United International University

School of Science and Engineering

Final Examination Trimester: Summer-2024

Course Title: Calculus and Linear Algebra

Course Code: MATH 2183 Marks: 50 Time: 2 Hours

Answer all the questions

1. a) Given that,

$$x + 2y - z = 1$$
$$-x + y - z = 0$$
$$3x + y - z = 2$$

i) Write the above system of linear equations in the form $AX = B$, where A, X and B are matrices. [1]

ii) Find the inverse of A by using matrix inversion algorithm and hence solve the above system of linear equations. [7]

iii) Find the unknown variables z by using Cramer's rule. [2]

b) Find the infinite solution of the following system of linear equations [5]

$$2x_1 + 2x_2 + x_3 - x_4 + 2x_5 = 0$$
$$-x_1 - x_2 + 3x_3 - x_4 - 4x_5 = 0$$
$$x_1 + 2x_2 - 5x_3 - x_4 + 2x_5 = 0$$

2. a) Solve $y'' + y' = e^{-2x} + cos\,2x - ln2 + 4^x + e^{-x}sin5x$. [8]

b) Solve the following second order ordinary differential equation [7]

i) $y'' - y' + 5y = 0$

ii) $2y'' - 11y' + 12y = 0 \qquad y(0) = 5 \qquad y'(0) = 15$

3. a) Find eigenvalues and eigenvectors of the Matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix}$. Also sketch the eigenspace in xy −coordinates. [8]

[6]

b)

Given $A = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}$, $B = [1 \quad -2 \quad 1 \quad 3]$

i) Find $\mathbf{det}(A)$ and $det(B)$

ii) Evaluate $AB + 5$.

iii) Find x, Such that $tr(AB) = 2x^2 - 3$

c) Solve $(2xy + y^2 + 1)dx + (x^2 + 2xy + 1)dy = 0$ [6]