Answer all the questions

1. For the function $f(x) = x^3 - 6x^2 + 2$ investigate,   [10]

   (i)   The x −intercept and y −intercept.
   (ii)   Discuss the behavior at the infinity.
   (iii)   The intervals on which $f(x)$ is increasing and decreasing.
   (iv)   The intervals on which $f(x)$ is concave up and concave down.
   (v)   The critical and inflection points.
   (vi)   The relative maximum and minimum values of $f(x)$ by using the 1st and 2nd derivative test.

   Finally, Sketch the graph of $f(x)$ using the above-mentioned information.

2. (a) For the given function $f(x, y) = \cos(xy^2 - 4x)$,   [6]
   (i) Find the slope of the surface $z = f(x, y)$ in the x and y direction at the point $(2, 1)$.
   (ii) Verify mixed second order partial derivatives are same or not?

   (b) Let $w = f(u)$ be a differentiable function and $u = \ln(xy) - x^2 - y^2$.   [2]
   Then, find $\frac{\partial w}{\partial x} - \frac{\partial w}{\partial y}$.

   (c) Using appropriate chain rule find $\frac{\partial w}{\partial \theta}$, where $w = 4x^3 + 4y^3 + z^3$ with the   [2]
   assumptions $x = \rho \sin \theta \cos \emptyset$, $y = \rho \sin \theta \sin \emptyset$, and $z = \rho \cos \theta$.

3. (a) Find the solution of the given differential equations: [7]
   (i) $t^3 y' + 3t^2 y = t^3 - t + 1$, $y(1) = 1$
   (ii) $y' = \frac{y \cos x}{1 + 2y^3}$, $y(0) = 1$

   (b) Determine all the values of r for which the second-order differential   [3]
   equation $t^2 y'' + 8ty' + 12y = 0$ has solutions of the form $t^r; t > 0$.