United International University
Department of Computer Science and Engineering
CSE 2213/CSI 219 Discrete Mathematics,
Final Exam, Summer 2021
Total Marks: 25, Time: 1 hour 15 minutes

Answer all the questions. Figures are in the right-hand margin indicate full marks. Any examinee found adopting unfair means will be expelled from the trimester / program as per UIU disciplinary rules.

1. (a) In a road network graph, vertices represent intersections and edges represent roads. If we want to model both one-way and two-way roads, what kind of graph that will be? Why? [1]

   (b) Suppose a graph has 11 vertices and 19 edges. Each of the odd-degree vertices has degree 3 and each of the even-degree vertices has degree 4. Find the numbers of the odd-degree and the even-degree vertices. [2]

   (c) Determine whether the following graph is a bipartite graph. [2]

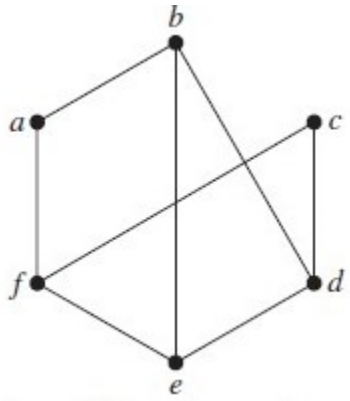

Figure for question 1 (c)

2. (a) Explain why the graph below is not a strongly connected graph. Also, explain why it is weakly connected. Determine the strongly connected components from the graph. [1.5]

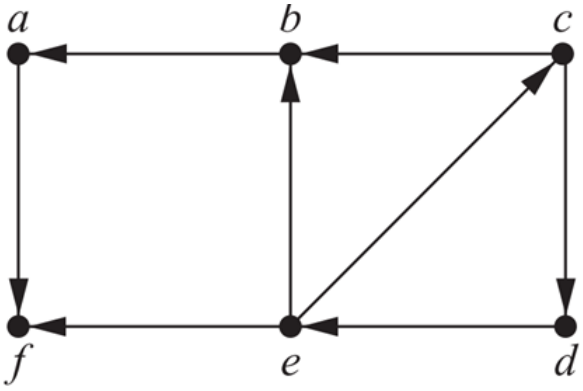

Figure for question 2 (a)

   (b) Draw a directed weakly connected graph of your choice. [1.5]

(c) The following matrix represents the adjacency matrix of a directed graph consisting of six vertices - a, b, c, d, e and f. [2]

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Draw the graph from the adjacency matrix.

3. (a) Draw a tree using the following information: [$0.25 \times 12 = 3$]

   i The parent of h is g.
   ii The ancestors of d are (from top to bottom) g, f and l.
   iii e, k and l are siblings.
   iv The descendants of h are a, b, i and j.
   v a, b, c and d are leaves at level 3.
   vi a and b has no other siblings.
   vii e, j and k are leaves.
   viii The tree is balanced.

   (b) A full m-ary tree has 136 vertices. Among them, 109 are leaves. Calculate the values of m, and the number of edges in the tree. [2]

4. (a) Considering the dictionary order, construct a Binary Search Tree from the strings given below. You must follow the order in which the strings are given. [2]

Binary, Search, Tree, Is, A, Very, Efficient, Data, Structure, For, Searching

   (b) Show the result of post-order traversal on the tree you constructed. [2]

   (c) Is the tree you constructed a balanced tree? Explain your answer in one sentence. [1]

5. (a) In a game of UNO, there are cards of 4 colors- red, green, blue and yellow. There are 25 cards for each color (there are some special cards, but we will not be considering them now). A player is dealt 7 cards in a round. [1 + 1 = 2]

   i Explain why there is no guarantee that a player will get at least 2 red cards.
   ii How many cards should be picked to ensure that he gets at least 2 red cards?

   (b) A coin is tossed 6 times such that every time it can land either on heads or tails. How many possible outcomes contain an odd number of heads? [1]

   (c) There are n_1 computer science courses and n_2 computer engineering courses available at a certain university. A student has to select r_1 courses from computer science courses and r_2 courses from computer engineering courses. If the order of the courses taken are important, then how many ways can a student complete the courses? [2]