United International **University**
School of Science and Engineering
Mid- term Examination Trimester: Spring-2024
Course Title: Calculus and Linear Algebra
Course Code: MATH 2183 Marks: 30 Time: 1 Hour 30 Minutes

Answer all the questions

1.	For the function $\boldsymbol{f(x) = x^3 - 4x^2 + 8}$ find, **(i)** It's critical, and inflection points. **(ii)** The intervals on which $\boldsymbol{f(x)}$ is increasing and decreasing. **(iii)** The intervals on which $\boldsymbol{f(x)}$ is concave up and down. **(iv)** It's relative maximum and minimum by using **1st** and **2nd** derivative test. **(v)** It's $\boldsymbol{x}$-intercepts and $\boldsymbol{y}$-intercepts. Finally draw the graph of $\boldsymbol{f(x)}$ using the above information.	**[10]**
2.	**(a)** For the given function $\boldsymbol{f(x,y) = sin(xy^2 - 4x)}$, **(i)** Find the slope of the surface $\boldsymbol{z = f(x,y)}$in the $\boldsymbol{x}$ and $\boldsymbol{y}$ direction at the point **(2,1)** **(ii)** Verify mixed second order partial derivatives [$\boldsymbol{f_{xy}(x,y) = f_{yx}(x,y)}$] are the same or not? **(b)** Using the chain rule find $\frac{\partial T}{\partial y}$, where $\boldsymbol{T = p^3qr + \sqrt{pqr}}$, $\boldsymbol{p = sin(xy)}$, $\boldsymbol{q = ln(x^2 + y^3)}$, $\boldsymbol{r = y + 2x}$ **(c)** Use implicit differentiation to find $\frac{dy}{dx}$, where $\boldsymbol{x^2 - y^3 + 5xy = 6}$	**[6]** **[2]** **[2]**
3.	Find the solution of the given differential equations **i)** $\boldsymbol{x\frac{dy}{dx} + 2y = x^2 - x + 1}$; $\boldsymbol{y(1) = \frac{1}{2}}$ **ii)** $\boldsymbol{y' - 6y = 4}$ **iii)** $\boldsymbol{y' = \frac{(1+y^2)\,sin2t}{2y}}$	**[5]** **[2.5]** **[2.5]**