United International University
Department of Computer Science and Engineering

CSE 2213/CSI 219: Discrete Mathematics
Mid-term Examination : Fall 2023
Total Marks: 30 Time: 1 hour and 45 minutes

Answer all the 5 questions. Numbers to the right of the questions denote their marks.
Any examinee found adopting unfair means will be expelled from the trimester / program as per UIU disciplinary rules.

1. (a) Consider the following propositions: [2]

p: John is smart
q: John is honest

Now using the logical operators formulate the following compound propositions:

i. Either John is smart, or he is not smart but honest
ii. John is honest if and only if he is not both smart and not honest.

(b) Prove $((p \wedge q) \rightarrow r) \rightarrow (\neg r \rightarrow (\neg p \vee \neg q))$ is a tautology using different logical equivalence laws. [2]

(c) Find the truth table of the following expressions to determine whether both are logically equivalent or not: $\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow \neg q$ [2]

2. (a) Consider the following predicates: [1 x 3 = 3]

B(x) : x is Bangladeshi
J(x) : x is Japanese
P(x) : x is polite
F(x, y): x and y are friends

Represent the following sentences using these predicates, appropriate quantifiers, and logical connectives. The domain of all the variables is the set of all people. Hint: A person is Bangladeshi-American means that he/she is both American and Bangladeshi.

i. All Japanese-Bangladeshis are polite.
ii. There exists a friendship between a Japanese and a Bangladeshi.
iii. Some Japanese are impolite.

(b) Determine the truth values of the following propositions. Here, the domain of each variable consists of all real numbers. [1 x 3 = 3]

i. $\forall x \exists y(x^2 + y^2 = 0)$
ii. $\neg\exists x((-x^2 + 2) = (x^2 + 1))$
iii. $\forall x(x^9 < 0)$

3. (a) Suppose you are given the following sets – [1 + 1 = 2]

$$A = \{x \in \mathbb{Z}^+ \mid x \text{ is even and } x \leq 14\}$$
$$B = \{8, 10, 12, 14\}$$

i. Find out the elements of set A.
ii. Determine $A - B$.

(b) Suppose that you have another set $C = \{1, 3\}$. [2 + 2 = 4]

i. Find out $(A - B) \times C$.
ii. Prove that $|(A - B) \times C| = |(A - B)| \cdot |C|$

4. (a) if f(x) = 5x + 7, g(x) = 3x + 5. Find fog(x) and gof(x), considering that both the domain and the co-domain consist of Real Numbers. [2]

(b) Evaluate the following functions to determine if they are one to one, onto or bijections. [2 + 2 = 4]

i. $f : R \rightarrow R, f(x) = -5x + 10$
ii. $g : R^+ \rightarrow R^+, g(x) = 5x^2 - 5$

5. (a) Using proof by contradiction show that, $\sqrt{5}$ is irrational. [4]

(b) Using direct proof show that, if a and b are odd integers then ab is also an odd integer. [2]