023 Trimester: Summer

Course: CSE 2213 Discrete Mathematics

Total Marks: 30, Time: 1 hour 45 minutes

(Any examinee found adopting unfair means will be expelled from the trimester / program as per UIU disciplinary rules)

There are FIVE questions. Answer all of them. Figures in the right-hand margin indicate the full marks.

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1. (a) Consider the statement: "If the user has not entered a valid password but has paid the subscription fee, then the access is granted." [2]
   - i. Determine the contrapositive version of the above statement.
   - ii. Consider the following propositional variables:
     p = "The user enters a valid password"
     q = "Access is granted"
     r = "The user has paid the subscription fee".
     Translate the above given statement into a compound proposition.

   (b) Construct the truth table for the following compound proposition and comment on whether the proposition is tautology, contradiction or neither of them. [2]

$$(x \vee \neg y) \vee (y \rightarrow \neg x)$$

   (c) Using propositional laws prove that: [2]

$$\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$$

2. (a) Consider the following predicates: [1X3=3]

M(x) ≡ x is a musician
R(x) ≡ x enjoys reading
S(x) ≡ x is a student
T(x) ≡ x likes to travel

   Represent the following sentences using these predicates, appropriate quantifiers, and logical connectives. The domain of all the variables is the set of all people.
   - i. Musicians like to travel.
   - ii. Some students do not enjoy reading.
   - iii. Everyone who likes to travel is a student

   (b) Determine whether the following propositions are true or false. Provide justification in favor of your answer. The domain of all the variables is the set of integers. [1X3=3]
   - i. $\exists x \exists y(x + y = 2 \ \wedge \ x - 2y = 8)$
   - ii. $\exists x \exists y(x^2+y^2=12)$
   - iii. $\forall x \exists y(y^2 - x < 100)$

3. (a) Use mathematical induction to prove the following summation formula for all non-negative integer values of n: [3]

$$1^2 + 3^2 + 5^2 + \ldots\ldots\ldots + (2n + 1)^2 = (n+1)(2n+1)(2n+3)/3$$

(b) Prove the following statement using the Proof by Contradiction technique: [3]

"If n is an integer and $n^4 + 7$ is even, then n is odd."

4. (a) Consider the students from the following houses in *Hogwarts* (the universal set) [1+2=3]

Gryffindor = {Oliver, Fred, Harry}
Slytherin = {Malfoy, Flint, Crabbe}
Ravenclaw = {Trelawney, Rowena}
Hufflepuff = {Justin}

i. Find out the power set of *Hufflepuff*
ii. Find the set *Ravenclaw* × *Hufflepuff* and its cardinality

(b) Two new students joined *Hogwarts* and the sorting hat bewitchedly (mistakenly) put them into multiple houses. Now the houses look like following: [1.5X2=3]

Gryffindor = {Oliver, Fred, Harry, **Newt, Isidora**}
Slytherin = {Malfoy, Flint, Crabbe, **Newt**}
Ravenclaw = {Trelawney, Rowena, **Isidora**}
Hufflepuff = {Justin}

i. Find the set: *Hogwarts* − ((*Gryffindor* ∩ *Ravenclaw*) ∪ *Hufflepuff*)
ii. Shade the following set in a Venn Diagram:

$$Hogwarts \cap \overline{Slytherin}$$

5. (a) Find $f \circ g(x)$ and $g \circ f(x)$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from R to R. [2X3=6]
(b) Find the value of $(f+g)(0)$ and $fg(0)$ for the functions $f(x)$ and $g(x)$ in 5a.
(c) Evaluate the following functions to determine if they are bijections or not, considering that both the domain and the codomain consist of Real Numbers:
i. $f(x) = x^5 + 1$
ii. $f(x) = -3x^2 + 7$