United International **University**

School of Science and Engineering

Final Examination Trimester: Spring-2024

Course Title: Fundamental Calculus

Course Code: Math 1151 Marks: 40 Time: 2 Hours

Answer <u>all</u> the questions. Answer all parts of a question together.

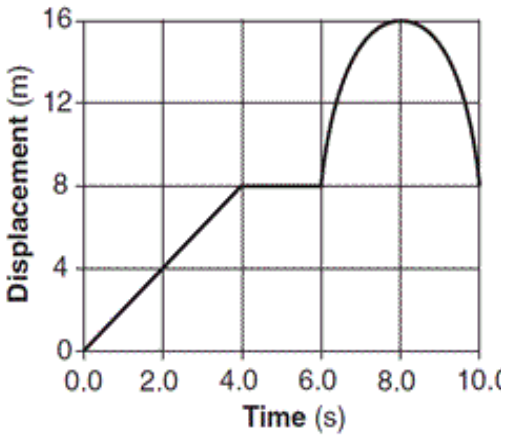
1. **(a)** The following figure represents a position function of a particle at time $\boldsymbol{t}$. **[5]**

(i) **Find** the average velocity over the time interval **[2, 6]**.

(ii) **What** is the instantaneous velocity at $\boldsymbol{t = 8}$. **Explain** the reason.

(iii) **Determine** whether the velocity doesn't exist. **Explain** the reason.

(iv) **Roughly sketch** the velocity graph of the particle.



(b) Consider the function **[5]**

$$f(x) = \begin{cases} 3 - x^2\,; & x < 1 \\ 4 - 2x\,; & x \geq 1 \end{cases}$$

(i) **Sketch** the graph of $\boldsymbol{f(x)}$.

(ii) **Determine** whether the function $\boldsymbol{f(x)}$ is continuous and differentiable at $\boldsymbol{x = 1}$.

2. **(a)** Consider, the function $\boldsymbol{f(x) = x^2 - 2}$.

(i) **Find** the slope of tangent line to the graph of $\boldsymbol{f(x)}$ at the point $\boldsymbol{x = 0}$. **[1]**

(ii) **Find** the equation tangent line to the graph of $\boldsymbol{f(x)}$ at the point $\boldsymbol{x = 0}$. **[2]**

(iii) **Draw** the graph of $\boldsymbol{f(x)}$ together with the tangent line from **(ii)** in the same axes. **[2]**

(b) **(i)** If $\boldsymbol{s = \cot w}$ and $\boldsymbol{w = t^3 - \frac{2}{\sqrt{t}} + 3}$, then find $\boldsymbol{\frac{ds}{dt}}$. **[2.5]**

(ii) If $\boldsymbol{y = \sec^3(2 - 3x^2 \cos x)}$, then find $\boldsymbol{\frac{dy}{dx}}$. **[2.5]**

Please Turn Over

3. **(a)** **Evaluate** the following integrals:

(i) $\int \frac{t^3 - t^2 \cos t - 2t + 3}{t^2} dt$ **[2]**

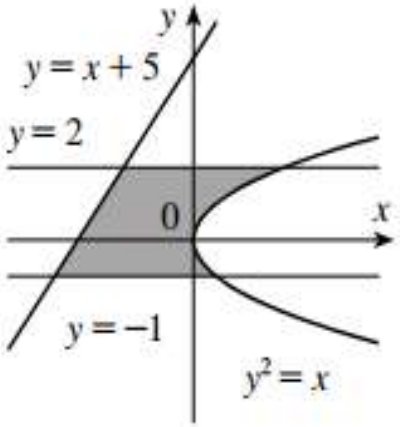
(ii) $\int \frac{x^3}{\sqrt{5 - 2x^4}} dx$ **[2]**

(iii) $\int \frac{x}{1 + x^4} dx$ **[2]**

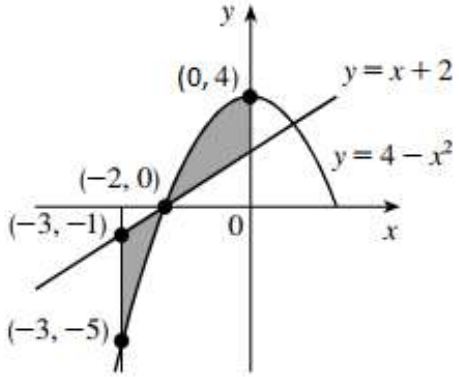
(b) **Evaluate** $\int_{-2}^{4} f(x) dx$, where $f(x) = \begin{cases} 2x + 1; & x \leq 1 \\ 5 - 2x; & x > 1 \end{cases}$ **[4]**

4. **(a)** Use a suitable method to **evaluate** the area of the following shaded regions: **[7]**

(i)



(ii)



(b) **Evaluate** the integral: $\int_0^{\pi} 2t \sin 2t \, dt$ **[3]**