



# United International University

## Department of Computer Science and Engineering

CSE 2213/CSI 219: Discrete Mathematics  
Midterm Examination : Spring 2024  
Total Marks: 30      Time: 1 hour 30 minutes

Answer all the 5 questions. Numbers to the right of the questions denote their marks.

Any examinee found adopting unfair means will be expelled from the trimester / program as per UIU disciplinary rules.

1. (a) Write down the contrapositive, converse and inverse of the following proposition: [1\*3=3]

"Your hard work is sufficient for your success."

- (b) Using logical equivalence laws, prove that the following expressions are logically equivalent: [3]

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$

2. (a) Consider the following predicates: [1\*3=3]

$C(x)$ : "x is a child"

$R(x)$ : "x is rational"

$P(x)$ : "x has a phone"

$K(x,y)$ : "x knows y"

Let the domain consist of all people in the world. Express each of the following statements using quantifiers, logical connectives, and the propositional functions given above.

- Children are irrational.
  - Nobody knows everybody.
  - Everyone has a phone or knows someone who has a phone.
- (b) Explain with reasoning whether the following propositions are true or false. The domain of all the variables is the set of real numbers. [1\*3=3]
- $\forall x \exists y \exists z (z = x * y)$
  - $(\exists x (x^2 = 2)) \rightarrow (\forall y (y^2 \geq y))$
  - $\exists x \forall y \forall z (xyz = 0)$

3. Suppose you are given the following sets:

$$S1 = \{x \in \mathbb{N} \mid x \text{ is a prime number and } x < 20\}$$

$$S2 = \{x \in \mathbb{Z}^+ \mid x \geq 5 \text{ and } x < 13\}$$

$$S3 = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$$

- (a) Based on the above sets S1, S2, and S3 - [1\*3=3]

- Find out the truth set of S1.
- Find out the truth set of S2.
- Express S3 in Set Builder Method.

- (b) Based on the determined elements of set S1, S2, and S3- [1.5\*2=3]

- Find out  $P((S2 \cap S3) - S1)$
- Find out  $(S1 \cap S3) - ((S1 \cap S2) \cup (S2 \cap S3))$

4. (a) Determine whether the following functions are injections or surjections or both. Give proper reasoning to justify your answer. [1\*2=2]

i.  $f: \mathbb{Z}^+ \rightarrow \mathbb{R}, f(x) = \ln(|x^2 + 3|)$       ii.  $f: \mathbb{Z} \rightarrow \mathbb{Z}^+, f(x) = \lceil \frac{x^2}{2} \rceil + 7$

- (b) Find out if the function f is invertible or not,  $f: \mathbb{R}^+ \rightarrow \mathbb{R}, f(p) = \frac{p^2+4}{p^3+2}$  [2]

- (c) Is the function  $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+, f(x) = \sqrt[3]{x^2}$  bijective? Explain with proper numerical examples. [2]

5. (a) Using proof by contraposition, prove that "For all integer n, if  $n^2 + 5$  is even, then n is odd" [4]

- (b) Using direct proof technique, prove that "If x is even and y is odd, then xy is even" [2]