UIU QUESTION BANK

MID-TERM QUESTION SOLUTIONS

PHYSICS

PHY 2105

SOLUTION BY

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*UPDATED TILL **SPRING 2024***

nurulalamador.github.io/**UIUQuestionBank**

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Spring 2024

1. a) Why we observe damped harmonic motion in RLC circuit?

Solution:

We know there are resistors (R) in RLC circuits. The resistor dissipates energy as heat, reducing the energy transfer between the capacitor and inductor. This energy loss causes the oscillations to gradually decrease in amplitude, which is characteristic of damped harmonic motion.

Therefore, we can say that resistors work as a damping factor in RLC circuits, and for this reason, we observe damped harmonic motion in RLC circuits.

1. b) **The equation of displacement of a simple harmonic oscillator is $x = A\cos(\omega t + \pi)$. Plot displacement vs. time and acceleration vs. time graphically. What is phase difference between displacement and acceleration?**

Solution:

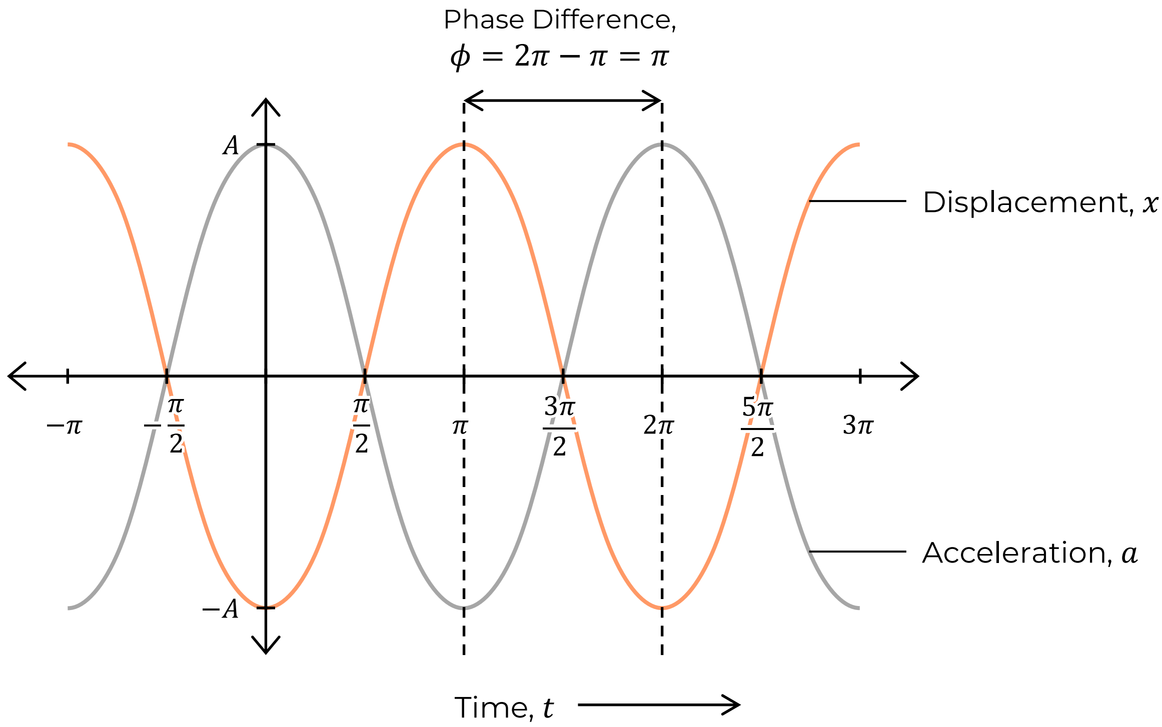
Here,

Displacement, $x = A\cos(\omega t + \pi)$

$\therefore$ Velocity, $v = \dfrac{dx}{dt} = -A\omega \sin(\omega t + \pi)$

$\therefore$ Acceleration, $a = \dfrac{dv}{dt} = -A\omega^2 \cos(\omega t + \pi)$

Displacement vs acceleration graph has been plotted below:



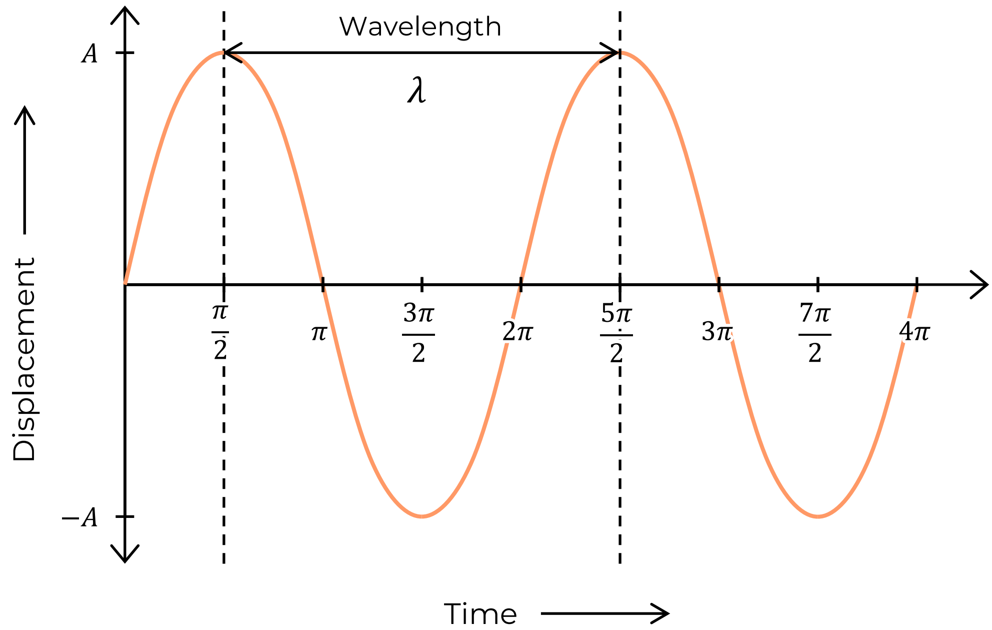
From the graph,

We can see the phase difference between displacement and acceleration is π.

1. c) Draw a transverse wave and show the wavelength on the wave.

Solution:

The transverse wave has been drawn below:



The wavelength has been shown on the graph.

2. a) Consider a mass-spring system oscillating in SHM and where the equation of displacement is:

$$y = 7\sin\left(8t - \frac{\pi}{4}\right)$$

If the block has mass m = 2kg, calculate:

i) time period of the oscillation

ii) the velocity at t = 0.3 sec

Consider all the units in S.I. unit system.

Solution:

Here,

$y = 7\sin\left(8t - \frac{\pi}{4}\right)$

$A = 7\ m$

$\omega = 8\ rad\ s^{-1}$

$T = ?$

$v(0.3) = ?$

i) We know,

Time period,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{8} = 0.785\ s$$

∴ Time period of the oscillation is $0.785\ s$.

ii) We know,

Velocity,

$$v = \frac{dy}{dt} = \frac{d}{dt}\left[7\sin\left(8t - \frac{\pi}{4}\right)\right]$$

$$= 7 \times 8\cos\left(8t - \frac{\pi}{4}\right)$$

$$= 56\cos\left(8t - \frac{\pi}{4}\right)$$

∴ Velocity at $t = 0.3\ s$,

$$\begin{aligned} v(0.3) &= 56\cos\left(8 \times 0.3 - \frac{\pi}{4}\right) \\ &= -2.45\ m\ s^{-1} \end{aligned}$$

∴ Velocity at $t = 0.3\ s$ is $-2.45\ m\ s^{-1}$.

2. b) A block attached to a spring is suspended vertically. If the block is pushed 7 cm upward from the equilibrium position and released at t = 0. The mass of the block is 5 kg and the spring constant is k = 22 N/m.

i) Calculate the potential energy at x = 3 cm.
ii) Calculate the kinetic energy at the same position.

Solution:

i) We know,
Potential energy,

$$\begin{aligned} P.E &= \frac{1}{2}kx^2 \\ &= \frac{1}{2} \times 22 \times (0.03)^2 \\ &= 0.0099\ J \end{aligned}$$

∴ Potential energy is $0.0099\ J$.

Here,
$A = 7\ cm = 0.07\ m$
$m = 5\ kg$
$k = 22\ N\ m^{-1}$
$x = 3\ cm = 0.03\ m$
$P.E = ?$
$K.E = ?$

ii) Velocity at x,

$$\begin{aligned} v &= \pm\omega\sqrt{A^2 - x^2} \\ \text{or, } v^2 &= \omega^2(A^2 - x^2) \\ \text{or, } v^2 &= \frac{k}{m}(A^2 - x^2) \\ \text{or, } v^2 &= \frac{22}{5} \times [(0.07)^2 - (0.03)^2] \\ \therefore v^2 &= 0.0176\ m\ s^{-1} \end{aligned}$$

∴ Kinetic energy at x,

$$\begin{aligned} K.E &= \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 5 \times 0.0176 \\ &= 0.044\ J \end{aligned}$$

∴ Kinetic energy is $0.044\ J$.

3. a) John constructed and RLC circuit with the value, C = 0.009 μF, L = 0.5 mH and R = 200Ω respectively.

i) Whether the circuit is oscillatory, calculate frequency of oscillation of the RLC circuit.
ii) What will be the value of resistance R if he wants to produce critical damping?

Solution:

i) Here,

$$\frac{1}{LC} = \frac{1}{0.009 \times 10^{-6} \times 0.5 \times 10^{-3}}$$
$$= 2.2222 \times 10^{11}$$

Now,

$$\frac{R^2}{4L^2} = \frac{(200)^2}{4 \times (0.5 \times 10^{-3})^2}$$
$$= 4 \times 10^{10}$$

Since $\frac{1}{LC} > \frac{R^2}{4L^2}$, therefore the circuit is oscillatory.

Here,

$$\omega_1 = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$
$$= \sqrt{2.2222 \times 10^{11} - 4 \times 10^{10}}$$
$$= 426874.95\ rad\ s^{-1}$$

We know,
Frequency,

$$f = \frac{\omega_1}{2\pi} = \frac{426874.95}{2\pi} = 67939.0995\ Hz$$

Here,
$C = 0.009\ \mu F$
$\quad = 0.009 \times 10^{-6}\ F$
$L = 0.5\ mH$
$\quad = 0.5 \times 10^{-3}\ H$
$R = 200\Omega$
$f = ?$

∴ Frequency of oscillation is $67939.0995\ Hz$.

ii) Let, R resistance will need to produce critical damping.

Now ,

$$\frac{R^2}{4L^2} = \frac{1}{LC}$$
$$\text{or, } R^2 = \frac{1}{LC} \times 4L^2$$
$$\text{or, } R = \sqrt{2.2222 \times 10^{11} \times 4(0.5 \times 10^{-3})^2}$$
$$\therefore R = 471.40\Omega$$

∴ Resistance $R = 471.40\Omega$ will produce critical damping.

3. b) For a damped oscillator, m = 0.30 kg, k = 19.6 N/m, and b = 0.00086 kg/s. The oscillator is released at t = 0 and the amplitude is 10 cm.

i) Calculate the frequency of oscillations of the oscillator.

ii) How long does it take for the amplitude of the damped oscillator to drop to one half of its initial value?

Solution:

i) Here,

$$\omega_1 = \sqrt{\omega^2 - \frac{\gamma^2}{4}}$$

$$= \sqrt{\frac{k}{m} - \frac{\left(\frac{b}{m}\right)^2}{4}}$$

$$= \sqrt{\frac{19.6}{0.30} - \frac{\left(\frac{0.00086}{0.30}\right)^2}{4}}$$

$$= 8.0829\ rad\ s^{-1}$$

We know,
Frequency,

$$f = \frac{\omega_1}{2\pi} = \frac{8.0829}{2\pi} = 1.286\ Hz$$

∴ Frequency of oscillation is $1.286\ Hz$.

ii) Let, amplitude will drop to one half of its initial value after t time.

Now ,

$$\frac{A_0}{2} = A_0 e^{-\frac{\gamma}{2}t}$$

$$\text{or, } \frac{1}{2} = e^{-\frac{\frac{b}{m}}{2}t}$$

$$\text{or, } \ln\left(\frac{1}{2}\right) = \ln(e^{-\frac{b}{2m}t})$$

$$\text{or, } -0.693 = -\frac{b}{2m}t$$

$$\text{or, } -0.693 = -\frac{0.00086}{2 \times 0.30}t$$

$$\text{or, } t = -0.693 \times -\frac{2 \times 0.30}{0.00086}$$

$$\therefore t = 483.488\ s$$

∴ The amplitude will drop to one half of its initial value after $483.488\ s$.

Here,
$m = 0.30\ kg$
$k = 19.6\ N\ m^{-1}$
$b = 0.00086\ kg\ s^{-1}$
$A = 10\ cm = 0.1\ m$
$f = ?$
$t = ?$

4. a) Show that, for a simple pendulum in SHM, $\frac{d^2\theta}{dt^2} + g\frac{\theta}{L} = 0$

Solution:

A simple pendulum consists of a particle of mass m, attached to a frictionless point by a cable of length L and negligible mass.

From the figure,
Restoring force for simple pendulum,

$$F = -mg\sin\theta$$

If the angle θ is very small $\sin\theta$ is very nearly equal to θ.
Therefore,

$$F = -mg\theta \text{ ...... (i)}$$

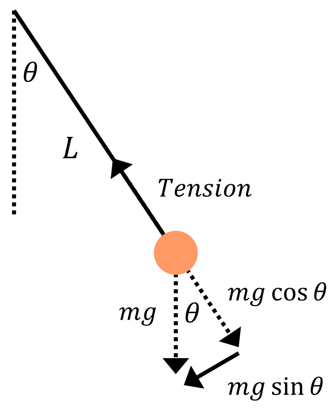


Fig: Simple Pendulum

Here, the displacement along the arc is $x = L\theta$

∴ Acceleration,

$$a = \frac{d^2x}{dt^2} = \frac{d^2}{dt^2}(L\theta) = L\frac{d\theta}{dt^2}$$

From equation (i),

$$F = -mg\theta$$

$$\text{or, } ma = -mg\theta \quad [\because F = ma]$$

$$\text{or, } mL\frac{d\theta}{dt^2} = -mg\theta$$

$$\text{or, } \frac{d\theta}{dt^2} = -\frac{mg\theta}{mL}$$

$$\text{or, } \frac{d\theta}{dt^2} = -g\frac{\theta}{L}$$

$$\therefore \frac{d\theta}{dt^2} + g\frac{\theta}{L} = 0 \quad \text{(Showed)}$$

4. b) For a mass-spring system oscillating in SHM, the equation of displacement is $x = A\sin\omega t$. Show that total energy of the oscillator is, $E = \frac{1}{2}kA^2$. Plot energy vs. displacement graph.

Solution:

Given,

Displacement, $x = A\sin\omega t$

∴ Velocity, $v = \frac{dx}{dt} = A\omega\cos\omega t$

Now,
Potential energy,

$$P.E = \frac{1}{2}kx^2$$
$$= \frac{1}{2}k(A\sin\omega t)^2$$
$$= \frac{1}{2}kA^2\sin^2\omega t$$

Kinetic energy,

$$K.E = \frac{1}{2}mv^2$$
$$= \frac{1}{2}m(A\omega\cos\omega t)^2$$

$$= \frac{1}{2} m A^2 \omega^2 \cos^2 \omega t$$
$$= \frac{1}{2} k A^2 \cos^2 \omega t \quad [\because k = m\omega^2]$$

We know,
Total energy,
$$E = K.E + P.E$$
$$= \frac{1}{2} k A^2 \cos^2 \omega t + \frac{1}{2} k A^2 \sin^2 \omega t$$
$$= \frac{1}{2} k A^2 (\cos^2 \omega t + \sin^2 \omega t)$$
$$= \frac{1}{2} k A^2 \quad [\because \cos^2 \theta + \sin^2 \theta = 1]$$
$$\therefore E = \frac{1}{2} k A^2 \quad \text{(Showed)}$$

5. a) Show that, the equation of displacement of particles of a medium for a progressive wave is $y = A \sin \frac{2\pi}{\lambda}(vt - x)$. Calculate the value of $\frac{d^2y}{dt^2}$. Here the symbols have their usual meanings.

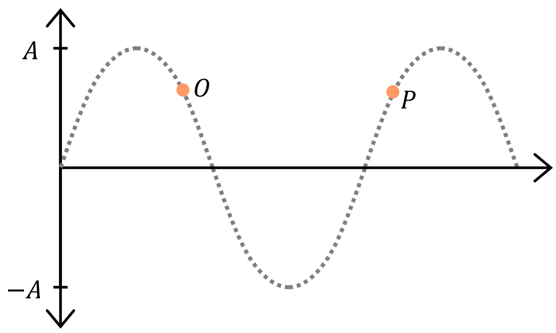
Solution:

Consider a particle P at a distance x from particle O on its right. Let the wave travel with a velocity v from left to right.

Now, Displacement of particle P is,

$$y = A \sin(\omega t - \phi) \text{ ...... (i)}$$

Where ϕ is the phase difference between the particles O and P.



We know that a path difference of ϕ corresponds to phase difference of 2π radians. Hence a path difference of x corresponds to a phase difference of,

$$\phi = \frac{2\pi}{\lambda} x$$

Now, From equation (i),

$$y = A \sin(\omega t - \phi)$$
$$\text{or, } y = A \sin\left(\omega t - \frac{2\pi}{\lambda} x\right)$$
$$\text{or, } y = A \sin\left(\frac{2\pi}{T}\, t - \frac{2\pi}{\lambda} x\right) \quad [\because \omega = 2\pi]$$
$$\text{or, } y = A \sin 2\pi \left(\frac{1}{T}\, t - \frac{1}{\lambda} x\right)$$
$$\text{or, } y = A \sin 2\pi \left(\frac{v}{\lambda} t - \frac{1}{\lambda} x\right) \quad \left[\because v = \frac{\lambda}{T}\ or,\ T = \frac{\lambda}{v}\right]$$
$$\therefore y = A \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{(Showed)}$$

Now,

$$\frac{d^2y}{dt^2} = \frac{d^2}{dt^2}\left(A\sin\frac{2\pi}{\lambda}(vt-x)\right)$$
$$= \frac{d}{dt}\left(A\frac{2\pi}{\lambda}v\cos\frac{2\pi}{\lambda}(vt-x)\right)$$
$$\therefore \frac{d^2y}{dt^2} = -A\frac{4\pi^2}{\lambda^2}v^2\sin\frac{2\pi}{\lambda}(vt-x)$$

5. b) Derive the differential equation of a mass-spring system oscillating in DHM. Write the conditions of three types of damped harmonic motion and graphically represent them by plotting displacement vs. time graphs.

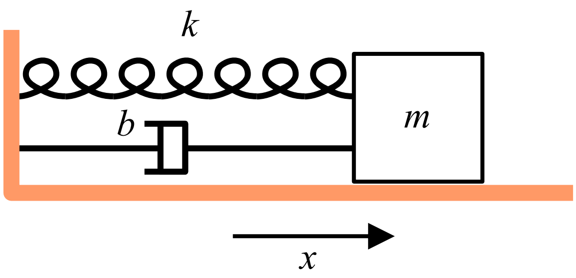
Solution:

We know,

From Hooks Law,
Restoring force,

$$\vec{F}_R \propto -\vec{x}$$
$$\text{or, } F_R = -kx$$

Here, k is spring constant.



In damped mass-spring system,
Damping force,

$$F' = -bv$$

∴ For horizontal force on the mass,

$$F_x = F_R + F'$$
$$\text{or, } ma = -kx - bv$$
$$\text{or, } m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$
$$\text{or, } \frac{d^2x}{dt^2} = -\frac{k}{m}x - \frac{b}{m}\frac{dx}{dt}$$
$$\text{or, } \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$
$$\text{or, } \frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega^2x = 0$$

This is the differential equation for DHM in spring-mass system,

Where, $\gamma = \frac{b}{m}$ and $\omega^2 = \frac{k}{m}$

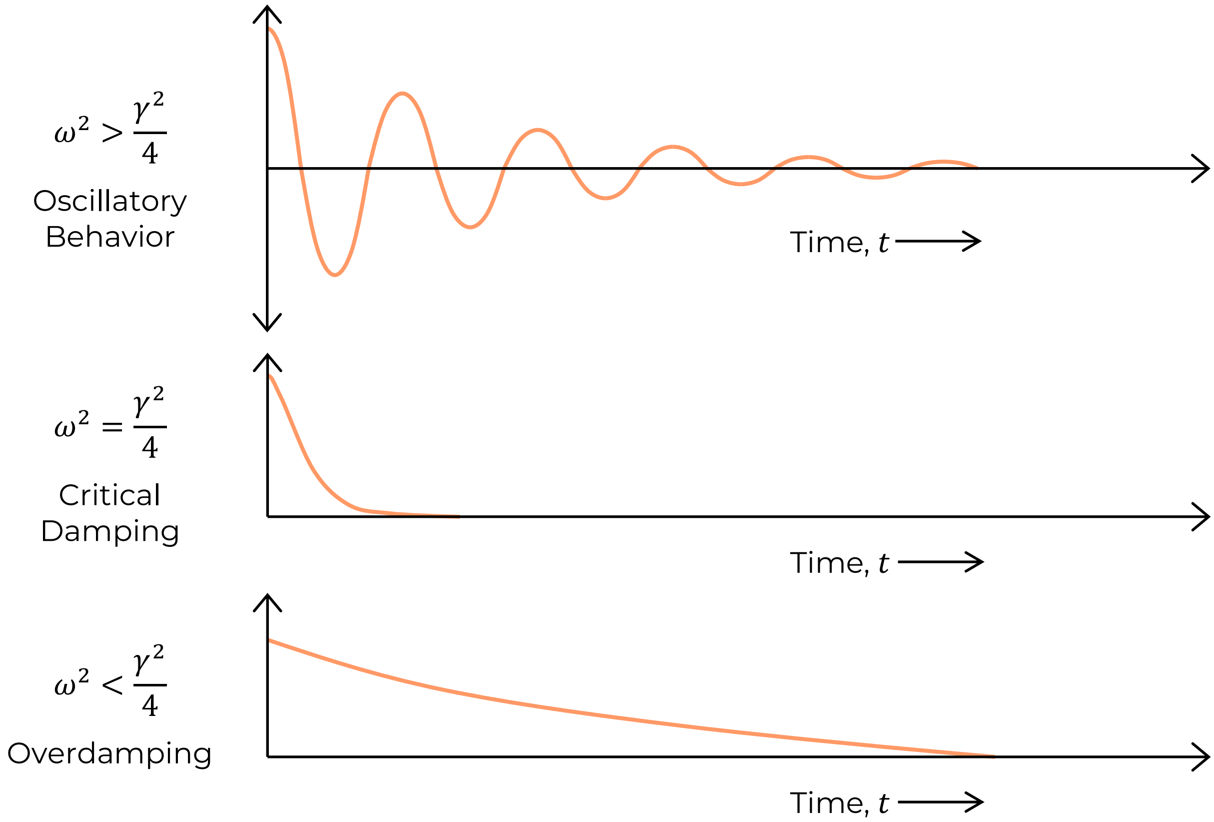
There are three types of conditions of damped harmonic motions. These are:

i) $\omega^2 < \frac{\gamma^2}{4}$: Overdamping

ii) $\omega^2 = \frac{\gamma^2}{4}$: Critical Damping

iii) $\omega^2 > \frac{\gamma^2}{4}$: Oscillatory Behavior

The graphical representation of these conditions are plotted below:



Fall 2023

1. a) **Why do we observe Damped Harmonic Motion (DHM) in a mass-spring system in our real life instead of Simple Harmonic Motion?**

Solution:

We see Dampened Harmonic Motion (DHM) in our real-life mass-spring systems because of various kinds of damping factors, including friction and air resistance. These damping factors oppose the motion, converting the system's mechanical energy into thermal energy (heat). This energy loss causes the oscillations to gradually decrease in amplitude, which is characteristic of Damped Harmonic Motion, unlike Simple Harmonic Motion (SHM).

In Simple Harmonic Motion, there would not be any energy loss, and the amplitude would remain constant. We cannot see Simple Harmonic Motion in real life because of friction, air resistance, and other damping factors.

1. b) **What type of change in current flow will be observed if we replace the resistor of an RLC circuit with a conducting wire? Explain briefly. (Consider the wire that has approximate no resistance)**

Solution:

Normally, the resistor works as a damping factor in an RLC circuit, which produces a damping harmonic motion (DHM) in the RLC circuit. The resistor dissipates energy as heat, reducing the current's amplitude over time, which is characteristic of damped harmonic motion.

If we replace the resistor with a conducting wire, our circuit will have approximately zero resistance, and there will be no energy loss due to heat. The energy will keep oscillating between the capacitor and inductor, resulting in a continuous back-and-forth current flow with constant amplitude, which is characteristic of simple harmonic motion (SHM).

Therefore, if we replace the resistor with a conducting wire, our RLC circuit will become an LC circuit, and it will produce simple harmonic motion instead of damped harmonic motion.

1. c) **The equation of displacement of a simple harmonic oscillator is $x = A\cos\left(\omega t - \frac{\pi}{4}\right)$. Graphically represent the displacement and velocity of the oscillator.**

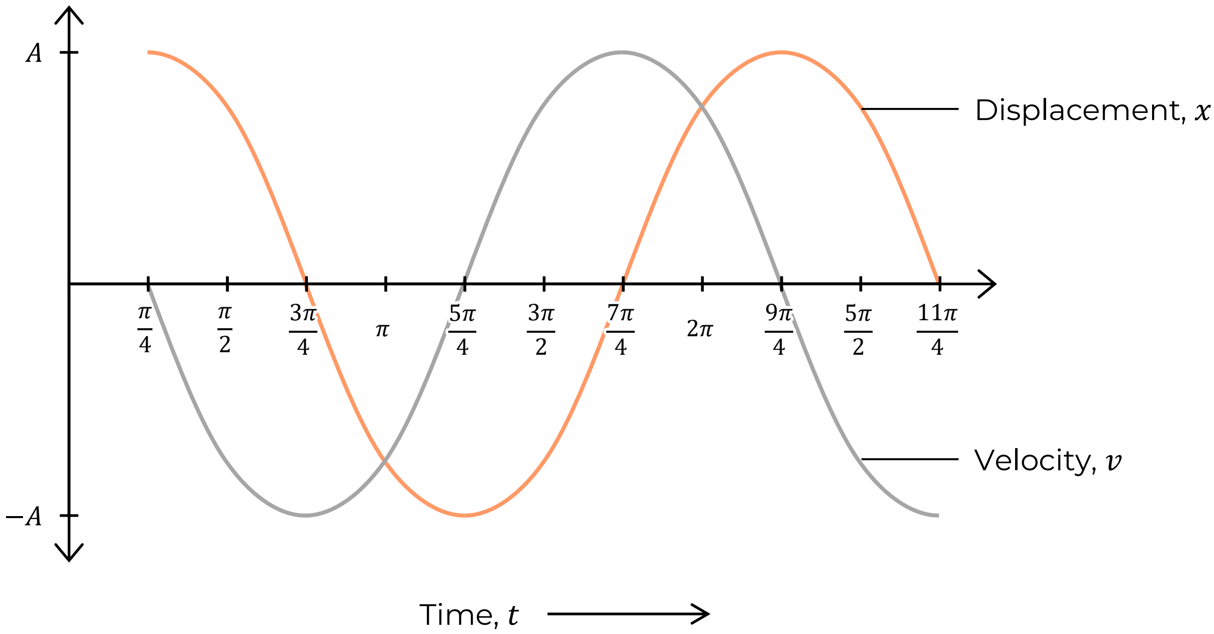
Solution:

Here,

Displacement, $x = A\cos\left(\omega t - \dfrac{\pi}{4}\right)$

$\therefore$ Velocity, $v = \dfrac{dx}{dt} = -A\omega \sin\left(\omega t - \dfrac{\pi}{4}\right)$

The displacement and velocity have been graphically represented below:



2. a) Suppose a spring block-system starts moving from the equilibrium as we apply force on it. The block has mass m = 6.4 kg and is designed to oscillate with a angular frequency ω = 56 rads^{-1} with amplitude 15 cm. Calculate:

i) the kinetic energy at x = 14 cm from the equilibrium point,

ii) mathematically calculate the position where the kinetic energy is 0.

Solution:

i) We know,

Kinetic energy,

$$K.E = \frac{1}{2}mv^2$$
$$= \frac{1}{2}m\omega^2(A^2 - x^2)$$
$$= \frac{1}{2} \times 6.4 \times (56)^2 \times [(0.15)^2 - (0.14)^2]$$
$$= 28.102\, J$$

Here,
$m = 6.4\, kg$
$\omega = 56\, rad\, s^{-1}$
$A = 15\, cm = 0.15\, m$
$x = 14\, cm = 0.14\, m$
$K.E = ?$

∴ Kinetic energy at $x = 14\, cm$ is $28.102\, J$.

ii) Let, kinetic energy will be 0 at position x.

Now,

$$K.E = 0$$
$$\text{or, } \frac{1}{2}mv^2 = 0$$
$$\text{or, } \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = 0$$
$$\text{or, } A^2 - x^2 = 0$$
$$\text{or, } x^2 = A^2$$
$$\therefore x = A$$

∴ Kinetic energy will be 0 when our displacement will be equal to amplitude.

2. b) A 3 kg block is attached to a spring and the spring constant is k = 19.6 N/m. The block is held 6 cm from equilibrium the released at t = 0.

i) Write an equation for x vs. time.
ii) Calculate the velocity at t = 3 and acceleration at t = 0.5 s.

Solution:

i) Here,
Angular frequency,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{19.6}{3}} = 2.556 \text{ rad s}^{-1}$$

According to question,
At $t = 0$,

$$0.06 = 0.06\cos(\omega \times 0 + \phi)$$
$$\text{or, } 1 = \cos(0 + \phi)$$
$$\text{or, } \cos^{-1}(1) = \phi$$
$$\therefore \phi = 0$$

∴ Equation for x vs t:

$$x(t) = 0.06\cos(2.556t)$$

ii) We know,
Velocity, $v(t) = -A\omega\sin(\omega t + \phi)$
Acceleration, $a(t) = -A\omega^2\cos(\omega t + \phi)$

Here,
$m = 3\ kg$
$k = 19.6\ N\ m^{-1}$
$A = 6\ cm = 0.06\ m$
$v(3) = ?$
$a(0.5) = ?$

∴ Velocity at $t = 3\ s$,

$$v(3) = -0.06 \times 2.556 \times \sin(2.556 \times 3)$$
$$= -0.15\ m\ s^{-1}$$

∴ Acceleration at $t = 0.5\ s$,

$$a(0.5) = -0.06 \times (2.556)^2 \times cos(2.556 \times 0.5)$$
$$= -0.113\ m\ s^{-2}$$

2. c) A particle with mass 50 g executes simple harmonic motion given by the equation $y = 3\sin\left(10t - \frac{\pi}{4}\right)$. Calculate the

i) velocity and acceleration at t = 5 s
ii) total energy at t = 3 s.

Solution:

i) Given,
Displacement, $y = \sin\left(10t - \frac{\pi}{4}\right)$

$$\therefore \text{Velocity, } v = \frac{dy}{dt} = \frac{d}{dt}\left[\sin\left(10t - \frac{\pi}{4}\right)\right]$$
$$= 10\cos\left(10t - \frac{\pi}{4}\right)$$

$$\therefore \text{Acceleration, } a = \frac{dv}{dt} = \frac{d}{dt}\left[10\cos\left(10t - \frac{\pi}{4}\right)\right]$$
$$= -10 \times 10\sin\left(10t - \frac{\pi}{4}\right)$$
$$= -100\sin\left(10t - \frac{\pi}{4}\right)$$

Here,
$m = 50\ g = 0.05\ kg$
$\omega = 10\ rad\ s^{-1}$
$v(5) = ?$
$a(5) = ?$
$E(3) = ?$

$\therefore$ Velocity at $t = 5\ s$,
$$v(5) = 10\cos\left(10 \times 5 - \frac{\pi}{4}\right)$$
$$= 4.968\ m\ s^{-1}$$

$\therefore$ Acceleration at $t = 5\ s$,
$$a(5) = -300\sin\left(10 \times 5 - \frac{\pi}{4}\right)$$
$$= 86.786\ m\ s^{-2}$$

ii) At $t = 3\ s$,
Kinetic energy,
$$K.E = \frac{1}{2}mv^2$$
$$= \frac{1}{2}m\left[10\cos\left(10t - \frac{\pi}{4}\right)\right]^2$$
$$= \frac{1}{2} \times 0.05 \times \left[10\cos\left(10 \times 3 - \frac{\pi}{4}\right)\right]^2$$
$$= 0.869\ J$$

Potential energy,
$$P.E = \frac{1}{2}kx^2$$
$$= \frac{1}{2}m\omega^2\left[\sin\left(10t - \frac{\pi}{4}\right)\right]^2$$
$$= \frac{1}{2} \times 0.05 \times 10^2 \times \left[\sin\left(10 \times 3 - \frac{\pi}{4}\right)\right]^2$$
$$= 1.631\ J$$

$\therefore$ Total energy,
$$E = P.E + K.E$$
$$= 0.869 + 1.631$$
$$= 2.50\ J$$

$\therefore$ Total energy at $t = 3\ s$ is $2.50\ J$

3. a) Labid wants to constructed an RLC circuit that produce critical damping. He has a capacitor and inductor with value, C = 0.05 μF, L = 0.2 mH respectively.

i) What is the value of resistance he must connect to make his desired circuit?
ii) If R = 500 Ω, is the circuit oscillatory? If oscillatory, find the frequency of oscillation.

Solution:

i) Here,

$$\frac{1}{LC} = \frac{1}{0.05 \times 10^{-6} \times 0.2 \times 10^{-3}}$$
$$= 1 \times 10^{11}$$

Let, R resistance will need to produce critical damping.

Now ,

$$\frac{R^2}{4L^2} = \frac{1}{LC}$$
$$\text{or, } R^2 = \frac{1}{LC} \times 4L^2$$
$$\text{or, } R = \sqrt{1 \times 10^{11} \times 4\left(0.2 \times 10^{-3}\right)^2}$$
$$\therefore R = 126.49\ \Omega$$

∴ Resistance $R = 126.49\ \Omega$ will produce critical damping.

Here,
$C = 0.05\ \mu F$
$= 0.05 \times 10^{-6}\ F$
$L = 0.2\ mH$
$= 0.2 \times 10^{-3}\ H$
$R = ?$
$f = ?$

ii) From (i),

$$\frac{1}{LC} = 1 \times 10^{11}$$

For $R = 500\ \Omega$,

$$\frac{R^2}{4L^2} = \frac{(500)^2}{4 \times (0.2 \times 10^{-3})^2}$$
$$= 1.56 \times 10^{12}$$

Since $\frac{1}{LC} < \frac{R^2}{4L^2}$, therefore the circuit is not oscillatory.

3. b) For a damped oscillator, m = 380 gm, k = 19.6 N/m, and b = 82 gm/s. The oscillator is released at t = 0 and the amplitude is 5 cm.

i) How long does it take for the amplitude of the damped oscillator to drop to one fourth of its initial value?

ii) How many complete oscillations be found after t = 6s?

Solution:

i) Let, amplitude will drop to one fourth of its initial value after t time.

Now ,

$$\frac{A_0}{4} = A_0 e^{-\frac{\gamma}{2}t}$$
$$\text{or, } \frac{1}{4} = e^{-\frac{\frac{b}{m}}{2}t}$$
$$\text{or, } \ln\left(\frac{1}{4}\right) = \ln(e^{-\frac{b}{2m}t})$$

$$\text{or, } -1.386 = -\frac{b}{2m}t$$
$$\text{or, } -1.386 = -\frac{0.082}{2 \times 0.38}t$$
$$\text{or, } t = -1.386 \times -\frac{2 \times 0.38}{0.082}$$
$$\therefore t = 12.85\ s$$

∴ The amplitude will drop to one fourth of its initial value after 12.85 s.

ii) Here,

$$\omega_1 = \sqrt{\omega^2 - \frac{\gamma^2}{4}}$$
$$= \sqrt{\frac{\mathrm{k}}{\mathrm{m}} - \frac{\left(\frac{b}{m}\right)^2}{4}}$$
$$= \sqrt{\frac{19.6}{0.38} - \frac{\left(\frac{0.082}{0.38}\right)^2}{4}}$$
$$= 7.181\ rad\ s^{-1}$$

∴ Time period,

$$t = \frac{2\pi}{\omega_1} = \frac{2\pi}{7.181} = 0.875\ s$$

∴ No of complete oscillation in $t = 6\ s$,

$$n = \frac{t}{T} = \frac{6}{0.875} = 6.857 \approx 6$$

∴ 6 complete oscillation can be found after $t = 6\ s$.

Here,
$m = 380\ g$
$= 0.38\ kg$
$k = 19.6\ N\ m^{-1}$
$b = 82\ g\ s^{-1}$
$= 0.082\ kg\ s^{-1}n$
$A_0 = 5\ cm$
$= 0.05\ m$
$t = ?$
$n = ?$

3. c) When a simple harmonic motion is propagated through a medium, the displacement of the particle at any instant of time is given by $y = 15\sin(5t - 0.66x)$. Calculate the

i) wavelength,
ii) wave velocity,
iii) amplitude and
iv) frequency.

Solution:

Given,

$$y = 15\sin(5t - 0.066x)$$
$$\text{or, } y = 15\sin\frac{2\pi}{2\pi}(5t - 0.066x)$$

$$\text{or, } y = 15 \sin \frac{2\pi}{2\pi} \times 0.066(75.76t - x)$$

$$\text{or, } y = 15 \sin \frac{2\pi}{30.30\pi}(75.76t - x)$$

Comparing with $y = A \sin \frac{2\pi}{\lambda}(vt - x)$

Here,

i) Wavelength, $\lambda = 30.30\pi = 95.19\, m$

ii) Wave velocity, $v = 75.76\, m\, s^{-1}$

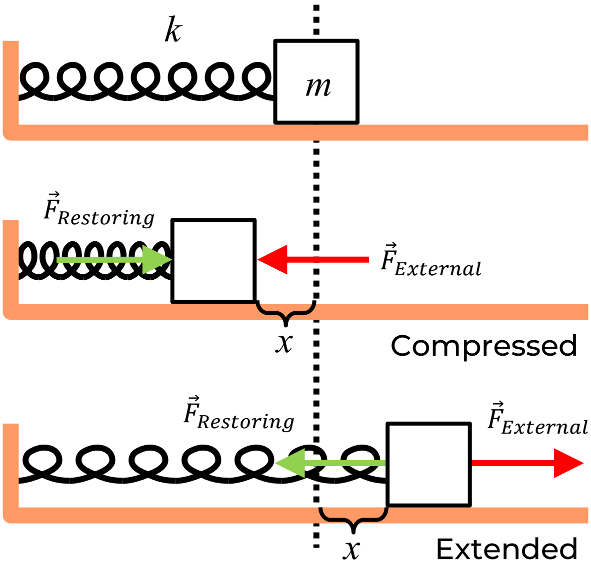
iii) Amplitude, $A = 15\, m$

iv) Frequency,

$$f = \frac{v}{\lambda} = \frac{75.76}{95.19} = 0.80\ Hz$$

4. a) Derive the differential equation of a mass-spring system oscillating in simple harmonic motion. Write down the possible solution of the differential equation and graphically plot it.

Solution:



We know,

From Hooks Law,
Restoring force,

$$\vec{F}_R \propto -\vec{x}$$

$$\text{or, } F_R = -kx$$

Here, k is spring constant.

From the momentum principle,

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

∴ For horizontal force on the mass,

$$\frac{dp_x}{dt} = -kx$$

$$\text{or, } \frac{d(mv)}{dt} = -kx \quad [\because p = mv]$$

$$\text{or, } \mathrm{m}\frac{dv}{dt} = -kx$$

$$\text{or, } \mathrm{m}\frac{d}{dt}\left(\frac{dx}{dt}\right) = -kx \quad \left[\because v = \frac{dx}{dt}\right]$$

$$\text{or, } \mathrm{m}\frac{d^2x}{dt^2} = -kx$$

$$\text{or, } \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\text{or, } \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

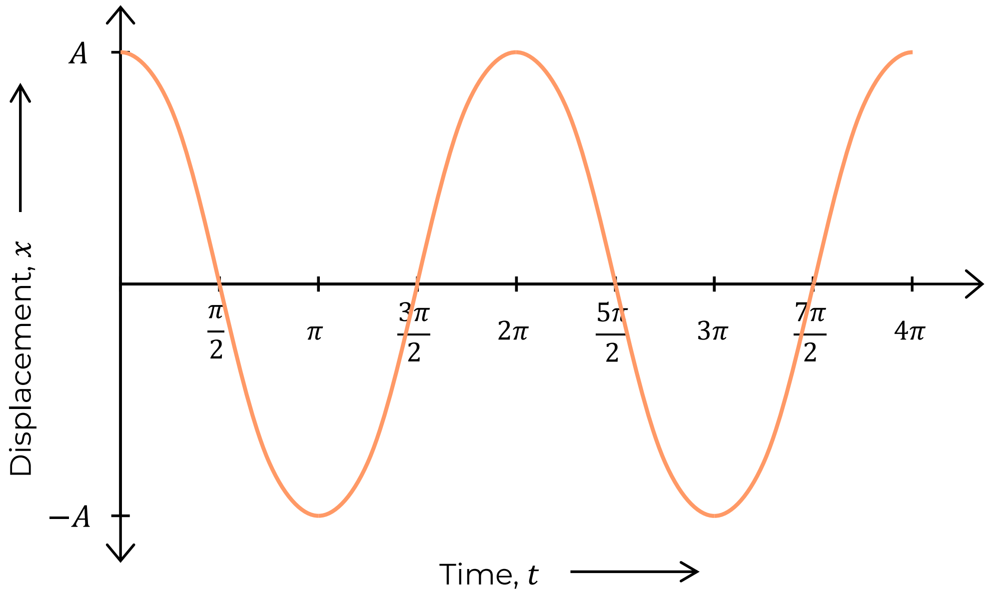
$$\text{or, } \frac{d^2x}{dt^2} + \omega^2 x = 0$$

This is the differential equation for simple harmonic motion in spring-mass system,

Where, $\omega^2 = \dfrac{k}{m}$

The possible solution for this differential equation is: $x(t) = A\cos(\omega t + \phi)$

The solution has been graphically plotted below:



4. b) For a body oscillating in simple harmonic motion, the equation of displacement is $y = A\cos\left(\omega t + \frac{\pi}{3}\right)$. Calculate the equation of velocity, acceleration, potential energy and kinetic energy. Graphically plot potential energy vs. time and kinetic energy vs. time graph.

Solution:

Given,

Displacement, $y = A\cos\left(\omega t + \dfrac{\pi}{3}\right)$

$\therefore$ Velocity, $v = \dfrac{dy}{dt} = -A\omega\sin\left(\omega t + \dfrac{\pi}{3}\right)$

$\therefore$ Acceleration, $a = \dfrac{dv}{dt} = -A\omega^2\cos\left(\omega t + \dfrac{\pi}{3}\right)$

Now,

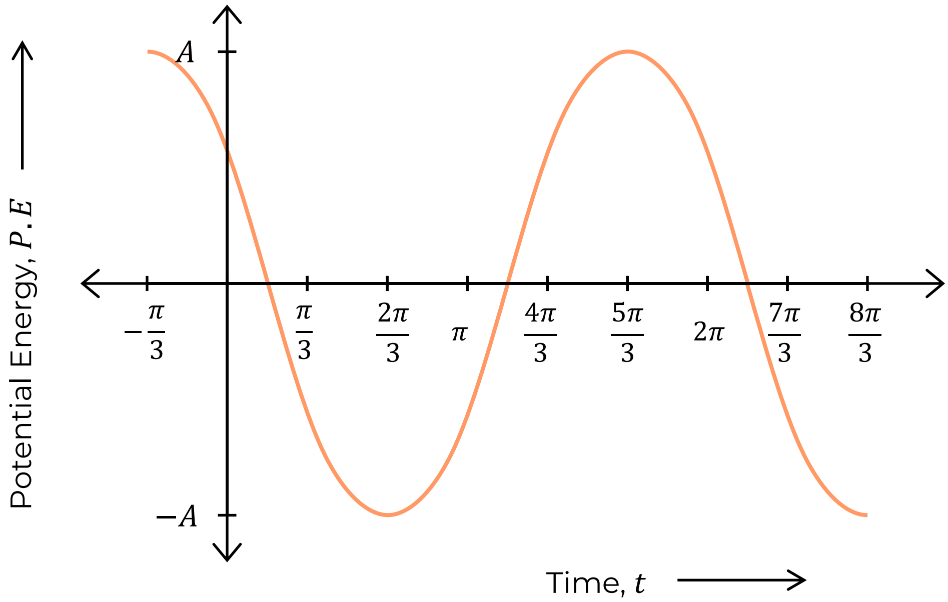
Potential energy,

$$
\begin{aligned}
P.E &= \frac{1}{2}ky^2 \\
&= \frac{1}{2}k\left[A\cos\left(\omega t + \frac{\pi}{3}\right)\right]^2 \\
&= \frac{1}{2}kA^2\cos^2\left(\omega t + \frac{\pi}{3}\right)
\end{aligned}
$$

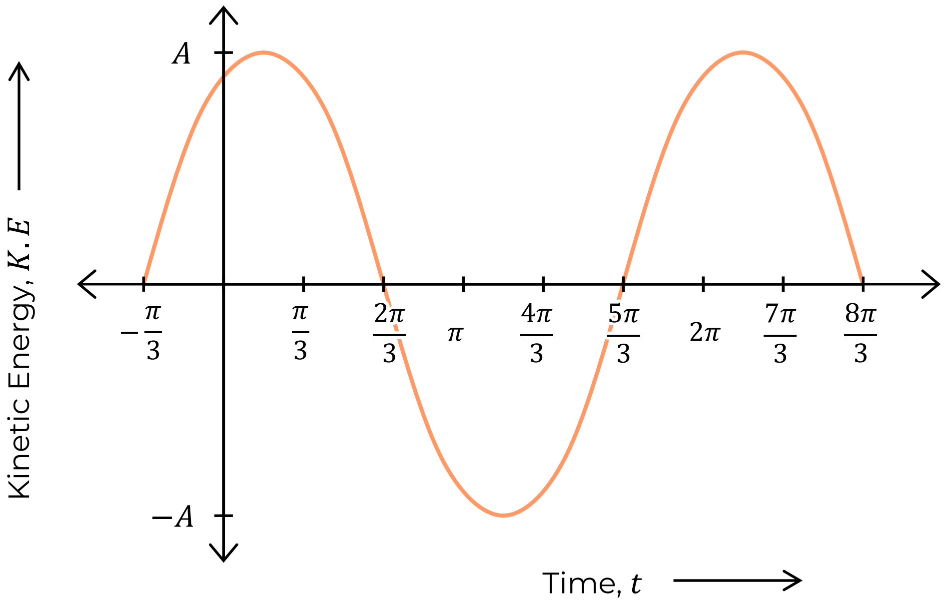
Kinetic energy,

$$
\begin{aligned}
K.E &= \frac{1}{2}mv^2 \\
&= \frac{1}{2}m\left[-A\omega\sin\left(\omega t+\frac{\pi}{3}\right)\right]^2 \\
&= \frac{1}{2}mA^2\omega^2\sin^2\left(\omega t+\frac{\pi}{3}\right) \\
&= \frac{1}{2}kA^2\sin^2\left(\omega t+\frac{\pi}{3}\right) \quad [\because k = m\omega^2]
\end{aligned}
$$

Potential energy vs time has been graphically plotted below:



Kinetic energy vs time has been graphically plotted below:



5. a) For a progressive wave show that, $y = A\sin\frac{2\pi}{\lambda}(vt - x)$ where the symbols have their usual meaning. Derive the equation of velocity of the particle from above equation.

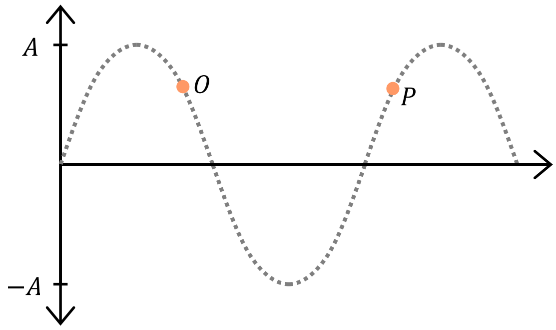
Solution:

Consider a particle P at a distance x from particle O on its right. Let the wave travel with a velocity v from left to right.

Now, Displacement of particle P is,

$$y = A\sin(\omega t - \phi) \text{ ...... (i)}$$

Where ϕ is the phase difference between the particles O and P.



We know that a path difference of ϕ corresponds to phase difference of 2π radians. Hence a path difference of x corresponds to a phase difference of,

$$\phi = \frac{2\pi}{\lambda}x$$

Now, From equation (i),

$$y = A\sin(\omega t - \phi)$$

$$\text{or, } y = A\sin\left(\omega t - \frac{2\pi}{\lambda}x\right)$$

$$\text{or, } y = A\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right) \quad [\because \omega = 2\pi]$$

$$\text{or, } y = A\sin 2\pi\left(\frac{1}{T}t - \frac{1}{\lambda}x\right)$$

$$\text{or, } y = A\sin 2\pi\left(\frac{v}{\lambda}t - \frac{1}{\lambda}x\right) \quad \left[\because v = \frac{\lambda}{T}\ or,\ T = \frac{\lambda}{v}\right]$$

$$\therefore y = A\sin\frac{2\pi}{\lambda}(vt - x) \quad \text{(Showed)}$$

Now,we will get particle velocity if we differentiate this equation with respect to t

$$\therefore \text{Particle velocity, } v_p = \frac{dy}{dt}$$

$$= \frac{d}{dt}\left(A\sin\frac{2\pi}{\lambda}(vt - x)\right)$$

$$\therefore v_p = A\frac{2\pi}{\lambda}v\cos\frac{2\pi}{\lambda}(vt - x)$$

5. b) **An inductor, a resistor and a charged capacitor are connected to a circuit. Derive differential equation for the circuits and write down the solution of the equation for oscillatory damping.**

Solution:

Since an inductor, a resistor and a charged capacitor are connected to that circuit, therefore it is an RLC circuit. The equation for the RLC circuit has been derived below.

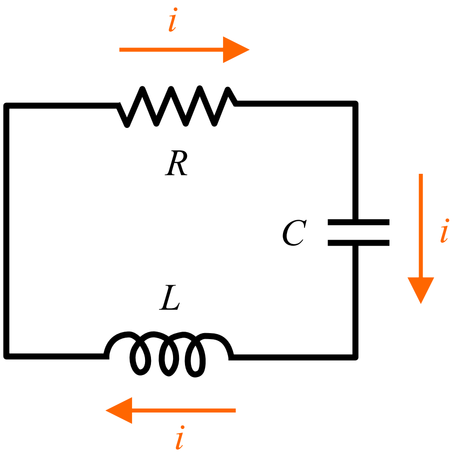
[P.T.O]

We know,
In a RLC circuit,

Voltage across resistance R, $V_R = iR$

Voltage across capacitor C, $V_C = \frac{Q}{C}$

Voltage across inductor L, $V_L = L\frac{di}{dt}$



Now, according to Kirchhoff's Voltage Law (KVL),

$$V_R + V_C + V_L = 0$$

$$\text{or, } \mathrm{iR} + \frac{\mathrm{Q}}{C} + L\frac{di}{dt} = 0$$

$$\text{or, } \mathrm{R}\frac{\mathrm{dQ}}{dt} + \frac{Q}{C} + L\frac{d}{dt}\left(\frac{dQ}{dt}\right) = 0$$

$$\text{or, } \mathrm{R}\frac{\mathrm{dQ}}{dt} + \frac{1}{C}Q + L\frac{d^2Q}{dt^2} = 0$$

$$\text{or, } L\frac{d^2Q}{dt^2} + \mathrm{R}\frac{\mathrm{dQ}}{dt} + \frac{1}{C}Q = 0$$

$$\text{or, } \frac{d^2Q}{dt^2} + \frac{\mathrm{R}}{L}\frac{\mathrm{dQ}}{dt} + \frac{1}{LC}Q = 0$$

$$\text{or, } \frac{d^2Q}{dt^2} + \gamma\frac{\mathrm{dQ}}{dt} + \omega^2 Q = 0$$

This is the differential equation for DHM in RLC circuit,
Where, $\gamma = \frac{R}{L}$ and $\omega^2 = \frac{1}{LC}$

The solution for this differential equation is:

$$Q(t) = Q_0 e^{-\frac{\gamma}{2}t}\cos(\omega_1 + \phi) = Q_0 e^{-\frac{R}{2L}t}\cos(\omega_1 + \phi)$$

Summer 2023

1. a) **Why does the amplitude of an oscillatory body decrease in Damped Harmonic Motion (DHM)?**

Solution:

In dampened harmonic motion (DHM), oscillations experience dampening forces like air resistance or friction. These forces act like brakes, opposing the motion and continuously sapping the system's energy. As this energy depletes, the oscillating body loses its ability to reach its peak displacement, resulting in a gradual decrease in the amplitude of its vibrations over time.

For these reasons, we notice a decrease in amplitude in damped harmonic motion.

1. b) **Does the total energy of an oscillatory body with SHM vary with time? Explain briefly.**

Solution:

No, the total energy of an oscillatory body with SHM does not vary with time.

In a simple harmonic motion (SHM) system, the total energy of the oscillating body remains constant throughout its motion. This is because of the continuous exchange between kinetic energy and potential energy. As the object moves away from its equilibrium position, it gains kinetic energy while losing potential energy, and vice versa when it returns. The total energy, which is the sum of both kinetic and potential energy, remains unchanged throughout the oscillation.

1. c) **The equation of displacement of a simple harmonic oscillator is $x = A\cos\left(\omega t + \frac{\pi}{6}\right)$. Graphically represent the displacement and acceleration with respect to time.**

Solution:

Here,

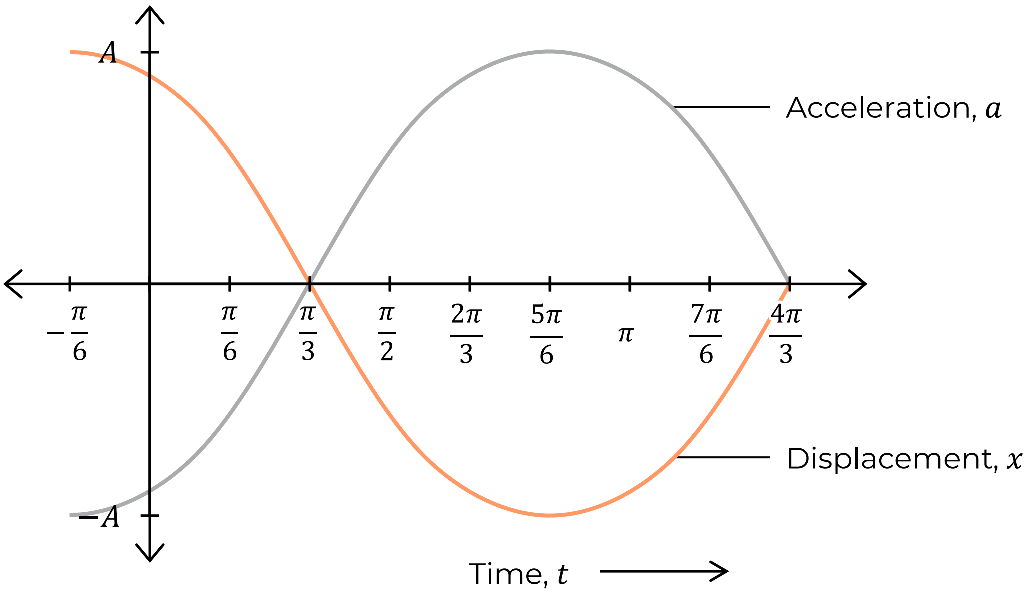
Displacement, $x = A\cos\left(\omega t + \frac{\pi}{6}\right)$

$\therefore$ Velocity, $v = \frac{dx}{dt} = -A\omega\sin\left(\omega t + \frac{\pi}{6}\right)$

$\therefore$ Acceleration, $a = \frac{dv}{dt} = -A\omega^2\cos\left(\omega t + \frac{\pi}{6}\right)$

The displacement and acceleration have been graphically represented below:

[P.T.O]



2. a) A 5 kg block is attached to a spring and the spring constant is k = 1400 N/m. The block is held a distance of 6 cm from equilibrium and released at t = 0.

i) Find the angular frequency w, the frequency f, and the period T.
ii) Write an equation for x vs. time.

Solution:

i) We know,
Angular frequency,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1400}{5}} = 16.73\ rad\ s^{-1}$$

Frequency,

$$f = \frac{\omega}{2\pi} = \frac{16.73}{2\pi} = 2.66\ Hz$$

Time period,

$$t = \frac{1}{f} = \frac{1}{2.66} = 0.38\ s$$

Here,
$m = 5\ kg$
$k = 1400\ N\ m^{-1}$
$A = 6\ cm = 0.06\ m$
$\omega = ?$
$f = ?$
$T = ?$

ii) At $t = 0$,

$$0.06 = 0.06\cos(\omega \times 0 + \phi)$$
$$\text{or, } \frac{0.06}{0.06} = \cos(0 + \phi)$$
$$\text{or, } 1 = \cos(\phi)$$
$$\text{or, } \phi = \cos^{-1}(1)$$
$$\therefore \phi = 0$$

$\therefore$ Equation for x vs t:

$$x(t) = 0.06\cos(16.73t)$$

2. b) Suppose a spring block-system move between top and bottom point of a tall building as a moving mass. The block has mass m = 5.7×10^3 kg and designed to oscillate at frequency f = 50 Hz with amplitude x_m = 15 cm. Calculate:

- **i)** the potential energy at the equilibrium point,
- **ii)** the block speed as it passes through the equilibrium point,
- **iii)** the maximum acceleration of the spring block-system.

Solution:

i) We know,
At equilibrium position, $x = 0$

∴ Potential energy,

$$P.E = \frac{1}{2}kx^2 = \frac{1}{2}k(0)^2 = 0$$

Here,
$m = 5.7 \times 10^3\ kg$
$f = 50\ Hz$
$A = x_{max} = 15\ cm = 0.15\ m$
$P.E = ?$
$v_{max} = ?$
$a_{max} = ?$

ii) We know,
At equilibrium position, SHM oscillation have its maximum speed.

∴ Maximum velocity,

$$\begin{aligned} v_{max} &= A\omega \\ &= A2\pi f \quad [\because \omega = 2\pi f] \\ &= 0.15 \times 2\pi \times 50 \\ &= 47.12\ \mathrm{m\ s^{-1}} \end{aligned}$$

iii) We know,
Maximum acceleration,

$$\begin{aligned} a &= -A\omega^2 \\ &= -A(2\pi f)^2 \quad [\because \omega = 2\pi f] \\ &= -0.15 \times (2\pi \times 50)^2 \\ &= 14804.48\ \mathrm{m\ s^{-2}} \end{aligned}$$

2. c) A particle executes simple harmonic motion given by equation $x = 3\sin\left(25t - \frac{3\pi}{4}\right)$. Calculate the

- **i)** displacement at t = 5 s
- **ii)** velocity and acceleration at t = 2.5 s.

Solution:

i) Given,

Displacement, $y = 3\sin\left(25t - \dfrac{3\pi}{4}\right)$

∴ Displacement at $t = 5\ s$

$$\begin{aligned} y(5) &= 3\sin\left(25 \times 5 - \frac{3\pi}{4}\right) \\ &= -0.36\ m \end{aligned}$$

ii) Given,

Displacement, $y = 3\sin\left(25t - \frac{3\pi}{4}\right)$

$\therefore$ Velocity, $v = \frac{dy}{dt} = 3 \times 25\cos\left(25t - \frac{3\pi}{4}\right)$

$= 75\cos\left(25t - \frac{3\pi}{4}\right)$

$\therefore$ Acceleration, $a = \frac{dv}{dt} = -75 \times 25\sin\left(25t - \frac{3\pi}{4}\right)$

$= -1875\sin\left(25t - \frac{3\pi}{4}\right)$

$\therefore$ Velocity at $t = 2.5\ s$,

$$v(2.5) = 75\cos\left(25 \times 2.5 - \frac{3\pi}{4}\right)$$
$$= -67.42\ m\ s^{-1}$$

$\therefore$ Acceleration at $t = 2.5\ s$,

$$a(2.5) = -1875\sin\left(25 \times 2.5 - \frac{3\pi}{4}\right)$$
$$= 821.54\ m\ s^{-2}$$

3. a) For a damped oscillator, m = 580 gm, k = 240 N/m, and b = 72 gm/s. The oscillator is stretched up to 8 cm from the equilibrium and release at t = 0.

i) What is period of the motion?

ii) How long does it take for the amplitude of the damped oscillator to drop to one third of its initial value?

Solution:

i) Here,

$$\omega_1 = \sqrt{\omega^2 - \frac{\gamma^2}{4}}$$
$$= \sqrt{\frac{\mathrm{k}}{\mathrm{m}} - \frac{\left(\frac{b}{m}\right)^2}{4}}$$
$$= \sqrt{\frac{240}{0.58} - \frac{\left(\frac{0.072}{0.58}\right)^2}{4}}$$
$$= 20.34\ rad\ s^{-1}$$

$\therefore$ Time period,

$$T = \frac{2\pi}{\omega_1} = \frac{2\pi}{20.34} = 0.31\ s$$

Here,

$m = 580\ g$

$= 0.58\ kg$

$k = 240\ N\ m^{-1}$

$b = 72\ g\ s^{-1}$

$= 0.072\ kg\ s^{-1}n$

$A_0 = 8\ cm$

$= 0.08\ m$

$T = ?$

$t = ?$

ii) Let, amplitude will drop to one third of its initial value after t time.

Now ,

$$\frac{A_0}{3} = A_0 e^{-\frac{\gamma}{2}t}$$

$$\text{or, } \frac{1}{3} = e^{-\frac{\frac{b}{m}}{2}t}$$

$$\text{or, } \ln\left(\frac{1}{3}\right) = \ln(e^{-\frac{b}{2m}t})$$

$$\text{or, } -1.099 = -\frac{b}{2m}t$$

$$\text{or, } -1.099 = -\frac{0.072}{2 \times 0.58}t$$

$$\text{or, } t = -1.099 \times -\frac{2 \times 0.58}{0.072}$$

$$\therefore t = 17.706\ s$$

∴ The amplitude will drop to one third of its initial value after $17.706\ s$.

3. b) Karim want to construct a RLC circuit that produces critical damping. He have a capacitor and inductor with value, C = 0.003 μF, L = 0.0001 H respectively.

i) What is the value of resistance he must connect to make his desired circuit?

ii) If R = 800 Ω, is the circuit oscillatory? If oscillatory, find the frequency of oscillation.

Solution:

i) Here,

$$\frac{1}{LC} = \frac{1}{0.003 \times 10^{-3} \times\ 0.0001}$$
$$= 3333333333.33$$

Let, R resistance will need to produce critical damping.

Now ,

$$\frac{R^2}{4L^2} = \frac{1}{LC}$$

$$\text{or, } R^2 = \frac{1}{LC} \times 4L^2$$

$$\text{or, } R = \sqrt{3333333333.33 \times 4(0.0001)^2}$$

$$\therefore R = 11.55\ \Omega$$

Here,

$C = 0.003\ mF$

$= 0.003 \times 10^{-3}\ F$

$L = 0.0001\ H$

$R = ?$

$f = ?$

∴ Resistance $R = 11.55\ \Omega$ will produce critical damping.

ii) From (i),

$$\frac{1}{LC} = 3333333333.33 = 3.33 \times 10^9$$

For $R = 800\ \Omega$,

$$\frac{R^2}{4L^2} = \frac{(800)^2}{4 \times (0.0001)^2}$$
$$= 1.6 \times 10^{13}$$

Since $\frac{1}{LC} < \frac{R^2}{4L^2}$, therefore the circuit is not oscillatory.

3. c) When a simple harmonic motion is propagated through a medium, the displacement of the particle at any instant of time is given by $y = 15\sin(5t - 0.066x)$. Calculate the

i) wavelength,
ii) wave velocity,
iii) amplitude and
iv) frequency.

Solution:

Given,

$$y = 15\sin(5t - 0.066x)$$
$$\text{or, } y = 15\sin\frac{2\pi}{2\pi}(5t - 0.066x)$$
$$\text{or, } y = 15\sin\frac{2\pi}{2\pi} \times 0.066(75.76t - x)$$
$$\text{or, } y = 15\sin\frac{2\pi}{30.30\pi}(75.76t - x)$$

Comparing with $y = A\sin\frac{2\pi}{\lambda}(vt - x)$

Here,

i) Wavelength, $\lambda = 30.30\pi = 95.19\ m$
ii) Wave velocity, $v = 75.76\ m\ s^{-1}$
iii) Amplitude, $A = 15\ m$
iv) Frequency,

$$f = \frac{v}{\lambda} = \frac{75.76}{95.19} = 0.80\ Hz$$

4. a) For a mass spring system oscillating in simple harmonic motion, the equation of displacement is $x = A\sin(\omega t + \phi)$. Calculate the potential and kinetic energy from the equation of displacement and graphically represent the potential and kinetic energy vs displacement..

Solution:

Given,

Displacement, $x = A\sin(\omega t + \phi)$

$\therefore$ Velocity, $v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$

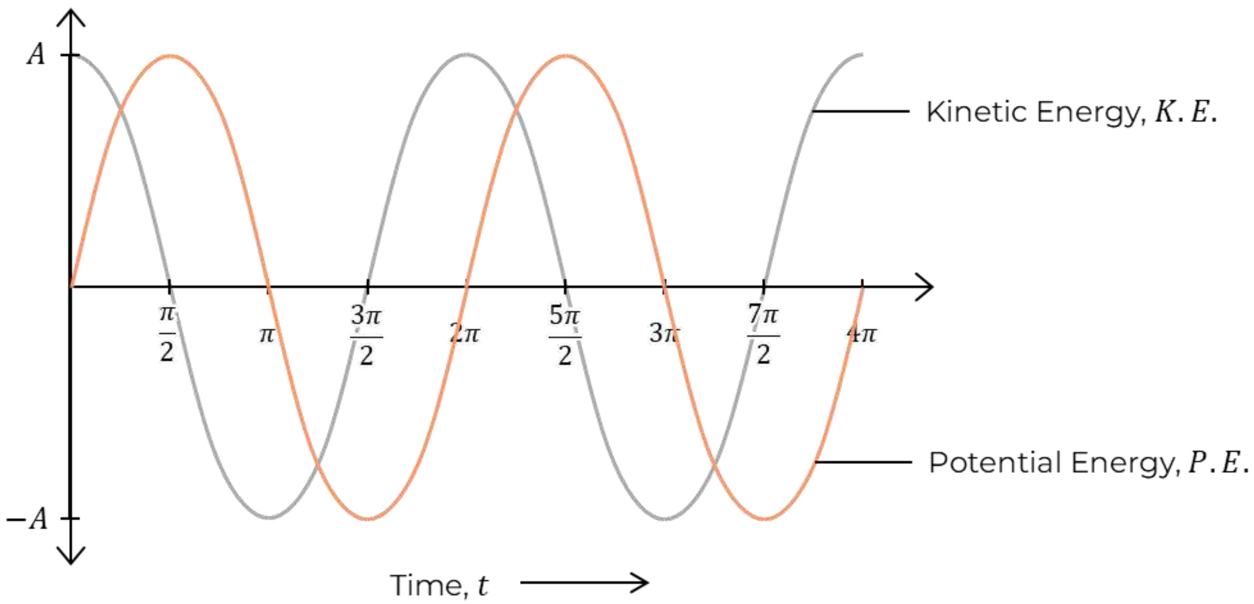
Now,
Potential energy,

$$\begin{aligned} P.E &= \frac{1}{2}kx^2 \\ &= \frac{1}{2}k[A\sin(\omega t + \phi)]^2 \\ &= \frac{1}{2}kA^2\sin^2(\omega t + \phi) \end{aligned}$$

Kinetic energy,

$$\begin{aligned} K.E &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}m[A\omega\cos(\omega t + \phi)]^2 \\ &= \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \phi) \\ &= \frac{1}{2}kA^2\cos^2(\omega t + \phi) \quad [\because k = m\omega^2] \end{aligned}$$

Potential energy vs time has been graphically plotted below:



4. a) Unavailable

5. a) Unavailable

5. b) Unavailable

Spring 2023

1. a) **How you can differentiate progressive wave and stationary wave?**

Solution:

The differences between progressive waves and stationary waves have been described below:

Progressive Waves	Stationary Waves
Transfer energy from one point to another.	No net transfer of energy.
All particle have same amplitude.	Particle have different amplitudes depending on position.
No specific points of zero or maximum displacement.	Well-defined points of zero (nodes) and maximum (antinodes) displacement.
All particles exhibit some form of movement.	Particles at nodes are at rest, others oscillate

1. b) **The equation of displacement of a simple harmonic oscillator is $x = A\cos\left(\omega t + \frac{\pi}{2}\right)$. Graphically represent the velocity and acceleration with respect to time.**

Solution:

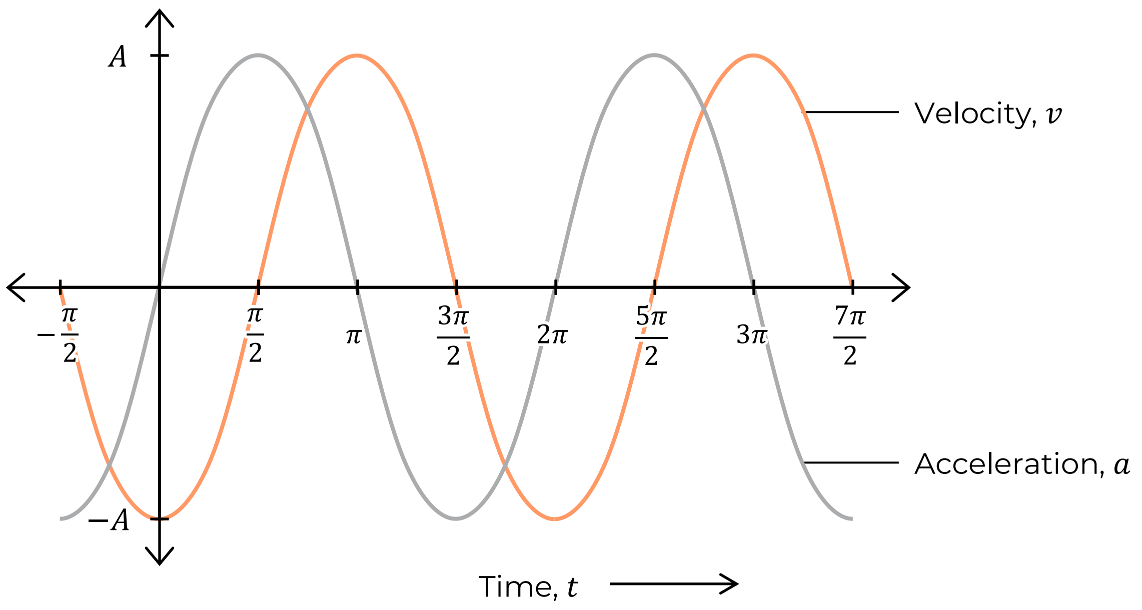
Here,

Displacement, $x = A\cos\left(\omega t + \dfrac{\pi}{2}\right)$

$\therefore$ Velocity, $v = \dfrac{dx}{dt} = -A\omega\sin\left(\omega t + \dfrac{\pi}{2}\right)$

$\therefore$ Acceleration, $a = \dfrac{dv}{dt} = -A\omega^2\cos\left(\omega t + \dfrac{\pi}{2}\right)$

The velocity and acceleration have been graphically represented below:



1. c) Why do we observe simple harmonic motion (SHM) in LC circuit and damped harmonic motion (DHM) in RLC circuit?

Solution:

In an LC circuit, energy flips back and forth between the capacitor and inductor, and there is no resistance to slow it down. This frictionless exchange creates simple harmonic motion (SHM) with constant-amplitude oscillations. However, adding a resistor (R) to create an RLC circuit introduces an energy drain. The resistor converts electrical energy into heat, causing the oscillations to gradually weaken over time. This energy dissipation transforms the SHM into damped harmonic motion (DHM), where the amplitude gradually decreases over time.

For these reasons, we observe simple harmonic motion in the LC circuit and damped harmonic motion in the RLC circuit.

2. a) A 0.12 kg body undergoes SHM of amplitude 8.5 cm and time period 0.20 s.

i) What is the magnitude of the maximum force acting on it?
ii) If the oscillations are produced by a spring, what is the spring constant?

Solution:

Here,
$m = 0.12\ kg$
$A = 8.5\ cm = 0.085\ m$
$T = 0.20\ s$
$F_{max} = ?$
$k = ?$

i) We know,
Maximum Force,

$$\begin{aligned} F_{max} &= kx_{max} \\ &= m\omega^2 A \quad [\because k = m\omega^2] \\ &= m(2\pi f)^2 A \quad [\because \omega = 2\pi f] \\ &= m\left(\frac{2\pi}{T}\right)^2 A \quad \left[\because f = \frac{1}{T}\right] \\ &= 0.12 \times \left(\frac{2\pi}{0.20}\right)^2 \times 0.085 \\ &= 10.067\ N \end{aligned}$$

ii) We know,
Spring constant,

$$\begin{aligned} k &= m\omega^2 \\ &= m(2\pi f)^2 \quad [\because \omega = 2\pi f] \\ &= m\left(\frac{2\pi}{T}\right)^2 \quad \left[\because f = \frac{1}{T}\right] \\ &= 0.12 \times \left(\frac{2\pi}{0.20}\right)^2 \\ &= 118.435\ N\ m^{-1} \end{aligned}$$

2. b) A body of mass 300 gm is attached with a spring of spring constant 5000 dynes/cm. The body is displaced by 7 cm from its equilibrium position and released. Then the
body executes SHM. Calculate the

i) frequency,
ii) angular frequency,
iii) total energy of the mass spring system.

Solution:

i) We know,
Frequency,

$$f = \frac{\omega}{2\pi}$$
$$= \frac{1}{2\pi} \times \sqrt{\frac{k}{m}}$$
$$= \frac{1}{2\pi} \times \sqrt{\frac{5000}{300}}$$
$$= 0.65\ Hz$$

Here,
$m = 300\ g$
$k = 5000\ dynes\ cm^{-1}$
$A = 7\ cm$
$f = ?$
$\omega = ?$
$E = ?$

ii) We know,
Angular frequency,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5000}{300}} = 4.082\ rad\ s^{-1}$$

iii) We know,
Total energy,

$$E = \frac{1}{2} kA^2$$
$$= \frac{1}{2} \times 5000 \times (7)^2$$
$$= -0.15 \times (2\pi \times 50)^2$$
$$= 122500\ erg$$

2. c) A particle executes SHM given by equation $x = 9 \sin\left(16t - \frac{\pi}{6}\right)$. Calculate

i) the maximum displacement,
ii) the maximum velocity, and
iii) the maximum acceleration.

Solution:

Here,

Displacement, $x = 9 \sin\left(16t - \frac{\pi}{6}\right)$

$\therefore$ Velocity, $v = \frac{dx}{dt} = 9 \times 16 \cos\left(16t - \frac{\pi}{6}\right)$
$$= 144 \cos\left(16t - \frac{\pi}{6}\right)$$

$\therefore$ Acceleration, $a = \frac{dv}{dt} = -144 \times 16 \sin\left(16t - \frac{\pi}{6}\right)$
$$= -2304 \sin\left(16t - \frac{\pi}{6}\right)$$

i) We will get maximum displacement when
$\sin\left(16t - \frac{\pi}{6}\right)$ will be equal 1.

∴ Maximum displacement,

$$x_{max} = A = 9\ m$$

ii) We will get maximum velocity when $\cos\left(16t - \frac{\pi}{6}\right)$ will be equal 1.

∴ Maximum velocity,

$$v_{max} = 144\ m\ s^{-1}$$

iii) We will get maximum acceleration when $\sin\left(16t - \frac{\pi}{6}\right)$ will be equal -1.

∴ Maximum acceleration,

$$a_{max} = 2304\ m\ s^{-2}$$

3. a) For a damped oscillator, m = 490 gm, k = 190 N/m, and b = 75 gm/s. The amplitude of the oscillator is 8 cm in initial time.

i) What is period of the motion?
ii) How long does it take for the amplitude of the damped oscillations to drop to one fourth of its initial value?
iii) What is life time of oscillation?

Solution:

Here,
$m = 490\ g$
$\quad = 0.49\ kg$
$k = 190\ N\ m^{-1}$
$b = 75\ g\ s^{-1}$
$\quad = 0.075\ kg\ s^{-1}n$
$A_0 = 8\ cm$
$\quad = 0.08\ m$
$T = ?$
$t = ?$
$\tau = ?$

i) Here,

$$\omega_1 = \sqrt{\omega^2 - \frac{\gamma^2}{4}}$$
$$= \sqrt{\frac{\mathrm{k}}{\mathrm{m}} - \frac{\left(\frac{b}{m}\right)^2}{4}}$$
$$= \sqrt{\frac{190}{0.49} - \frac{\left(\frac{0.075}{0.49}\right)^2}{4}}$$
$$= 19.69\ rad\ s^{-1}$$

∴ Time period,

$$T = \frac{2\pi}{\omega_1} = \frac{2\pi}{19.69} = 0.32\ s$$

ii) Let, amplitude will drop to one fourth of its initial value after t time.

Now,

$$\frac{A_0}{4} = A_0 e^{-\frac{\gamma}{2}t}$$
$$\text{or, } \frac{1}{4} = e^{-\frac{\frac{b}{m}}{2}t}$$

$$\text{or, } \ln\left(\frac{1}{4}\right) = \ln(e^{-\frac{b}{2m}t})$$

$$\text{or, } -1.386 = -\frac{b}{2m}t$$

$$\text{or, } -1.386 = -\frac{0.075}{2 \times 0.49}t$$

$$\text{or, } t = -1.386 \times -\frac{2 \times 0.49}{0.075}$$

$$\therefore t = 18.11\ s$$

∴ The amplitude will drop to one fourth of its initial value after $18.11\ s$.

iii) We know,
Life time,

$$\tau = \frac{1}{\gamma} = \frac{1}{\frac{b}{m}} = \frac{1}{\frac{0.075}{0.49}} = 0.153\ s$$

3. b) Find whether the discharge of capacitor through the following inductive series circuit is oscillatory or not. Given, C = 0.0005 mF, L = 0.1 h. and R = 250 Ω. If oscillatory, find the frequency of oscillation.

Solution:

Here,

$$\frac{1}{LC} = \frac{1}{0.0005 \times 10^{-3} \times 0.01}$$
$$= 2 \times 10^7$$

For $R = 250\ \Omega$,

$$\frac{R^2}{4L^2} = \frac{(250)^2}{4 \times (0.1)^2}$$
$$= 1.5625 \times 10^6$$

Since $\frac{1}{LC} > \frac{R^2}{4L^2}$, therefore the circuit is oscillatory.

Here,

$$\omega_1 = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$
$$= \sqrt{2 \times 10^7 - 1.5625 \times 10^6}$$
$$= 4293.891\ rad\ s^{-1}$$

Here,
$C = 0.0005\ mF$
$= 0.0005 \times 10^{-3}\ F$
$L = 0.1\ H$
$R = 250\ \Omega$
$f = ?$

We know,
Frequency,

$$f = \frac{\omega_1}{2\pi} = \frac{4293.891}{2\pi} = 683.39\ Hz$$

3. c) When a simple harmonic wave is propagated through a medium, the displacement of the particle at any instant of time is given by $y = 4\sin\pi(250t - 0.25x)$. Calculate the

a) wave velocity,
b) wave length,
c) frequency of particle of the medium.

Solution:

Given,

$$y = 4\sin\pi(250t - 0.25x)$$
$$\text{or, } y = 4\sin\frac{2\pi}{2}\times 0.25(1000t - x)$$
$$\text{or, } y = 4\sin\frac{2\pi}{8}(1000t - x)$$

Comparing with $y = A\sin\frac{2\pi}{\lambda}(vt - x)$

Here,

a) Wave velocity, $v = 1000\ m\ s^{-1}$
b) Wavelength, $\lambda = 8\ m$
c) Frequency,

$$f = \frac{v}{\lambda} = \frac{1000}{8} = 125\ Hz$$

4. a) For a mass spring system oscillating in SHM, the equation of displacement is $x = A\cos(\omega t)$. Show that, potential and kinetic energy depends on time but total energy of the system is constant.

Solution:

Given,

Displacement, $x = A\cos(\omega t)$

$\therefore$ Velocity, $v = \frac{dx}{dt} = -A\omega\sin(\omega t)$

Now,

Potential energy,

$$P.E = \frac{1}{2}kx^2$$
$$= \frac{1}{2}k[A\cos(\omega t)]^2$$
$$= \frac{1}{2}kA^2\cos^2(\omega t) \quad \text{......... (i)}$$

Kinetic energy,

$$K.E = \frac{1}{2}mv^2$$
$$= \frac{1}{2}m[-A\omega\sin(\omega t)]^2$$
$$= \frac{1}{2}mA^2\omega^2\sin^2(\omega t)$$

$$= \frac{1}{2}kA^2 \sin^2(\omega t) \text{ .... (ii)} \quad [\because k = m\omega^2]$$

∴ Total energy,

$$E = K.E + P.E$$

$$= \frac{1}{2}kA^2 \sin^2(\omega t) + \frac{1}{2}kA^2 \cos^2(\omega t)$$

$$= \frac{1}{2}kA^2[\sin^2(\omega t) + \cos^2(\omega t)]$$

$$= \frac{1}{2}kA^2 \text{ .... (iii)} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

From the equations (i) and (ii), we can see that potential energy and kinetic energy depends on time. On the other hand, from equation (iii), we can see that total energy does not depend on time since the spring constant (k) and amplitude (A) are constants for a specific mass spring system.

4. b) Derive differential equation for LC circuits. Write down the solution of the differential equation.

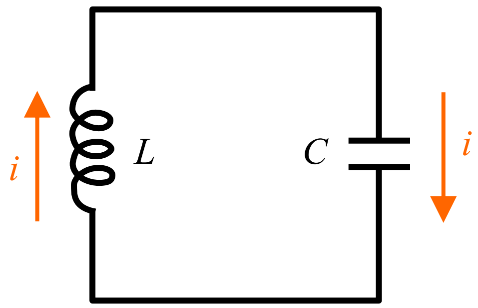
Solution:

We know,

In a LC circuit,

Voltage across capacitor C, $V_C = \frac{Q}{C}$

Voltage across inductor L, $V_L = L\frac{di}{dt}$



Now,

According to Kirchhoff's Voltage Law (KVL),

$$V_L + V_C = 0$$

$$\text{or, } L\frac{di}{dt} + \frac{Q}{C} = 0$$

$$\text{or, } L\frac{d}{dt}\left(\frac{dQ}{dt}\right) + \frac{1}{C}Q = 0 \quad \left[\because i = \frac{dQ}{dt}\right]$$

$$\text{or, } L\frac{d^2Q}{dt^2} + \frac{1}{C}Q = 0$$

$$\text{or, } \frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

$$\text{or, } \frac{d^2Q}{dt^2} + \omega^2 Q += 0$$

This is the differential equation for SHM in LC circuit,

Where, $\omega^2 = \frac{1}{LC}$

The solution for this differential equation is:

$$Q(t) = Q_0 \cos(\omega + \phi)$$

5. a) Derive the differential equation of DHM for a mass-spring system. With proper conditions, graphically represent the types of damping that may be observed in the system.

Solution:

Repeat of Spring 2024 Question 5(b)

5. b) Unavailable

1. a) The displacement of a Simple Harmonic Motion (SHM) is $y = A\sin\left(\omega t + \frac{\pi}{2}\right)$. Graphically show that the displacement and acceleration are out of phase to each other.

Solution:

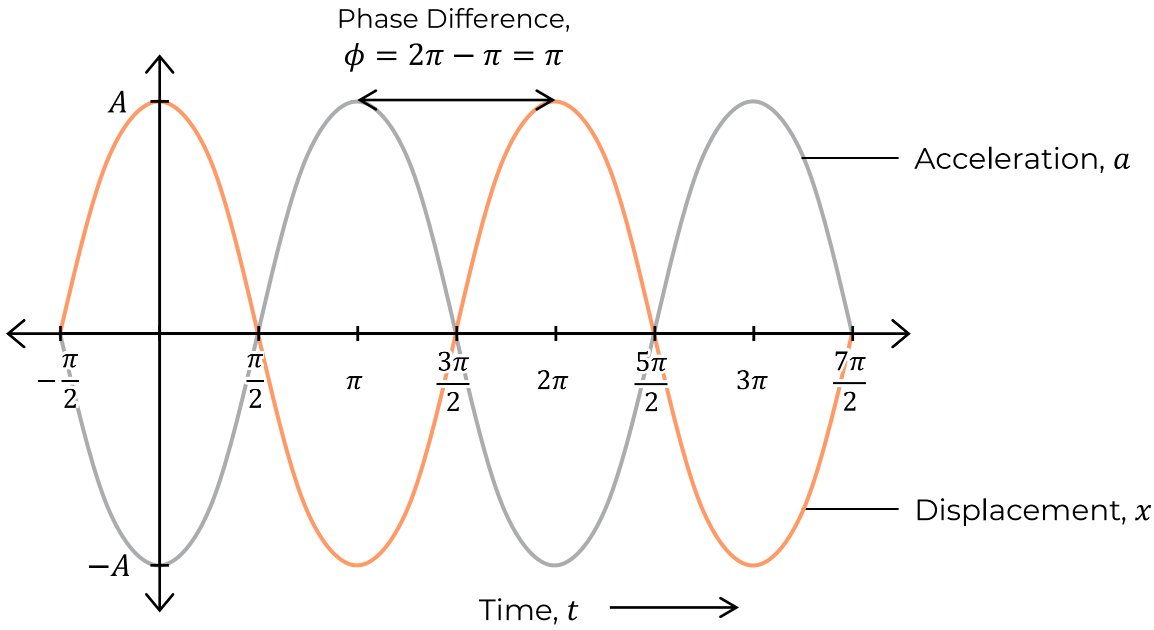
Here,

$$\text{Displacement, } x = A\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\therefore \text{Velocity, } v = \frac{dx}{dt} = A\omega\cos\left(\omega t + \frac{\pi}{2}\right)$$

$$\therefore \text{Acceleration, } a = \frac{dv}{dt} = -A\omega^2\sin\left(\omega t + \frac{\pi}{2}\right)$$

The displacement and acceleration have been graphically represented below:



We know, when two sinusoidal wave have phase difference of π, then we say that these two sinusoidal wave are out phase to each other. Since, phase difference between in the acceleration and displacement is π, therefore displacement and acceleration is out phase to each other.

1. b) Why do $\omega^2 > \gamma^2/4$ is oscillatory? Draw displacement vs. time graphs for (i) $\omega/\gamma = 5$ and (ii) $\omega/\gamma = 0.005$.

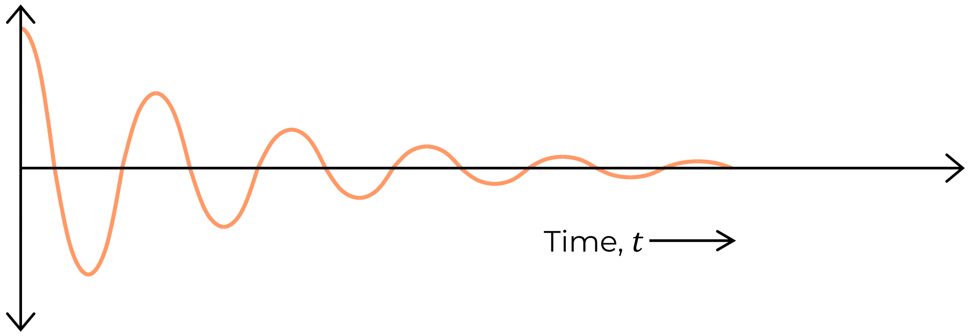
Solution:

In a damped harmonic motion, ω^2 represents the system's natural tendency to oscillate, and $\gamma^2/4$ represents the dampening effect. If ω^2 is greater than $\gamma^2/4$, the system has enough oscillatory strength to vibrate back and forth, although with decreasing amplitude due to damping. However, if the damping force is too strong ($\gamma^2/4 > \omega^2$), it overpowers the restoring force, and the system will not oscillate.

The displacement vs time graphs for given conditions are plotted below:

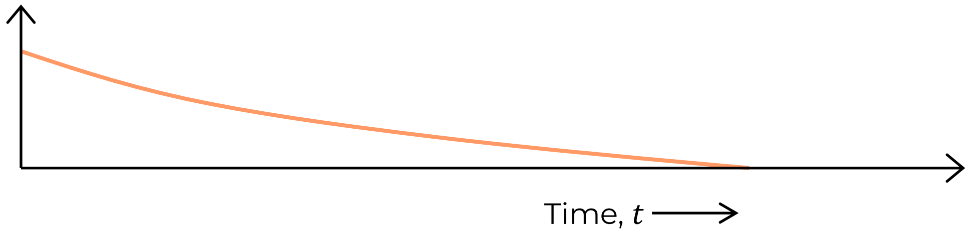
(i)

$\frac{\omega}{\gamma} = 5$



(ii)

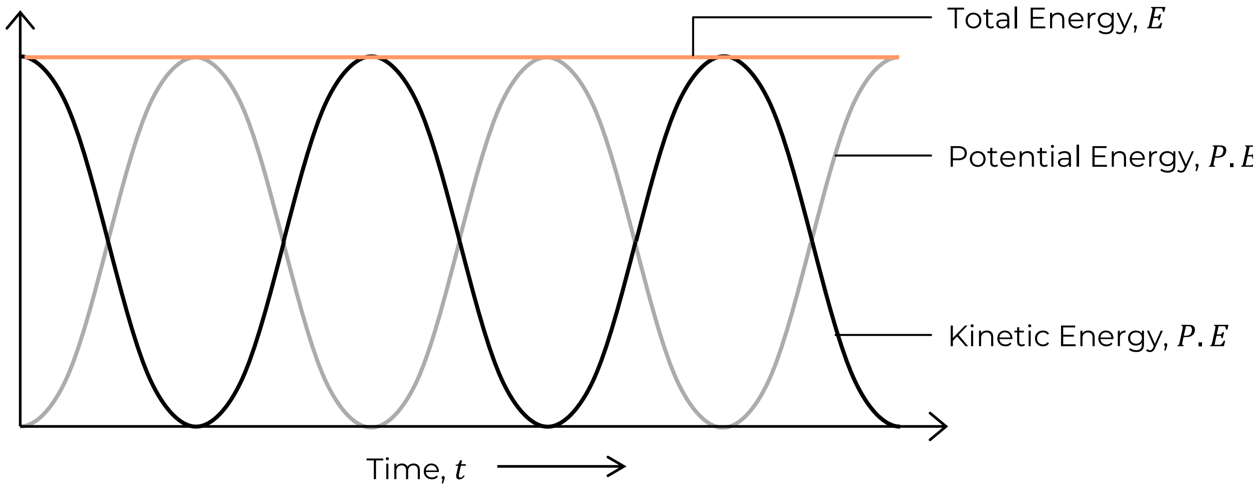
$\frac{\omega}{\gamma} = 0.005$



1. c) Graphically show that even though the potential and kinetic energies for Simple harmonic motion (SHM) vary with time, the total energy remains constant.

Solution:

The graph of potential energy, kinetic energy and total energy vs time has been plotted below:



In the graph, we can see that potential energy and kinetic energy vary with time, but total energy remains constant.

This is because of the continuous exchange between kinetic energy and potential energy. As the object moves away from its equilibrium position, it gains kinetic energy while losing potential energy, and vice versa when it returns. However, the total sum of these two energies remains constant.

2. a) A 4.0kg block extends a spring 16cm from its equilibrium position. The block is removed and 0.5kg block is hung from the same spring. If the spring stretched and released, what is the period of motion?

Solution:

We know,
Restoring force,

$$F_{\text{Restoring}} = -kx$$
$$\text{or, } -F_{\text{External}} = -kx$$
$$[\because F_{\text{Restoring}} = -F_{\text{External}}]$$
$$\text{or, } -m_1 g = -kx$$
$$\text{or, } k = \frac{m_1 g}{x}$$
$$\text{or, } k = \frac{4.0 \times 9.8}{0.16}$$
$$\therefore k = 245\ N\ m^{-1}$$

Here,
$m_1 = 4.0\ kg$
$x = 16\ cm = 0.16\ m$
$m_2 = 0.5\ kg$
$k = ?$
$T = ?$

For $m_2 = 0.5\ kg$,
Time period,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.5}{245}} = 0.28\ s$$

2. b) Suppose the block has $m = 2.75 \times 10^5\ kg$ and is designed to oscillate at frequency $f = 10.0\ Hz$ and amplitude $A = 20.0\ cm$.

i) What is the total energy E of the spring-block system?
ii) What is the KE and PE at $x = 10\ cm$
iii) At what position $KE = PE$?

Solution:

i) Here,

$$\omega = 2\pi f$$
$$= 2\pi \times 10$$
$$= 62.83\ rad\ s^{-1}$$

And,

$$k = m\omega^2$$
$$= 2.75 \times 10^5 \times (62.83)^2$$
$$= 1.086 \times 10^9\ N\ m^{-1}$$

Here,
$m = 2.75 \times 10^5\ kg$
$f = 10\ Hz$
$A = 20\ cm = 0.2\ m$
$E = ?$
$KE = ?$
$PE = ?$
$x = ?$

We know,
Total energy,

$$E = \frac{1}{2}\text{kA}^2$$
$$= \frac{1}{2} \times 1.086 \times 10^9 \times (0.2)^2$$
$$= 2.172 \times 10^7\ J$$

ii) Velocity when $x = 10\ cm = 0.1\ m$,

$$v = \pm\omega\sqrt{A^2 - x^2}$$
$$\text{or, } v^2 = \omega^2(A^2 - x^2)$$
$$\text{or, } v^2 = (62.83)^2\,[(0.2)^2 - (0.1)^2]$$
$$\therefore v^2 = 118.43\ m\ s^{-1}$$

Now,
Kinetic energy at $x = 10\ cm$,

$$KE = \frac{1}{2} mv^2$$
$$= \frac{1}{2} \times 2.75 \times 10^5 \times 118.43$$
$$= 1.628 \times 10^7\ J$$

Potential energy at $x = 10\ cm$,

$$PE = \frac{1}{2} kx^2$$
$$= \frac{1}{2} \times 1.086 \times 10^9 \times (0.1)^2$$
$$= 5.43 \times 10^6\ J$$

iii) Let, KE will equal to PE at displacement x.
Now,

$$KE = PE$$
$$\text{or, } \frac{1}{2}mv^2 = \frac{1}{2}kx^2$$
$$\text{or, } \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2}kx^2$$
$$[\because \mathrm{v}^2 = \omega^2(A^2 - x^2)]$$
$$\text{or, } \frac{1}{2}k\left(A^2 - x^2\right) = \frac{1}{2}kx^2 \quad [\because k = m\omega^2]$$
$$\text{or, } A^2 - x^2 = x^2$$
$$\text{or, } A^2 = 2x^2$$
$$\text{or, } x = \sqrt{\frac{A^2}{2}}$$
$$\text{or, } x = \sqrt{\frac{(0.2)^2}{2}}$$
$$\therefore x = 0.14\ m$$

2. c) An oscillating block has kinetic energy equal to potential energy of $25\ J$ ($KE = PE = 25\ J$) when the block is at $x = +0.50\ m$.

i) What is the amplitude of oscillation?
ii) What is the kinetic energy when the block is $x = 0$?

Solution:

i) Given,
Potential energy at $x = 0.5\ m$,

$$PE = \frac{1}{2}kx^2$$
$$\text{or, } 25 = \frac{1}{2}k(0.5)^2$$
$$\text{or, } 25 \times 2 = k \times \frac{1}{4}$$
$$\therefore k = 200\ N\ m^{-1}$$

We know,
Total energy,

$$E = PE + KE$$
$$\text{or, } \frac{1}{2}kA^2 = 25 + 25$$
$$\text{or, } \frac{1}{2} \times 200 \times A^2 = 50$$
$$\text{or, } 100 \times A^2 = 50$$
$$\text{or, } A = \sqrt{\frac{50}{100}}$$
$$\therefore A = 0.707\ m$$

Here,
$PE = 25\ J$
$KE = 25\ J$
$x = 0.5\ m$
$A = ?$

ii) We know,
At $x = 0$, potential energy is 0 and kinetic energy is equal to total energy.

Total energy at $x = 0.5\ m$,

$$E = PE + KE = 25 + 25 = 50\ J$$

$\therefore$ Kinetic energy at $x = 0$,

$$KE = E - PE = 50 - 0 = 50\ J$$

3. a) For the damped oscillator of $m = 250\ gm$, $k = 85\ N/m$, and $b = 70\ gm/s$. If it is oscillatory find the frequency of oscillator. If the initial amplitude of the system is $10\ cm$, find out amplitude after 10 oscillation. What is life time of oscillation?

Solution:

i) Here,

$$\omega^2 = \frac{k}{m} = \frac{85}{0.25} = 340$$

And,

$$\frac{\gamma^2}{4} = \frac{\left(\frac{b}{m}\right)^2}{4} = \frac{\left(\frac{0.07}{0.25}\right)^2}{4} = 0.0196$$

Since $\omega^2 > \frac{\gamma^2}{4}$, therefore it is oscillatory.

Now,

$$\omega_1 = \sqrt{\omega^2 - \frac{\gamma^2}{4}}$$
$$= \sqrt{340 - 0.0196}$$
$$= 18.44\ rad\ s^{-1}$$

Here,
$m = 250\ g$
$= 0.25\ kg$
$k = 85\ N\ m^{-1}$
$b = 70\ g\ s^{-1}$
$= 0.07\ kg\ s^{-1}n$
$A_0 = 10\ cm$
$= 0.1\ m$
$T = ?$
$A_{10} = ?$
$\tau = ?$

$\therefore$ Time period,

$$T = \frac{2\pi}{\omega_1} = \frac{2\pi}{18.44} = 0.34\ s$$

ii) Time to complete 10 oscillation,

$$t = T \times 10 = 0.34 \times 10 = 3.4\ s$$

∴ Amplitude after 10 oscillation,

$$A_{10} = A_0 e^{-\frac{\gamma}{2}t} \cos(\omega_1 t + \phi)$$
$$= A_0 e^{-\frac{b}{2m}t} \cos(\omega_1 t) \quad \text{[Considering } \phi = 0\text{]}$$
$$= 0.1 \times e^{-\frac{0.07}{2\times 0.25}\times 3.4} \times \cos(18.44 \times 3.4)$$
$$= 0.1 \times 0.62 \times 0.99$$
$$= 0.061\ m$$

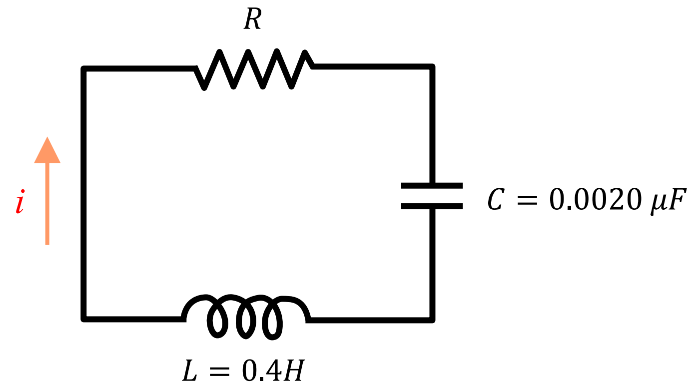
iii) We know,
Life time,

$$\tau = \frac{1}{\gamma} = \frac{1}{\frac{b}{m}} = \frac{1}{\frac{0.07}{0.25}} = 3.57\ s$$

3. b) Draw an LRC series circuit using L = 0.4 h, C = 0.0020 μF components. What is the maximum resistance for which circuit will be oscillatory?

Solution:

The RLC circuit has been drawn below:



Here,
$C = 0.0020\ \mu F$
$= 0.002 \times 10^{-6}\ F$
$L = 0.4\ H$
$R = ?$

Let, this circuit will be oscillatory for maximum resistance R.

Now,

$$\frac{R^2}{4L^2} = \frac{1}{LC}$$
$$\text{or, } R^2 = \frac{4L^2}{LC}$$
$$\text{or, } R = \sqrt{\frac{4L}{C}}$$
$$\text{or, } R = \sqrt{\frac{4 \times 0.4}{0.002 \times 10^{-6}}}$$
$$\therefore R = 28284.27\ \Omega$$

3. c) An oscillator consists of a block attached to a spring ($k = 400\ N/m$). At some time t, the position, velocity, and acceleration of the block are $x = 0.100\ m$, $v = -13.6\ m/s$, and $a = -123\ m/s^2$. Calculate

a) the mass of the block and
b) the amplitude of the motion.

Solution:

a) We know,

$$F = -kx$$
$$\text{or, } ma = -kx$$
$$\text{or, } m = -\frac{kx}{a}$$
$$\text{or, } m = -\frac{400 \times 0.1}{-123}$$
$$\therefore m = 0.325\ kg$$

b) We know,

$$v = \pm\omega\sqrt{A^2 - x^2}$$
$$\text{or, } v^2 = \omega^2\left(A^2 - x^2\right)$$
$$\text{or, } \frac{v^2}{\omega^2} = A^2 - x^2$$
$$\text{or, } \frac{v^2}{\frac{k}{m}} + x^2 = A^2$$
$$\text{or, } A = \sqrt{v^2 \times \frac{m}{k} + x^2}$$
$$\text{or, } A = \sqrt{(-13.6)^2 \times \frac{0.325}{400} + (0.1)^2}$$
$$\text{or, } A = \sqrt{(-13.6)^2 \times \frac{0.325}{400} + (0.1)^2}$$
$$\therefore A = 0.400\ m$$

Here,
$k = 400\ N\ m^{-1}$
$x = 0.1\ m$
$v = -13.6\ m\ s^{-1}$
$a = -123\ m\ s^{-2}$
$m = ?$
$A = ?$

4. a) Show that for a particle executing SHM, the instantaneous velocity is $\omega\sqrt{A^2 - x^2}$ and the maximum velocity is $\sqrt{2E/m}$, where symbols have their usual meanings

Solution:

We know,

$$E = PE + KE$$
$$\text{or, } \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$
$$\text{or, } \frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$
$$\text{or, } \frac{1}{2}mv^2 = \frac{1}{2}k\left(A^2 - x^2\right)$$
$$\text{or, } mv^2 = k\left(A^2 - x^2\right)$$
$$\text{or, } v^2 = \frac{k}{m}\left(A^2 - x^2\right)$$

Here,
E = Total energy
PE = Potential energy
KE = Kinetic energy
k = Spring constant
A = Amplitude
x = Displacement
v = Instantaneous velocity

$$\text{or, } v^2 = \omega^2(A^2 - x^2) \qquad \left[\because \omega^2 = \frac{k}{m}\right]$$

$$\text{or, } v = \sqrt{\omega^2\left(A^2 - x^2\right)}$$

$$\therefore v = \sqrt{\omega^2(A^2 - x^2)} \qquad \text{(Showed)}$$

We know,
We got maximum velocity, v_{max} at $x = 0$.
And, potential energy, PE is 0 at $x = 0$.

Now,

$$E = PE + KE$$

$$\text{or, } E = 0 + \frac{1}{2}mv_{max}^2 \quad [\because PE = 0]$$

$$\text{or, } 2E = mv_{max}^2$$

$$\text{or, } \frac{2E}{m} = v_{max}^2$$

$$\therefore v_{max} = \sqrt{\frac{2E}{m}} \qquad \text{(Showed)}$$

4. b) **Show that the phase difference between acceleration and displacement of a body executing SHM is π and that of displacement and velocity $\frac{\pi}{2}$**

Solution:

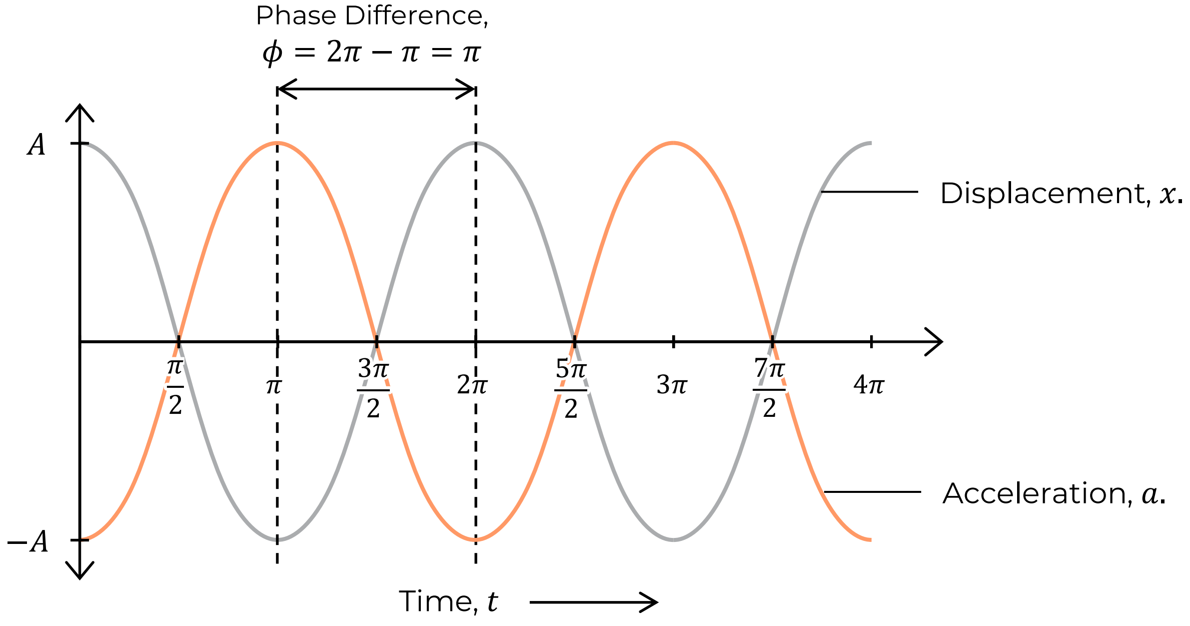
Let,

Displacement, $x = A\cos(\omega t)$

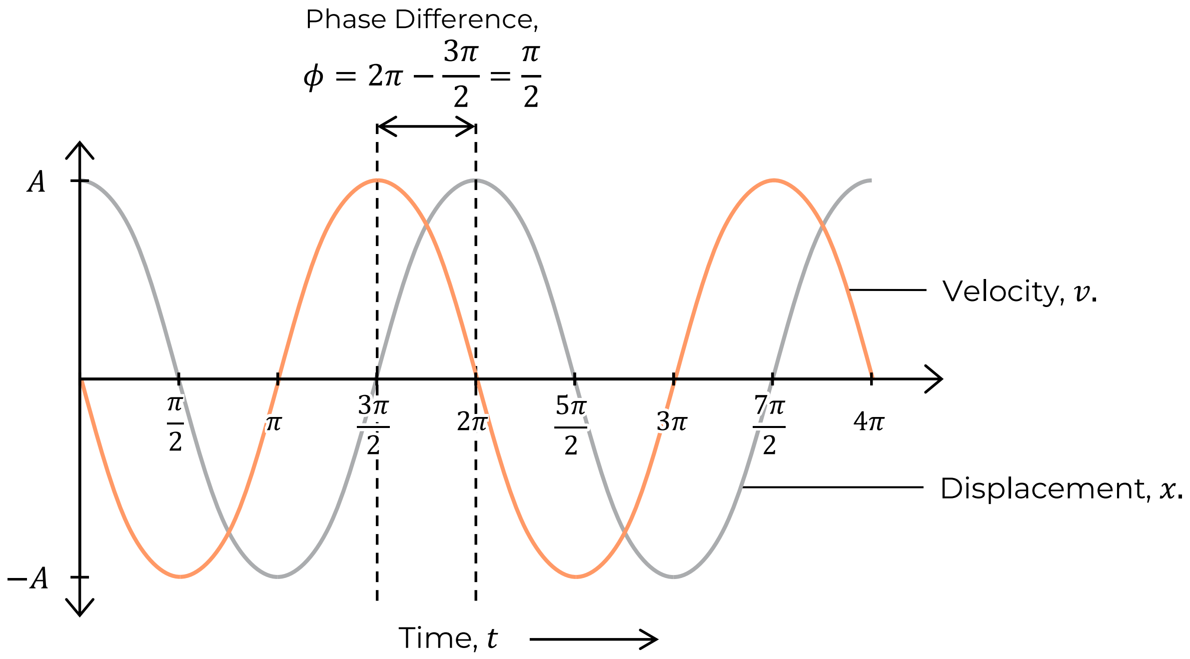
$\therefore$ Velocity, $v = \frac{dx}{dt} = -A\omega\sin(\omega t)$

$\therefore$ Acceleration, $a = \frac{dv}{dt} = -A\omega^2\cos(\omega t)$

The displacement and acceleration vs time graphs have been plotted below:



The displacement and velocity vs time graphs have been plotted below:



From the displacement and acceleration vs time graphs, we can see that phase difference between acceleration and displacement is π. And, from the displacement and velocity vs time graphs, we can see that phase difference between velocity and displacement is $\pi/2$.

∴ Phase difference between acceleration and displacement of a body executing SHM is π and that of displacement and velocity $\pi/2$. (Showed)

5. a) Derive differential equation for Simple pendulum. Find out expression for frequency of oscillation.

Solution:

A simple pendulum consists of a particle of mass m, attached to a frictionless point by a cable of length L and negligible mass.

From the figure,

Restoring force for simple pendulum,

$$F = -mg\sin\theta$$

If the angle θ is very small $\sin\theta$ is very nearly equal to θ.

Therefore,

$$F = -mg\theta \quad \text{...... (i)}$$

Here, the displacement along the arc is $x = L\theta$

∴ Acceleration,

$$a = \frac{d^2x}{dt^2} = \frac{d^2}{dt^2}(L\theta) = L\frac{d\theta}{dt^2}$$

From equation (i),

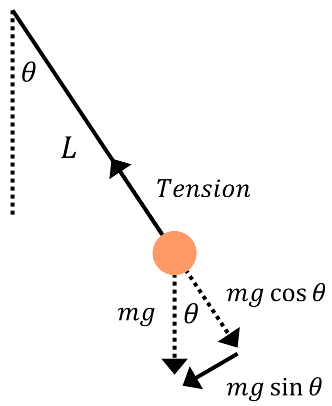


Fig: Simple Pendulum

$$\text{or, } mL\frac{d\theta}{dt^2} = -mg\theta$$

$$\text{or, } \frac{d\theta}{dt^2} = -\frac{mg\theta}{mL}$$

$$\text{or, } \frac{d\theta}{dt^2} = -\frac{g}{L}\theta$$

$$\text{or, } \frac{d\theta}{dt^2} + \frac{g}{L}\theta = 0$$

This is the differential equation for SHM in Simple Pendulum,

Where, $\omega^2 = \frac{g}{L}$

We know,

$$\omega = 2\pi f$$

$$\text{or, } \frac{\omega}{2\pi} = f$$

$$\text{or, } \frac{\sqrt{\frac{g}{L}}}{2\pi} = f$$

$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

∴ Frequency of oscillation, $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

5. b) Derive the differential equation of DHM for a mass-spring system. With proper condition, graphically represent the types of damping that may be observed in the system.

Solution:

Repeat of ***Spring 2024 Question 5(b)***