UIU QUESTION BANK

FINAL QUESTION SOLUTIONS

THEORY OF COMPUTATION

CSE 2233

SOLUTION BY

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UPDATED TILL ***FALL 2023***

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Fall 2023

1. Answer the question based on the given CFG:

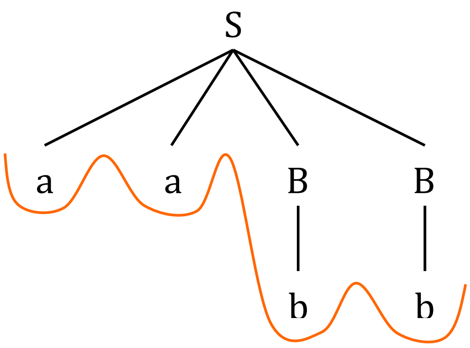
$S \rightarrow aaBB \mid aCB$
$B \rightarrow b \mid \varepsilon$
$C \rightarrow AB$
$A \rightarrow a \mid \varepsilon$

a) With the help of **Parse Tree** show that the CFG is ambiguous for the string '**aabb**'.

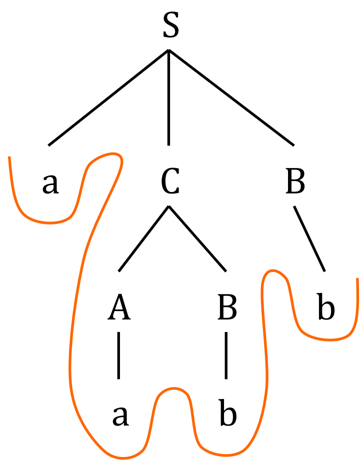
b) Modify the CFG to remove the ambiguity for the said string.

Solution:

a) First Parse Tree:



Second Parse Tree:



Since there are two different parse tree for same input string,
∴ The given CFG is ambiguous for the string 'aabb'.

b) **Removing Ambiguity:**

The language of following grammar is:
$L = \{ a^n b^m \mid n \geq 1, m \geq 0 \text{ and } n, m \leq 2 \}$

∴ After removing ambiguity, the grammar becomes:

$S \rightarrow aCB$
$B \rightarrow b \mid \varepsilon$
$C \rightarrow AB$
$A \rightarrow a \mid \varepsilon$

2. Design **CFGs** that generate the following languages:

a) $L = \{ a^n b^m c^m d^n \mid n, m \geq 1 \ \Sigma = \{a, b, c, d\} \}$

b) $L = \{ ww^R \mid w \in \{a, b\}^+ \}$

c) $L = \{ w \in \{a, b\}^+ \mid w \text{ contains at least three } 1s \}$

Solution:

a) $L = \{ a^n b^m c^m d^n \mid n, m \geq 1 \ \Sigma = \{a, b, c, d\} \}$

$$\underline{\text{CFG:}} \quad \begin{aligned} S &\rightarrow aSd \mid aAd \\ A &\rightarrow bAc \mid bc \end{aligned}$$

b) $L = \{ ww^R \mid w \in \{a, b\}^+ \}$

$$\underline{\text{CFG:}} \quad S \rightarrow aSa \mid bSb \mid aa \mid bb$$

c) $L = \{ w \in \{a, b\}^+ \mid w \text{ contains at least three } 1s \}$

$$\underline{\text{RE:}} \quad \underbrace{(0|1)^*}_{A}\ 1\ \underbrace{(0|1)^*}_{A}\ 1\ \underbrace{(0|1)^*}_{A}\ 1\ \underbrace{(0|1)^*}_{A}$$

$$\underline{\text{CFG:}} \quad \begin{aligned} S &\rightarrow A1A1A1A \\ A &\rightarrow 0A \mid 1A \mid \varepsilon \end{aligned}$$

3. Showing all necessary steps, convert the following CFGs into their equivalent **Chomsky Normal Form (CNF)**.

a)

$$\begin{aligned} S &\rightarrow ABC \mid BaB \\ A &\rightarrow aA \mid BaC \mid aaa \\ C &\rightarrow bBb \mid a \mid D \\ D &\rightarrow \varepsilon \end{aligned}$$

b)

$$\begin{aligned} S &\rightarrow BAC \mid B \\ B &\rightarrow 0B1 \mid 01 \\ A &\rightarrow aAb \mid \varepsilon \\ C &\rightarrow Bc \end{aligned}$$

Solution:

a) Given CFG:

$$\begin{aligned} S &\rightarrow ABC \mid BaB \\ A &\rightarrow aA \mid BaC \mid aaa \\ C &\rightarrow bBb \mid a \mid D \\ D &\rightarrow \varepsilon \end{aligned}$$

Step 1: Skipping this step as no starting variable is appearing on the right side.

Step 2: Removing null production:

Removing $D \rightarrow \varepsilon$:

$$\begin{aligned} S &\rightarrow ABC \mid BaB \\ A &\rightarrow aA \mid BaC \mid aaa \\ C &\rightarrow bBb \mid a \mid \varepsilon \end{aligned}$$

Removing $C \rightarrow \varepsilon$:

$$\begin{aligned} S &\rightarrow ABC \mid BaB \mid AB \\ A &\rightarrow aA \mid BaC \mid aaa \mid Ba \\ C &\rightarrow bBb \mid a \end{aligned}$$

Step 3: Skipping this step as there is no unit production left.

Step 4: Reducing rules that have length>2:

Let, $X \to AB$, $Y \to Ba$, $Z \to bB$, $W \to aa$

$$
\begin{aligned}
S &\to XC \mid YB \mid AB \\
A &\to aA \mid YC \mid Wa \mid Ba \\
C &\to Zb \mid a \\
X &\to AB \\
Y &\to Ba \\
Z &\to bB \\
W &\to aa
\end{aligned}
$$

Step 5: Bring the rules to CNF form:

Let, $P \to a$, $Q \to b$, $R \to B$

$$
\begin{aligned}
S &\to XC \mid YB \mid AB \\
A &\to PA \mid YC \mid WP \mid BP \\
C &\to ZQ \mid a \\
X &\to AB \\
Y &\to BP \\
Z &\to QB \\
W &\to PP \\
P &\to a \\
Q &\to b \\
R &\to B
\end{aligned}
$$

This is our final Chomsky Normal Form (CNF).

b) Given CFG:

$$
\begin{aligned}
S &\to BAC \mid B \\
B &\to 0B1 \mid 01 \\
A &\to aAb \mid \varepsilon \\
C &\to Bc
\end{aligned}
$$

Step 1: Skipping this step as no starting variable is appearing on the right side.

Step 2: Removing null production:

Removing $A \to \varepsilon$:

$$
\begin{aligned}
S &\to BAC \mid B \mid BC \\
B &\to 0B1 \mid 01 \\
A &\to aAb \mid ab \\
C &\to Bc
\end{aligned}
$$

Step 3: Removing unit production:

Removing $S \to B$:

$$
\begin{aligned}
S &\to BAC \mid BC \mid 0B1 \mid 01 \\
B &\to 0B1 \mid 01 \\
A &\to aAb \mid ab \\
C &\to Bc
\end{aligned}
$$

Step 4: Reducing rules that have length>2:

Let, $X \rightarrow BA$, $Y \rightarrow 0B$, $Z \rightarrow aA$

$$
\begin{aligned}
S &\rightarrow XC \mid BC \mid Y1 \mid 01 \\
B &\rightarrow Y1 \mid 01 \\
A &\rightarrow Zb \mid ab \\
C &\rightarrow Bc \\
X &\rightarrow BA \\
Y &\rightarrow 0B \\
Z &\rightarrow aA
\end{aligned}
$$

Step 5: Bring the rules to CNF form:

Let, $P \rightarrow 0$, $Q \rightarrow 1$, $T \rightarrow a$, $U \rightarrow b$, $V \rightarrow c$

$$
\begin{aligned}
S &\rightarrow XC \mid BC \mid YQ \mid PQ \\
B &\rightarrow YQ \mid PQ \\
A &\rightarrow ZU \mid TU \\
C &\rightarrow BV \\
X &\rightarrow BA \\
Y &\rightarrow PB \\
Z &\rightarrow TA \\
P &\rightarrow 0 \\
Q &\rightarrow 1 \\
T &\rightarrow a \\
U &\rightarrow b \\
V &\rightarrow c
\end{aligned}
$$

This is our final Chomsky Normal Form (CNF).

4. Draw the **Push Down Automata (PDA)** for the following languages:

a) $L = \{\, x^m \# y^n z^w \mid m = \frac{n}{2} \ or \ w = \frac{m}{3} \ and \ m, n, w > 0 \,\}$

b) $L = \{\, a^i b^j c^k \mid i + j = 2k \ and \ i, j, k \geq 0 \,\}$

Solution:

a) $L = \{\, x^m \# y^n z^w \mid m = \frac{n}{2} \ or \ w = \frac{m}{3} \ and \ m, n, w > 0 \,\}$

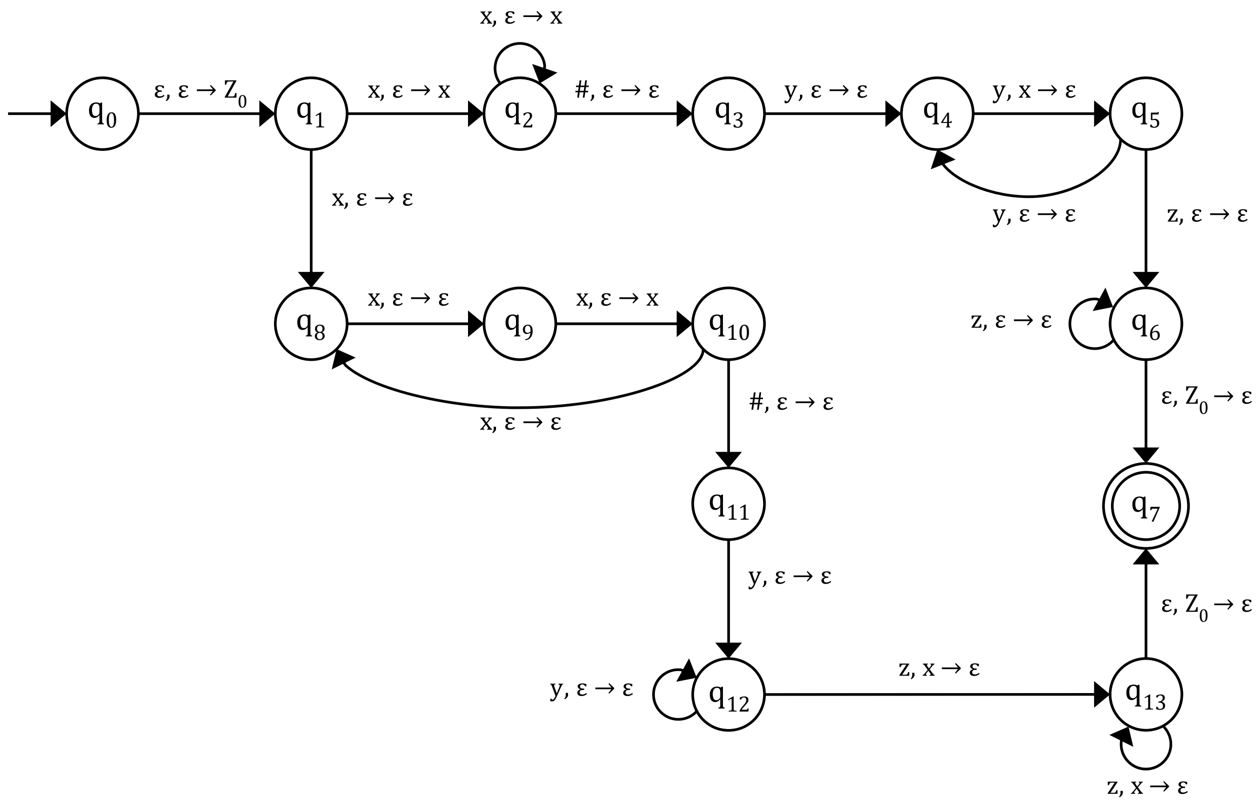
Here, $m = \dfrac{n}{2}$ or, $w = \dfrac{m}{3}$

$\therefore n = 2m$ $\qquad$ $\therefore m = 3w$

$\downarrow$ $\qquad$ $\downarrow$

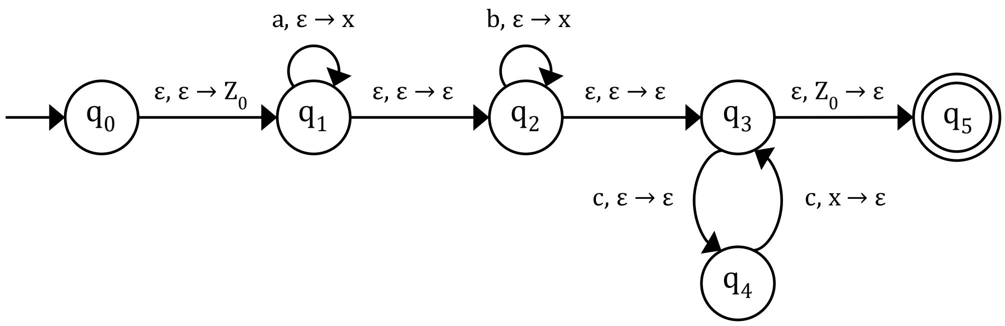
$x^m \# y^{2m} z^w$ $\qquad$ $x^{3w} \# y^n z^w$

[P.T.O]



b) $L = \{ a^i b^j c^k \mid i + j = 2k \ and \ i, j, k \geq 0 \}$

Here, our idea is we will push for 'x' for 'a' and 'b' both and pop one 'x' for every two 'c'.



5. Draw a **Turing Machine** for the following language and show the **Tape Traversal** to validate the given input:

$L = \{ a^p b^r c^q d^x \mid r = x - p \ and \ q = p + r \ and \ p, q, r, x \geq 1 \}$

Input String: $aabbbcccccddddd$

Solution:

$L = \{ a^p b^r c^q d^x \mid r = x - p \ and \ q = p + r \ and \ p, q, r, x \geq 1 \}$

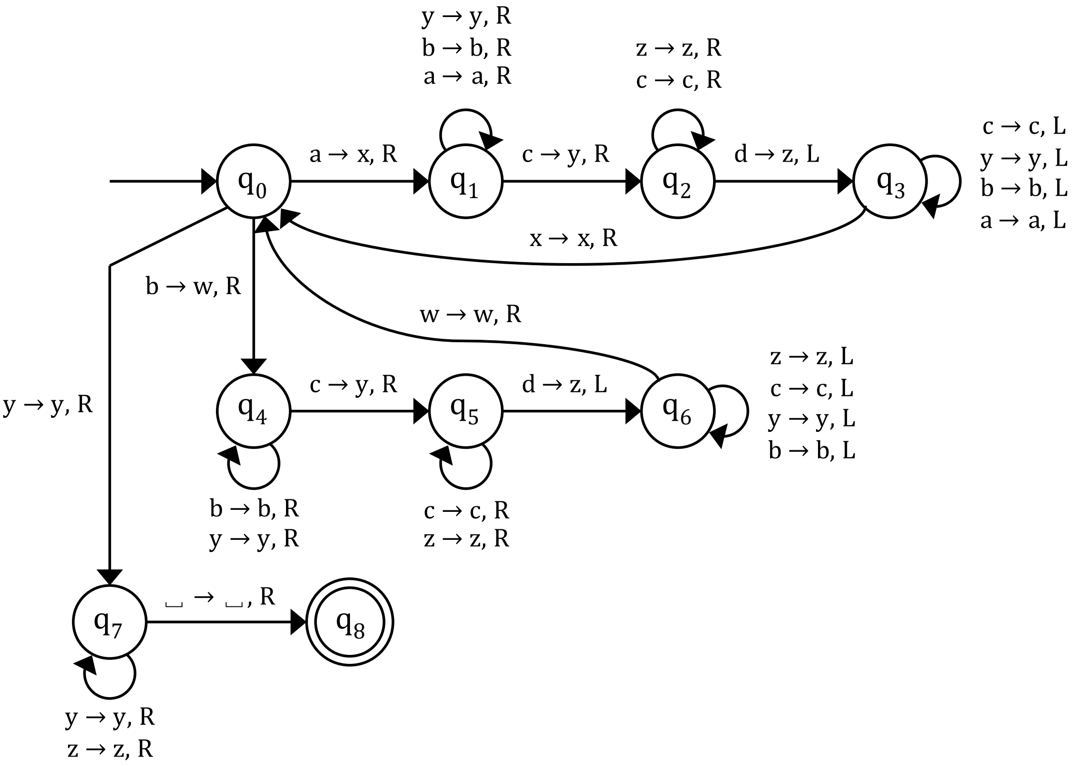
Here, $r = x - p$ and, $q = p + r$

$\therefore x = r + p$

$$a^p b^r c^q d^x$$
$$\downarrow$$
$$a^p b^r c^p c^r d^r d^p \rightarrow a^p b^r c^p c^r d^p d^r$$

[P.T.O]

Turing Machine:



Tape Traversal:

```
a a  b b b  c c  c c c  d d  d d d  ␣
↑
x a  b b b  c c  c c c  d d  d d d  ␣
  ↑
x a  b b b  c c  c c c  d d  d d d  ␣
            ↑
x a  b b b  y c  c c c  d d  d d d  ␣
              ↑
x a  b b b  y c  c c c  d d  d d d  ␣
                        ↑
x a  b b b  y c  c c c  z d  d d d  ␣
                     ↑
x a  b b b  y c  c c c  z d  d d d  ␣
↑
x a  b b b  y c  c c c  z d  d d d  ␣
  ↑
x x  b b b  y c  c c c  z d  d d d  ␣
     ↑
x x  b b b  y c  c c c  z d  d d d  ␣
              ↑
x x  b b b  y y  c c c  z d  d d d  ␣
                 ↑
x x  b b b  y y  c c c  z d  d d d  ␣
                          ↑
x x  b b b  y y  c c c  z z  d d d  ␣
                        ↑
```

```
x x  b b b  y y  c c c  z z  d d d  ␣
  ↑
x x  b b b  y y  c c c  z z  d d d  ␣
     ↑
x x  w b b  y y  c c c  z z  d d d  ␣
       ↑
x x  w b b  y y  c c c  z z  d d d  ␣
                 ↑
x x  w b b  y y  y c c  z z  d d d  ␣
                   ↑
x x  w b b  y y  y c c  z z  d d d  ␣
                             ↑
x x  w b b  y y  y c c  z z  z d d  ␣
                          ↑
x x  w b b  y y  y c c  z z  z d d  ␣
     ↑
x x  w b b  y y  y c c  z z  z d d  ␣
       ↑
x x  w w b  y y  y c c  z z  z d d  ␣
         ↑
x x  w w b  y y  y c c  z z  z d d  ␣
                   ↑
x x  w w b  y y  y y c  z z  z d d  ␣
                     ↑
x x  w w b  y y  y y c  z z  z d d  ␣
                               ↑
```

[P.T.O]

$$x\,x\;\;w\,w\;b\;\;y\,y\;\;y\,y\,c\;\;z\,z\;\;\underset{\uparrow}{z}\,z\,d\;\sqcup$$

$$x\,x\;\;w\,\underset{\uparrow}{w}\;b\;\;y\,y\;\;y\,y\,c\;\;z\,z\;\;z\,z\,d\;\sqcup$$

$$x\,x\;\;w\,w\;\underset{\uparrow}{b}\;\;y\,y\;\;y\,y\,c\;\;z\,z\;\;z\,z\,d\;\sqcup$$

$$x\,x\;\;w\,w\,w\;\;\underset{\uparrow}{y}\,y\;\;y\,y\,c\;\;z\,z\;\;z\,z\,d\;\sqcup$$

$$x\,x\;\;w\,w\,w\;\;y\,y\;\;y\,y\,\underset{\uparrow}{c}\;\;z\,z\;\;z\,z\,d\;\sqcup$$

$$x\,x\;\;w\,w\,w\;\;y\,y\;\;y\,y\,y\;\;\underset{\uparrow}{z}\,z\;\;z\,z\,d\;\sqcup$$

$$x\,x\;\;w\,w\,w\;\;y\,y\;\;y\,y\,y\;\;z\,z\;\;z\,z\,\underset{\uparrow}{d}\;\sqcup$$

$$x\,x\;\;w\,w\,w\;\;y\,y\;\;y\,y\,y\;\;z\,z\;\;z\,\underset{\uparrow}{z}\,z\;\sqcup$$

$$x\,x\;\;w\,w\,\underset{\uparrow}{w}\;\;y\,y\;\;y\,y\,y\;\;z\,z\;\;z\,z\,z\;\sqcup$$

$$x\,x\;\;w\,w\,w\;\;\underset{\uparrow}{y}\,y\;\;y\,y\,y\;\;z\,z\;\;z\,z\,z\;\sqcup$$

$$x\,x\;\;w\,w\,w\;\;y\,y\;\;y\,y\,y\;\;z\,z\;\;z\,z\,\underset{\uparrow}{z}\;\sqcup$$

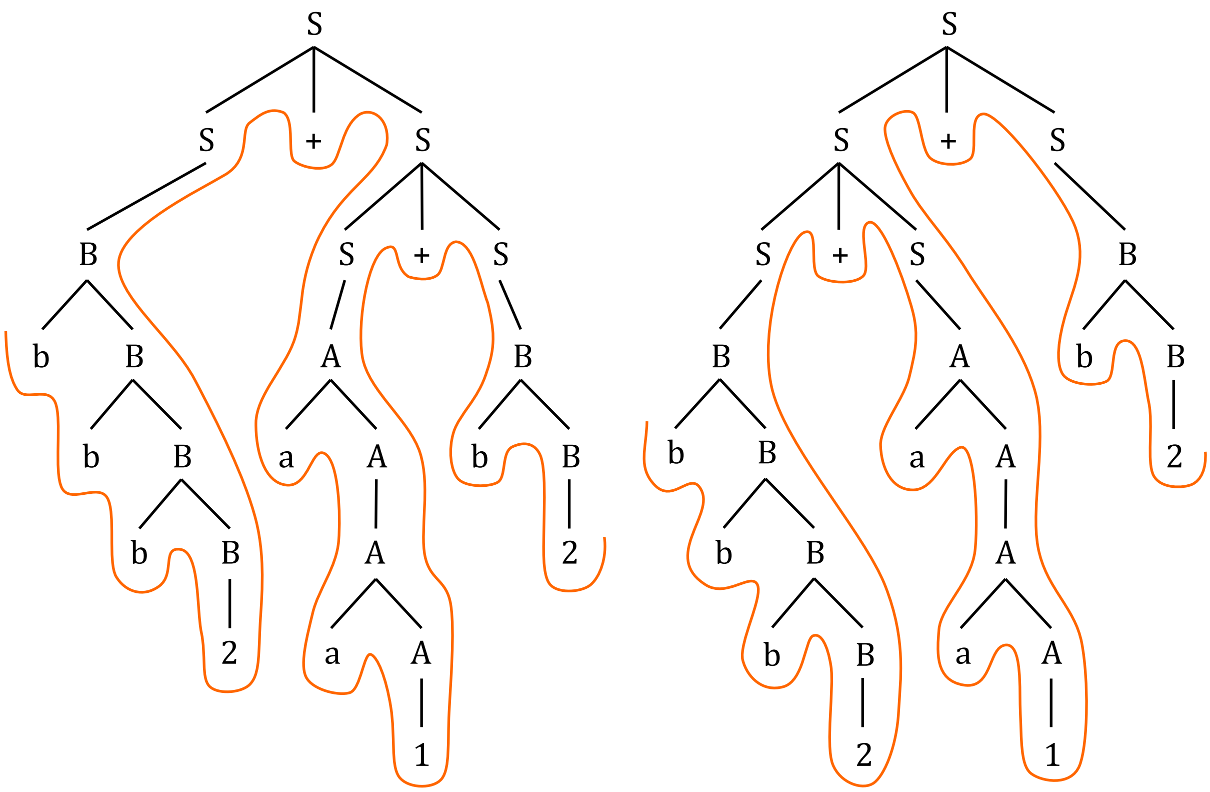
$$x\,x\;\;w\,w\,w\;\;y\,y\;\;y\,y\,y\;\;z\,z\;\;z\,z\,z\;\underset{\uparrow}{\sqcup}$$

1. Consider the following ***Context-free grammars (CFG)*** and answer according to it:

a)	S → S + S | S * S | A | B A → aA | 1 B → bB | 2	With the help of **Top-Down Parse Trees**, find-out if the grammar is Ambiguous or not for the string "bbb2 + aa1 + b2"
b)	S → S + S | S – S | (S) | T T → X * X | X % X | X X → x | y | z | Y Y → 0 | 1 | 2 | 3	With the help of **Leftmost derivation**, derive the following string "(x + 2*y) – (3*z + 1)"

Solution:

a) First Parse Tree: Second Parse Tree:



Since there are two different parse tree for same input string,
∴ The given CFG is ambiguous for the string "bbb2 + aa1 + b2".

b) **Leftmost Derivation:**

S → S – S
→ (S) – S
→ (S + S) – S
→ (T + S) – S
→ (X + S) – S
→ (x + S) – S
→ (x + T) – S
→ (x + X * X) – S
→ (x + Y * X) – S
→ (x + 2 * X) – S
→ (x + 2 * y) – S
→ (x + 2 * y) – (S)

[P.T.O]

$\rightarrow$ (x + 2 * y) – (S + S)
$\rightarrow$ (x + 2 * y) – (T + S)
$\rightarrow$ (x + 2 * y) – (X * X + S)
$\rightarrow$ (x + 2 * y) – (Y * X +S)
$\rightarrow$ (x + 2 * y) – (3 * X + S)
$\rightarrow$ (x + 2 * y) – (3 * z + S)
$\rightarrow$ (x + 2 * y) – (Y * z + T)
$\rightarrow$ (x + 2 * y) – (3 * z + X)
$\rightarrow$ (x + 2 * y) – (3 * z + Y)
$\rightarrow$ (x + 2 * y) – (3 * z + 1)

∴ "(x + 2*y) – (3*z + 1)" has been derived using Leftmost Derivation.

2. Find ***CFGs that generates*** the following languages.

a) $L = \{\, a^{m+n} c^{3n} d^{2m} \mid n,m \geq 2 \,\}$

b) $L = \{\, w$ is considered of $\{0,1\} \mid w$ is of odd length & w start and ends with same smybol $\}$

c) $L = \{\, a^i b^j c^k \mid 2i + 3j \geq 6$ and $4i - 8j \geq -16$ and $k \geq 1 \,\}$

Solution:

a) $L = \{\, a^{m+n} c^{3n} d^{2m} \mid n,m \geq 2 \,\}$

Here, $a^{m+n} c^{3n} d^{2m} \rightarrow a^m a^n c^{3n} d^{2m}$

<u>CFG:</u>
S $\rightarrow$ aaAdddd
A $\rightarrow$ aAdd | aBccc
B $\rightarrow$ aBccc | accc

b) $L = \{\, w$ is considered of $\{0,1\} \mid w$ is of odd length & w start and ends with same smybol $\}$

<u>RE:</u> $0\ \underset{A}{(0|1)}\ (\underbrace{\underset{A}{(0|1)}\underset{A}{(0|1)}}_{B})^*\ 0\ \mid\ 1\ \underset{A}{(0|1)}\ (\underbrace{\underset{A}{(0|1)}\underset{A}{(0|1)}}_{B})^*\ 1\ \mid\ 0\ \mid\ 1$

<u>CFG:</u>
S $\rightarrow$ 0AB0 | 1AB1 | 0 | 1
A $\rightarrow$ 0 | 1
B $\rightarrow$ AAB | ε

c) $L = \{\, a^i b^j c^k \mid 2i + 3j \geq 6$ and $4i - 8j \geq -16$ and $k \geq 1 \,\}$

Here, $2i + 3j \geq 6$ (i)
$4i - 8j \geq -16$ (ii)

Solving equation (i) and (ii): $i \geq 0,\ j \geq 2$

<u>CFG:</u>
S $\rightarrow$ ABC
A $\rightarrow$ aA | ε
B $\rightarrow$ bB | bb
C $\rightarrow$ cC | c

3. Convert the following ***CFG's*** into equivalent ***Chomsky Normal Form (CNF)***
[Show all the Steps]

[P.T.O]

a)
$$\begin{aligned} S &\rightarrow YXZ \mid Y \\ Y &\rightarrow 0Y1 \mid 01 \\ X &\rightarrow aXb \mid \varepsilon \\ Z &\rightarrow bZ \end{aligned}$$

b)
$$\begin{aligned} S &\rightarrow ASB \\ A &\rightarrow aAS \mid a \mid \varepsilon \\ B &\rightarrow SbS \mid A \mid bb \end{aligned}$$

Solution:

a) Given CFG:
$$\begin{aligned} S &\rightarrow YXZ \mid Y \\ Y &\rightarrow 0Y1 \mid 01 \\ X &\rightarrow aXb \mid \varepsilon \\ Z &\rightarrow bZ \end{aligned}$$

Step 1: Skipping this step as no starting variable is appearing on the right side.

Step 2: Removing null production:

Removing $X \rightarrow \varepsilon$:
$$\begin{aligned} S &\rightarrow YXZ \mid Y \mid YZ \\ Y &\rightarrow 0Y1 \mid 01 \\ X &\rightarrow aXb \mid ab \\ Z &\rightarrow bZ \end{aligned}$$

Step 3: Removing unit production:

Removing $S \rightarrow Y$:
$$\begin{aligned} S &\rightarrow YXZ \mid YZ \mid 0Y1 \mid 01 \\ Y &\rightarrow 0Y1 \mid 01 \\ X &\rightarrow aXb \mid ab \\ Z &\rightarrow bZ \end{aligned}$$

Step 4: Reducing rules that have length>2:

Let, $P \rightarrow YX, Q \rightarrow 0Y, R \rightarrow aX$
$$\begin{aligned} S &\rightarrow PZ \mid YZ \mid Q1 \mid 01 \\ Y &\rightarrow Q1 \mid 01 \\ X &\rightarrow Rb \mid ab \\ Z &\rightarrow bZ \\ P &\rightarrow YX \\ Q &\rightarrow 0Y \\ R &\rightarrow aX \end{aligned}$$

Step 5: Bring the rules to CNF form:

Let, $A \rightarrow a, B \rightarrow b, C \rightarrow 0, D \rightarrow 1$
$$\begin{aligned} S &\rightarrow PZ \mid YZ \mid QD \mid CD \\ Y &\rightarrow QD \mid CD \end{aligned}$$

$X \rightarrow RB \mid AB$
$Z \rightarrow BZ$
$P \rightarrow YX$
$Q \rightarrow CY$
$R \rightarrow AX$
$A \rightarrow a$
$B \rightarrow b$
$C \rightarrow 0$
$D \rightarrow 1$

This is our final Chomsky Normal Form (CNF).

b) Given CFG:

$S \rightarrow ASB$
$A \rightarrow aAS \mid a \mid \varepsilon$
$B \rightarrow SbS \mid A \mid bb$

Step 1: Adding new starting variable:

$S_0 \rightarrow S$
$S \rightarrow ASB$
$A \rightarrow aAS \mid a \mid \varepsilon$
$B \rightarrow SbS \mid A \mid bb$

Step 2: Removing null production:

Removing $A \rightarrow \varepsilon$:

$S_0 \rightarrow S$
$S \rightarrow ASB \mid SB$
$A \rightarrow aAS \mid a \mid aS$
$B \rightarrow SbS \mid A \mid bb \mid \varepsilon$

Removing $B \rightarrow \varepsilon$:

$S_0 \rightarrow S$
$S \rightarrow ASB \mid SB \mid AS \mid S$
$A \rightarrow aAS \mid a \mid aS$
$B \rightarrow SbS \mid A \mid bb$

Step 3: Removing unit production:

Removing $S_0 \rightarrow S$, $S \rightarrow S$, $B \rightarrow A$:

$S_0 \rightarrow ASB \mid SB \mid AS$
$S \rightarrow ASB \mid SB \mid AS$
$A \rightarrow aAS \mid a \mid aS$
$B \rightarrow SbS \mid bb \mid aAS \mid a \mid aS$

Step 4: Reducing rules that have length>2:

Let, $X \rightarrow AS$, $Y \rightarrow Sb$

$S_0 \rightarrow XB \mid SB \mid AS$
$S \rightarrow XB \mid SB \mid AS$
$A \rightarrow aX \mid a \mid aS$
$B \rightarrow YS \mid bb \mid aX \mid a \mid aS$
$X \rightarrow AS$

Y → Sb

Step 5: Bring the rules to CNF form:

Let, P → a, Q → b

S_0 → XB | SB | AS
S → XB | SB | AS
A → PX | a | PS
B → YS | QQ | PX | a | PS
X → AS
Y → SQ
P → a
Q → b

This is our final Chomsky Normal Form (CNF).

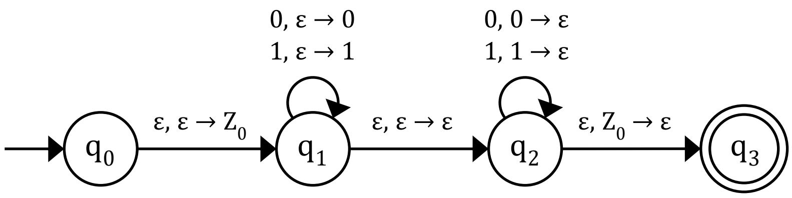
4. Draw ***Push Down Automata (PDA)*** for the following Languages

a) $L = \{ ww^R \mid w \in \{a, b\}^* \}$

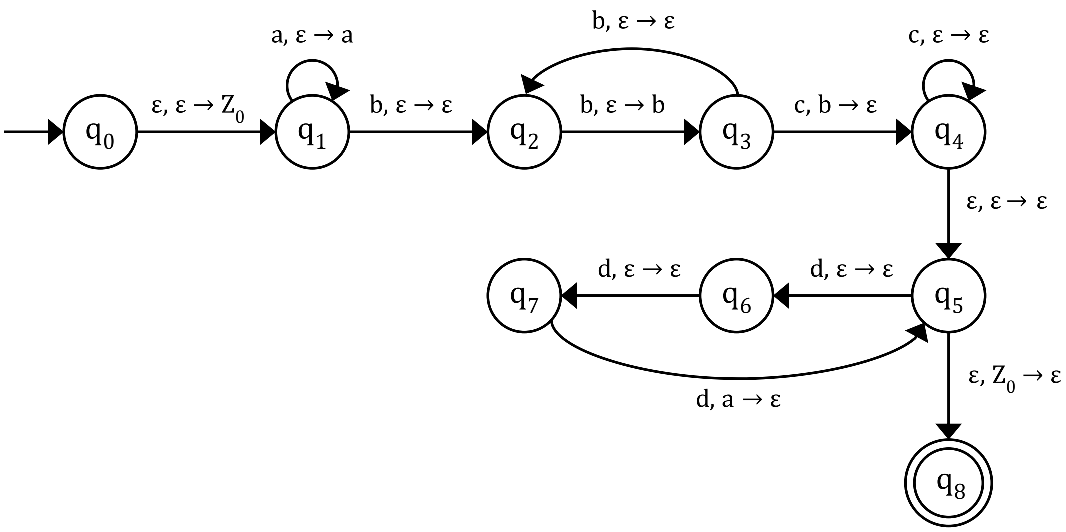
b) $L = \{ a^m b^{2n} c^n d^{3m} \mid m \geq 0, n \geq 1 \}$

Solution:

a) $L = \{ ww^R \mid w \in \{a, b\}^* \}$



b) $L = \{ a^m b^{2n} c^n d^{3m} \mid m \geq 0, n \geq 1 \}$



5. Draw ***Turing Machine*** for the following Languages and Show the ***Tape Traversal*** to ***validate*** the given input:

$L = \{ \, x^a y^b z^c \mid \text{where } a = b - c \text{ and } a, b, c \geq 1 \, \}$

Input String: *xxxyyyyyyyyyzzzzzz*

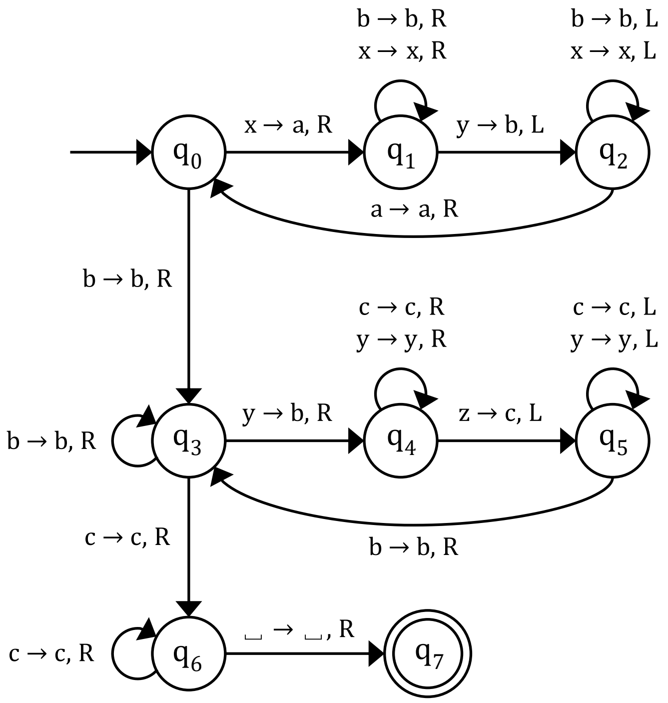
Solution:

$L = \{ \, x^a y^b z^c \mid \text{where } a = b - c \text{ and } a, b, c \geq 1 \, \}$

Here, $\quad a = b - c \;\therefore b = a + c$

$$\downarrow$$
$$x^a y^{a+c} z^c$$
$$\downarrow$$
$$x^a y^a y^c z^c$$

Turing Machine:



Tape Traversal:

```
x x x  y y y  y y y y y y  z z z z z z ␣
↑
a x x  y y y  y y y y y y  z z z z z z ␣
  ↑
a x x  y y y  y y y y y y  z z z z z z ␣
       ↑
a x x  b y y  y y y y y y  z z z z z z ␣
    ↑
a x x  b y y  y y y y y y  z z z z z z ␣
↑
a x x  b y y  y y y y y y  z z z z z z ␣
  ↑
a a x  b y y  y y y y y y  z z z z z z ␣
    ↑
a a x  b y y  y y y y y y  z z z z z z ␣
         ↑
```

```
a a x  b b y  y y y y y y  z z z z z z ␣
       ↑
a a x  b b y  y y y y y y  z z z z z z ␣
  ↑
a a x  b b y  y y y y y y  z z z z z z ␣
    ↑
a a a  b b y  y y y y y y  z z z z z z ␣
       ↑
a a a  b b y  y y y y y y  z z z z z z ␣
           ↑
a a a  b b b  y y y y y y  z z z z z z ␣
         ↑
a a a  b b b  y y y y y y  z z z z z z ␣
    ↑
a a a  b b b  y y y y y y  z z z z z z ␣
       ↑
```

[P.T.O]

```
a a a b b b y y y y y y z z z z z z ␣
            ↑
a a a b b b y y y y y y z z z z z z ␣
            ↑
a a a b b b b y y y y y z z z z z z ␣
              ↑
a a a b b b b y y y y y z z z z z z ␣
                        ↑
a a a b b b b y y y y y c z z z z z ␣
                      ↑
a a a b b b b y y y y y c z z z z z ␣
            ↑
a a a b b b b y y y y y c z z z z z ␣
              ↑
a a a b b b b b y y y y c z z z z z ␣
                ↑
a a a b b b b b y y y y c z z z z z ␣
                          ↑
a a a b b b b b y y y y c c z z z z ␣
                        ↑
a a a b b b b b y y y y c c z z z z ␣
              ↑
a a a b b b b b y y y y c c z z z z ␣
                ↑
a a a b b b b b b y y y c c z z z z ␣
                  ↑
a a a b b b b b b y y y c c z z z z ␣
                            ↑
a a a b b b b b b y y y c c c z z z ␣
                          ↑
a a a b b b b b b y y y c c c z z z ␣
                ↑
a a a b b b b b b y y y c c c z z z ␣
                  ↑
```

```
a a a b b b b b b b y y c c c z z z ␣
                    ↑
a a a b b b b b b b y y c c c z z z ␣
                              ↑
a a a b b b b b b b y y c c c c z z ␣
                            ↑
a a a b b b b b b b y y c c c c z z ␣
                  ↑
a a a b b b b b b b y y c c c c z z ␣
                    ↑
a a a b b b b b b b b y c c c c z z ␣
                      ↑
a a a b b b b b b b b y c c c c z z ␣
                                ↑
a a a b b b b b b b b y c c c c c z ␣
                              ↑
a a a b b b b b b b b y c c c c c z ␣
                    ↑
a a a b b b b b b b b y c c c c c z ␣
                      ↑
a a a b b b b b b b b b c c c c c z ␣
                        ↑
a a a b b b b b b b b b c c c c c z ␣
                                  ↑
a a a b b b b b b b b b c c c c c c ␣
                                ↑
a a a b b b b b b b b b c c c c c c ␣
                      ↑
a a a b b b b b b b b b c c c c c c ␣
                        ↑
a a a b b b b b b b b b c c c c c c ␣
                                  ↑
a a a b b b b b b b b b c c c c c c ␣
                                    ↑
```

Spring 2023

1.

E → E + E | E – E | E = E
E → MNV | MN
M → – | ε
N → 0 | 1 | 2 | 3 |4 | 5 | 6 | 7 | 8 | 9 | NN
V → x | y | z

a) With the help of **Top-Down Parse Trees** figure out if the grammar is Ambiguous or not for the string "**x + y + z = 2**".

b) Show the **Right Most Derivation** for the string **"-26x + 3y – 8z = -83".**

Solution:

a) *Not Possible.* [Most probably, printing mistake in the question]

b) **Rightmost Derivation:**

S → E = E
→ E = MN
→ E = MNN
→ E = MN3
→ E = M83
→ E = –83
→ E – E = –83
→ E – M□V□= –83
→ E – MNz = –83
→ E – M8z = –83
→ E – ε8z = –83
→ E + E – ε8z = –83
→ E + MNV – ε8z = –83
→ E + MNV –8z = –83
→ E + MNy – 8z = –83
→ E + M3y – 8z = –83
→ E + ε3y – 8z = –83
→ MNV + 3y – 8z = –83
→ MNx + 3y – 8z = –83
→ MNNx + 3y – 8z = –83
→ MN6x + 3y – 8z = –83
→ M26x + 3y – 8z = –83
→ –26x + 3y – 8z = –83

∴ "-26x + 3y – 8z = -83" has been derived using Rightmost Derivation.

2. Define a Context Free Grammar for the following languages:

a) $L = \{\, x^i y^j z^{k+1} \mid k = 2j \text{ and } i \geq 0, j > 0 \,\}$

b) $L = \{\, a^m b^n c^u d^v \mid m = \frac{n}{2}, v = \frac{u}{4}, m, n, u, v > 0 \,\}$

c) $L = \{\, c^p \# d^q g^r h \mid q = 4p, p, q \geq 0 \text{ and } r > 2 \,\}$

Solution:

a) $L = \{\, x^i y^j z^{k+1} \mid k = 2j \text{ and } i \geq 0, j > 0 \,\}$

Here, $k = 2j$

$\therefore x^i y^j z^{k+1} \rightarrow x^i y^j z^{2j+1}$

CFG:

$$
\begin{aligned}
S &\rightarrow ABz \\
A &\rightarrow xA \mid \varepsilon \\
B &\rightarrow yBzz \mid yzz
\end{aligned}
$$

b) $L = \{\, a^m b^n c^u d^v \mid m = \frac{n}{2}, v = \frac{u}{4}, m, n, u, v > 0 \,\}$

Here, $m = \dfrac{n}{2}$ and, $v = \dfrac{u}{4}$

$\therefore n = 2m$ $\therefore u = 4v$

$\therefore a^m b^n c^u d^v \rightarrow a^m b^{2m} c^{4v} d^v$

CFG:

$$
\begin{aligned}
S &\rightarrow AB \\
A &\rightarrow aAbb \mid abb \\
B &\rightarrow ccccBd \mid ccccd
\end{aligned}
$$

c) $L = \{\, c^p \# d^q g^r h \mid q = 4p, p, q \geq 0 \text{ and } r > 2 \,\}$

Here, $q = 4p$

$\therefore c^p \# d^q g^r h \rightarrow c^p \# d^{4p} g^r h$

CFG:

$$
\begin{aligned}
S &\rightarrow XGh \\
X &\rightarrow cXdddd \mid \# \\
G &\rightarrow gG \mid ggg
\end{aligned}
$$

3. Convert the following Context Free Grammars to Chomsky Normal Form (CNF)

a)

$$
\begin{aligned}
S &\rightarrow ASA \mid aB \\
A &\rightarrow B \mid S \\
B &\rightarrow b \mid \varepsilon
\end{aligned}
$$

b)

$$
\begin{aligned}
S &\rightarrow S + S \mid S - S \mid (S) \mid T \\
T &\rightarrow x \mid y \mid z \mid X \\
X &\rightarrow X * X \mid X \% X \mid Y \\
Y &\rightarrow 0 \mid 1
\end{aligned}
$$

c)

$$
\begin{aligned}
S &\rightarrow ASB \\
A &\rightarrow aAS \mid a \mid \varepsilon \\
B &\rightarrow SbS \mid A \mid bb
\end{aligned}
$$

Solution:

a) Given CFG:

$$
\begin{aligned}
S &\rightarrow ASA \mid aB \\
A &\rightarrow B \mid S \\
B &\rightarrow b \mid \varepsilon
\end{aligned}
$$

Step 1: Adding new starting variable:

$$
\begin{aligned}
S_0 &\rightarrow S \\
S &\rightarrow ASA \mid aB
\end{aligned}
$$

$A \rightarrow B \mid S$

$B \rightarrow b \mid \varepsilon$

Step 2: Removing null production:

Removing $B \rightarrow \varepsilon$:

$S_0 \rightarrow S$

$S \rightarrow ASA \mid aB \mid a$

$A \rightarrow B \mid S \mid \varepsilon$

$B \rightarrow b$

Removing $A \rightarrow \varepsilon$:

$S_0 \rightarrow S$

$S \rightarrow ASA \mid aB \mid a \mid AS \mid SA \mid S$

$A \rightarrow B \mid S$

$B \rightarrow b$

Step 3: Removing unit production:

Removing $S_0 \rightarrow S$, $S \rightarrow S$, $A \rightarrow B$, $A \rightarrow S$:

$S_0 \rightarrow ASA \mid aB \mid a \mid AS \mid SA$

$S \rightarrow ASA \mid aB \mid a \mid AS \mid SA$

$A \rightarrow b \mid ASA \mid aB \mid a \mid AS \mid SA$

$B \rightarrow b$

Step 4: Reducing rules that have length>2:

Let, $X \rightarrow AS$

$S_0 \rightarrow XA \mid aB \mid a \mid AS \mid SA$

$S \rightarrow XA \mid aB \mid a \mid AS \mid SA$

$A \rightarrow b \mid XA \mid aB \mid a \mid AS \mid SA$

$B \rightarrow b$

$X \rightarrow AS$

Step 5: Bring the rules to CNF form:

Let, $Y \rightarrow a$

$S_0 \rightarrow XA \mid YB \mid a \mid AS \mid SA$

$S \rightarrow XA \mid YB \mid a \mid AS \mid SA$

$A \rightarrow b \mid XA \mid YB \mid a \mid AS \mid SA$

$B \rightarrow b$

$X \rightarrow AS$

$Y \rightarrow a$

This is our final Chomsky Normal Form (CNF).

b) Given CFG:

$S \rightarrow S + S \mid S - S \mid (S) \mid T$

$T \rightarrow x \mid y \mid z \mid X$

$X \rightarrow X * X \mid X \% X \mid Y$

$Y \rightarrow 0 \mid 1$

Step 1: Adding new starting variable:

$S_0 \rightarrow S$

$S \rightarrow S + S \mid S - S \mid (S) \mid T$

$T \rightarrow x \mid y \mid z \mid X$
$X \rightarrow X * X \mid X \% X \mid Y$
$Y \rightarrow 0 \mid 1$

Step 2: We will skip this step as there is no null production.

Step 3: Removing unit production:

Removing $S_0 \rightarrow S,\ S \rightarrow T,\ T \rightarrow X,\ X \rightarrow Y$:

$S_0 \rightarrow S + S \mid S - S \mid (S) \mid x \mid y \mid z \mid X * X \mid X \% X \mid 0 \mid 1$
$S \rightarrow S + S \mid S - S \mid (S) \mid x \mid y \mid z \mid X * X \mid X \% X \mid 0 \mid 1$
$T \rightarrow x \mid y \mid z \mid X * X \mid X \% X \mid 0 \mid 1$
$X \rightarrow X * X \mid X \% X \mid 0 \mid 1$
$Y \rightarrow 0 \mid 1$

Step 4: Reducing rules that have length>2:

Let, $A \rightarrow S +,\ B \rightarrow S -,\ C \rightarrow (S,\ D \rightarrow X *,\ E \rightarrow X \%$

$S_0 \rightarrow AS \mid BS \mid C) \mid x \mid y \mid z \mid DX \mid EX \mid 0 \mid 1$
$S \rightarrow AS \mid BS \mid C) \mid x \mid y \mid z \mid DX \mid EX \mid 0 \mid 1$
$T \rightarrow x \mid y \mid z \mid DX \mid EX \mid 0 \mid 1$
$X \rightarrow DX \mid EX \mid 0 \mid 1$
$Y \rightarrow 0 \mid 1$
$A \rightarrow S +$
$B \rightarrow S -$
$C \rightarrow (S$
$D \rightarrow X *$
$E \rightarrow X \%$

Step 5: Bring the rules to CNF form:

Let, $M \rightarrow (,\ N \rightarrow),\ O \rightarrow *,\ P \rightarrow +,\ Q \rightarrow -,\ R \rightarrow \%$

$S_0 \rightarrow AS \mid BS \mid CN \mid x \mid y \mid z \mid DX \mid EX \mid 0 \mid 1$
$S \rightarrow AS \mid BS \mid CN \mid x \mid y \mid z \mid DX \mid EX \mid 0 \mid 1$
$T \rightarrow x \mid y \mid z \mid DX \mid EX \mid 0 \mid 1$
$X \rightarrow DX \mid EX \mid 0 \mid 1$
$Y \rightarrow 0 \mid 1$
$A \rightarrow SP$
$B \rightarrow S$
$C \rightarrow MS$
$D \rightarrow XO$
$E \rightarrow XR$
$M \rightarrow ($
$N \rightarrow)$
$O \rightarrow *$
$P \rightarrow +$
$Q \rightarrow -$

R $\rightarrow$ %

This is our final Chomsky Normal Form (CNF).

c) *Repeat of* ***Summer 2023 Question 3(b)***

4. Draw Push Down Automata (PDA) for the following Languages

a) $L = \{\, a^p b^q c^{2r} \mid p \neq q \text{ and } p, q, r \geq 0 \,\}$

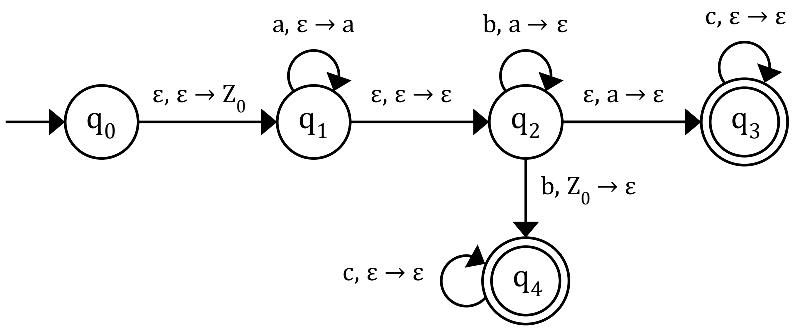
b) $L = \left\{\, 0^i 1^j 2^k \,\middle|\, (i = 3j \text{ or } j = k) \text{ and } i, j, k \geq 1 \,\right\}$

Solution:

a) $L = \{\, a^p b^q c^{2r} \mid p \neq q \text{ and } p, q, r \geq 0\}$

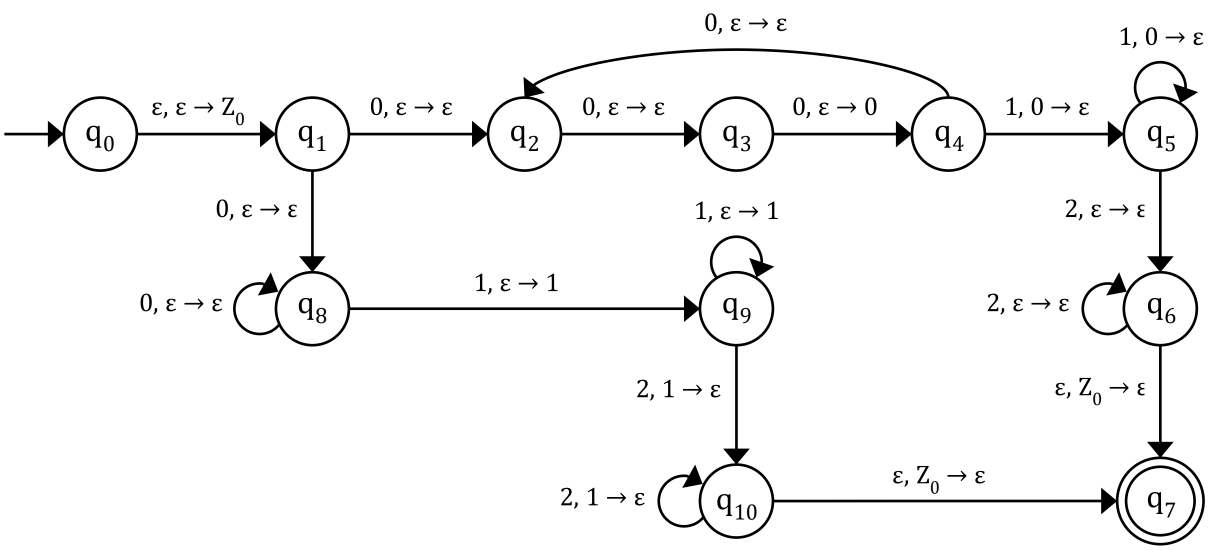
Here, $p \neq q$ ↓ $p > q$ or, $p < q$

Now, our idea is, we will push 'a' for every 'a' in the stack. If there $p > q$, then at least one 'a' will left in the stack after popping 'a' for every 'b'. If there $p < q$, then we will get Z_0 while popping 'a' for every 'b'. Here, confirming 'c' for final state is not necessary since $r \geq 0$.



b) $L = \left\{\, 0^i 1^j 2^k \,\middle|\, (i = 3j \text{ or } j = k) \text{ and } i, j, k \geq 1 \,\right\}$

Here, $i = 3j$ ↓ $0^{3j} 1^j 2^k$ or, $j = k$ ↓ $0^i 1^j 2^j$



Fall 2022

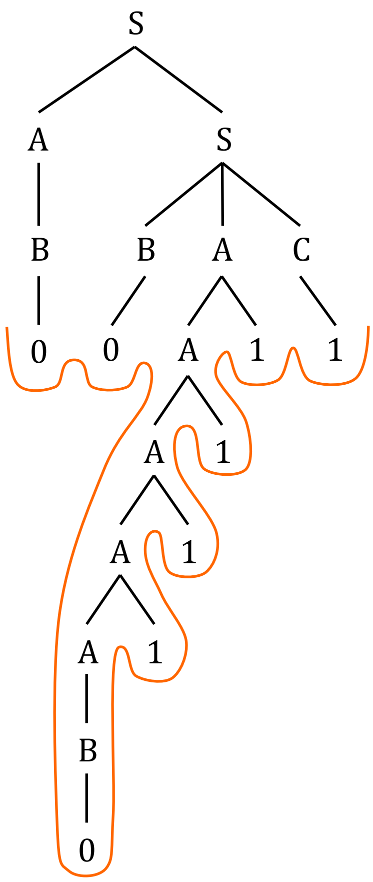
1. Consider the following ***Context-free grammars (CFG)*** and answer according to it:

a)	$S \rightarrow AS \mid BAC$ $A \rightarrow A1 \mid 0A1 \mid 0B1 \mid B$ $B \rightarrow 0B \mid 0 \mid \varepsilon$ $C \rightarrow 1 \mid \varepsilon$	With the help of ***Top-Down Parse Trees***, find-out if the grammar is ***Ambiguous*** or not for the string **00011111**
b)	$E \rightarrow E{+}E \mid E{-}E \mid (E) \mid V$ $V \rightarrow p \mid q \mid r \mid X$ $X \rightarrow X * X \mid X \% X \mid Y$ $Y \rightarrow 0 \mid 1$	With the help of **Leftmost derivation**, find-out if the grammar is ***Ambiguous*** or not for the string **p+(0*1%0)–r**

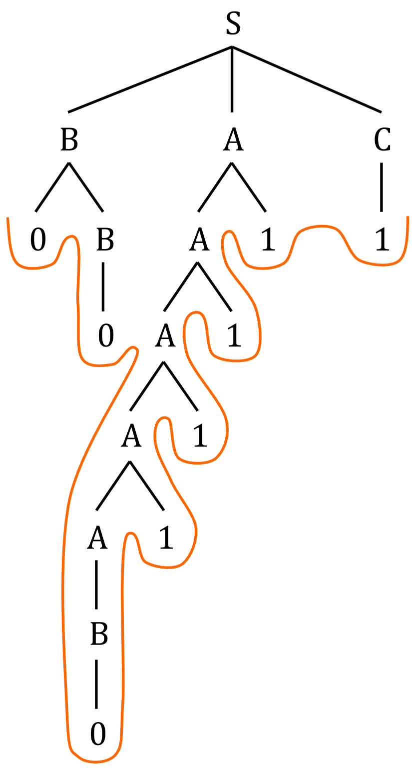
Solution:

a)

First Parse Tree:



Second Parse Tree:



Since there are two different parse tree for same input string,
∴ The given CFG is ambiguous for the string "00011111".

b) [P.T.O]

First Leftmost Derivation:

$$
\begin{aligned}
E &\rightarrow E+E\\
&\rightarrow V+E\\
&\rightarrow p+E\\
&\rightarrow p+E-E\\
&\rightarrow p+(E)-E\\
&\rightarrow p+(V)-E\\
&\rightarrow p+(X)-E\\
&\rightarrow p+(X*X)-E\\
&\rightarrow p+(Y*X)-E\\
&\rightarrow p+(0*X)-E\\
&\rightarrow p+(0*X\%X)-E\\
&\rightarrow p+(0*Y\%X)-E\\
&\rightarrow p+(0*1\%X)-E\\
&\rightarrow p+(0*1\%Y)-E\\
&\rightarrow p+(0*1\%0)-E\\
&\rightarrow p+(0*1\%0)-V\\
&\rightarrow p+(0*1\%0)-r
\end{aligned}
$$

Second Leftmost Derivation:

$$
\begin{aligned}
E &\rightarrow E-E\\
&\rightarrow E+E-E\\
&\rightarrow V+E-E\\
&\rightarrow p+E-E\\
&\rightarrow p+(E)-E\\
&\rightarrow p+(V)-E\\
&\rightarrow p+(X)-E\\
&\rightarrow p+(X*X)-E\\
&\rightarrow p+(Y*X)-E\\
&\rightarrow p+(0*X)-E\\
&\rightarrow p+(0*X\%X)-E\\
&\rightarrow p+(0*Y\%X)-E\\
&\rightarrow p+(0*1\%X)-E\\
&\rightarrow p+(0*1\%Y)-E\\
&\rightarrow p+(0*1\%0)-E\\
&\rightarrow p+(0*1\%0)-V\\
&\rightarrow p+(0*1\%0)-r
\end{aligned}
$$

Since there are two different left derivation for same input string,
∴ The given CFG is ambiguous for the string "p+(0*1%0)−r".

2. Find ***CFGs that generates*** the following languages.

a) $L = \{\, x^{2n}\#y^{3m} \mid n,m \geq 1 \,\}$, Here $\sum = \{\, x, y, \# \,\}$

b) $L = \{\, w$ is considered of $\{0,1\} \mid w$ is of even length & w start and ends with different smybol $\}$

c) $L = \{\, a^i b^j c^k \mid$ where $i \neq j$ and $k \geq 1 \,\}$

Solution:

a) $L = \{\, x^{2n}\#y^{3m} \mid n,m \geq 1 \,\}$

CFG:

$$
\begin{aligned}
S &\rightarrow A\#B\\
A &\rightarrow xA \mid xx\\
B &\rightarrow yB \mid yyy
\end{aligned}
$$

b) $L = \{\, w$ is considered of $\{0,1\} \mid w$ is of even length & w start and ends with different smybol $\}$

RE: $0\ (\underbrace{\underbrace{(0|1)}_{B}\underbrace{(0|1)}_{B}}_{A})^*\ 1 \mid 1\ (\underbrace{\underbrace{(0|1)}_{B}\underbrace{(0|1)}_{B}}_{A})^*\ 0$

CFG:

$$
\begin{aligned}
S &\rightarrow 0A1 \mid 1A0\\
A &\rightarrow BBA \mid \varepsilon\\
B &\rightarrow 0 \mid 1
\end{aligned}
$$

c) $L = \{ a^i b^j c^k \mid \text{where } i \neq j \text{ and } k \geq 1 \}$

Here, $i \neq j$

$\downarrow$

$i > j$ or, $i < j$

<u>CFG:</u>

$S \rightarrow S_{i>j} \mid S_{i<j}$
$S_{i>j} \rightarrow aXC$
$S_{i<j} \rightarrow YbC$
$X \rightarrow aXb \mid A$
$Y \rightarrow aYb \mid B$
$A \rightarrow aA \mid \varepsilon$
$B \rightarrow bB \mid \varepsilon$
$C \rightarrow cC \mid c$

3. Convert the following CFG's into equivalent ***Chomsky Normal Form (CNF)*** [Show all the Steps]

a)

$S \rightarrow aSBcD \mid BC$
$A \rightarrow AbCd \mid a$
$B \rightarrow CBA \mid \varepsilon$
$C \rightarrow c \mid \varepsilon$
$D \rightarrow d$

b)

$S \rightarrow xP \mid yQ \mid y \mid RRz$
$P \rightarrow Qxx \mid xyR \mid \varepsilon$
$Q \rightarrow yPPy \mid xy \mid zR$
$R \rightarrow x \mid y \mid PR \mid \varepsilon$

Solution:

a) Given CFG:

$S \rightarrow aSBcD \mid BC$
$A \rightarrow AbCd \mid a$
$B \rightarrow CBA \mid \varepsilon$
$C \rightarrow c \mid \varepsilon$
$D \rightarrow d$

Step 1: Adding new starting variable:

$S_0 \rightarrow S$
$S \rightarrow aSBcD \mid BC$
$A \rightarrow AbCd \mid a$
$B \rightarrow CBA \mid \varepsilon$
$C \rightarrow c \mid \varepsilon$
$D \rightarrow d$

Step 2: Removing null production:

Removing $B \rightarrow \varepsilon$:

$$
\begin{aligned}
S_0 &\rightarrow S \\
S &\rightarrow aSBcD \mid BC \mid aScD \mid C \\
A &\rightarrow AbCd \mid a \\
B &\rightarrow CBA \mid CA \\
C &\rightarrow c \mid \varepsilon \\
D &\rightarrow d
\end{aligned}
$$

Removing $C \rightarrow \varepsilon$:

$$
\begin{aligned}
S_0 &\rightarrow S \\
S &\rightarrow aSBcD \mid BC \mid aScD \mid C \mid B \mid \varepsilon \\
A &\rightarrow AbCd \mid a \\
B &\rightarrow CBA \mid CA \mid BA \mid A \\
C &\rightarrow c \\
D &\rightarrow d
\end{aligned}
$$

Removing $S \rightarrow \varepsilon$:

$$
\begin{aligned}
S_0 &\rightarrow S \\
S &\rightarrow aSBcD \mid BC \mid aScD \mid C \mid B \mid aBcD \mid acD \\
A &\rightarrow AbCd \mid a \\
B &\rightarrow CBA \mid CA \\
C &\rightarrow c \\
D &\rightarrow d
\end{aligned}
$$

Step 3: Removing unit production:

Removing $S_0 \rightarrow S$, $S \rightarrow C$, $S \rightarrow B$, $S_0 \rightarrow B$:

$$
\begin{aligned}
S_0 &\rightarrow aSBcD \mid BC \mid aScD \mid aBcD \mid acD \mid c \mid CBA \mid CA \\
S &\rightarrow aSBcD \mid BC \mid aScD \mid aBcD \mid acD \mid c \mid CBA \mid CA \\
A &\rightarrow AbCd \mid a \\
B &\rightarrow CBA \mid CA \\
C &\rightarrow c \\
D &\rightarrow d
\end{aligned}
$$

Step 4: Reducing rules that have length>2:

Let, $X \rightarrow YB$, $Y \rightarrow aS$, $Z \rightarrow cD$, $W \rightarrow aB$, $M \rightarrow CB$, $N \rightarrow Ab$, $O \rightarrow Cd$

$$
\begin{aligned}
S_0 &\rightarrow XZ \mid BC \mid YZ \mid WZ \mid aZ \mid c \mid MA \mid CA \\
S &\rightarrow XZ \mid BC \mid YZ \mid WZ \mid aZ \mid c \mid MA \mid CA \\
A &\rightarrow NO \mid a \\
B &\rightarrow MA \mid CA \\
C &\rightarrow c \\
D &\rightarrow d \\
X &\rightarrow YB \\
Y &\rightarrow aS \\
Z &\rightarrow cD \\
W &\rightarrow aB \\
M &\rightarrow CB \\
N &\rightarrow Ab \\
O &\rightarrow Cd
\end{aligned}
$$

Step 5: Bring the rules to CNF form:

Here, $C \rightarrow c$, $D \rightarrow d$

Let, $P \rightarrow a$, $Q \rightarrow b$

$S_0 \rightarrow XZ \mid BC \mid YZ \mid WZ \mid PZ \mid c \mid MA \mid CA$
$S \rightarrow XZ \mid BC \mid YZ \mid WZ \mid PZ \mid c \mid MA \mid CA$
$A \rightarrow NO \mid a$
$B \rightarrow MA \mid CA$
$C \rightarrow c$
$D \rightarrow d$
$X \rightarrow YB$
$Y \rightarrow PS$
$Z \rightarrow CD$
$W \rightarrow PB$
$M \rightarrow CB$
$N \rightarrow AQ$
$O \rightarrow CD$
$P \rightarrow a$
$Q \rightarrow b$

This is our final Chomsky Normal Form (CNF).

b) Given CFG:

$S \rightarrow xP \mid yQ \mid y \mid RRz$
$P \rightarrow Qxx \mid xyR \mid \varepsilon$
$Q \rightarrow yPPy \mid xy \mid zR$
$R \rightarrow x \mid y \mid PR \mid \varepsilon$

Step 1: Skipping this step as no starting variable is appearing on the right side.

Step 2: Removing null production:

Removing $P \rightarrow \varepsilon$:

$S \rightarrow xP \mid yQ \mid y \mid RRz \mid x$
$P \rightarrow Qxx \mid xyR$
$Q \rightarrow yPPy \mid xy \mid zR \mid yPy \mid yy$
$R \rightarrow x \mid y \mid PR \mid \varepsilon \mid R$

Removing $R \rightarrow \varepsilon$:

$S \rightarrow xP \mid yQ \mid y \mid RRz \mid x \mid Rz \mid z$
$P \rightarrow Qxx \mid xyR \mid xy$
$Q \rightarrow yPPy \mid xy \mid zR \mid yPy \mid yy \mid z$
$R \rightarrow x \mid y \mid PR \mid P$

Step 3: Removing unit production:

Removing $R \rightarrow P$:

$S \rightarrow xP \mid yQ \mid y \mid RRz \mid x \mid Rz \mid z$
$P \rightarrow Qxx \mid xyR \mid xy$
$Q \rightarrow yPPy \mid xy \mid zR \mid yPy \mid yy \mid z$
$R \rightarrow x \mid y \mid PR \mid Qxx \mid xyR \mid xy$

Step 4: Reducing rules that have length>2:

Let, $A \rightarrow RR$, $B \rightarrow Qx$, $C \rightarrow xy$, $D \rightarrow yP$, $E \rightarrow Py$

$S \rightarrow xP \mid yQ \mid y \mid Az \mid x \mid Rz \mid z$
$P \rightarrow Bx \mid CR \mid xy$
$Q \rightarrow DE \mid xy \mid zR \mid Dy \mid yy \mid z$

R → x | y | PR | Bx | CR | xy
A → RR
B → Qx
C → xy
D → yP
E → Py

Step 5: Bring the rules to CNF form:
Let, X → x, Y → y, Z → z

S → XP | YQ | Y | AZ | x | RZ | z
P → BX | CR | XY
Q → DE | XY | ZR | DY | YY | z
R → x | y | PR | BX | CR | XY
A → RR
B → QX
C → XY
D → YP
E → PY
X → x
Y → y
Z → z

This is our final Chomsky Normal Form (CNF).

4. Draw the **Push Down Automata (PDA)** for the following languages:

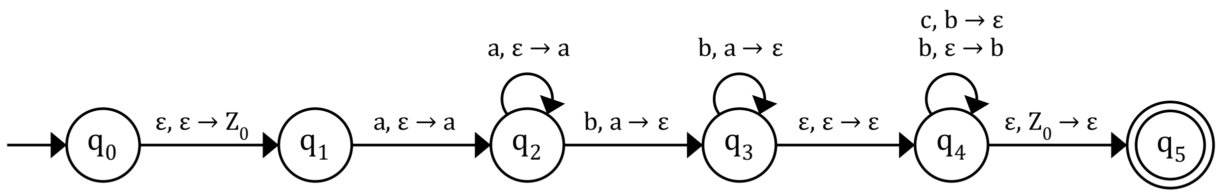
a) $L = \{a^p b^q c^r \mid \text{Where } p = q - r \text{ and } p, q > 0 \text{ and } r \geq 0\}$

b) $L = \{x^m \# y^n z^w \mid m = 2n \text{ or } w = 2m \text{ and } m, n, w > 0\}$

Solution:

a) $L = \{a^p b^q c^r \mid \text{Where } p = q - r \text{ and } p, q > 0 \text{ and } r \geq 0\}$

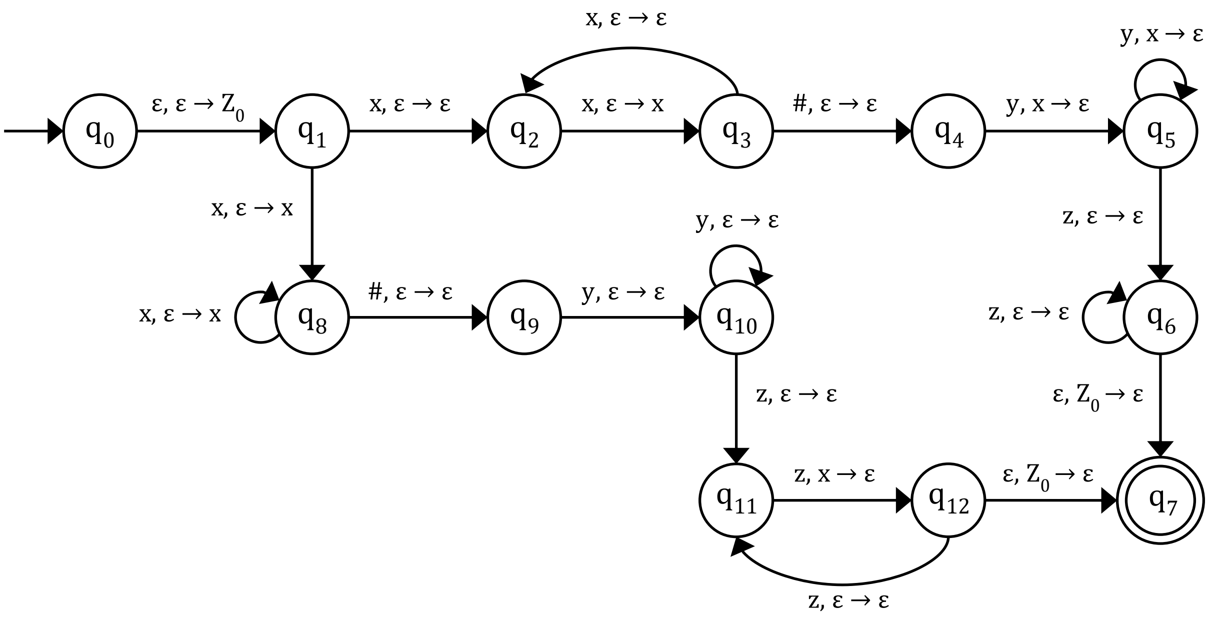
Here, $p = q - r$
$\therefore q = p + r$
$\downarrow$
$a^p b^q c^r \rightarrow a^p b^p b^r c^r$



b) $L = \{x^m \# y^n z^w \mid m = 2n \text{ or } w = 2m \text{ and } m, n, w > 0\}$

[P.T.O]

Here, $m = 2n$ $\downarrow$ $x^{2n}\#y^n z^w$ or, $w = 2m$ $\downarrow$ $x^m\#y^n z^{2m}$



5. Draw ***Turing Machine*** for the following Languages and Show the ***Tape Traversal*** for the Given input:

a) $L = \{\, a^l b^m c^n d^k \mid \text{where } k = (m+n) * l \text{ and } l, m, n, k \geq 0 \,\}$ **| Input String:** $\boldsymbol{aabccdddddd}$

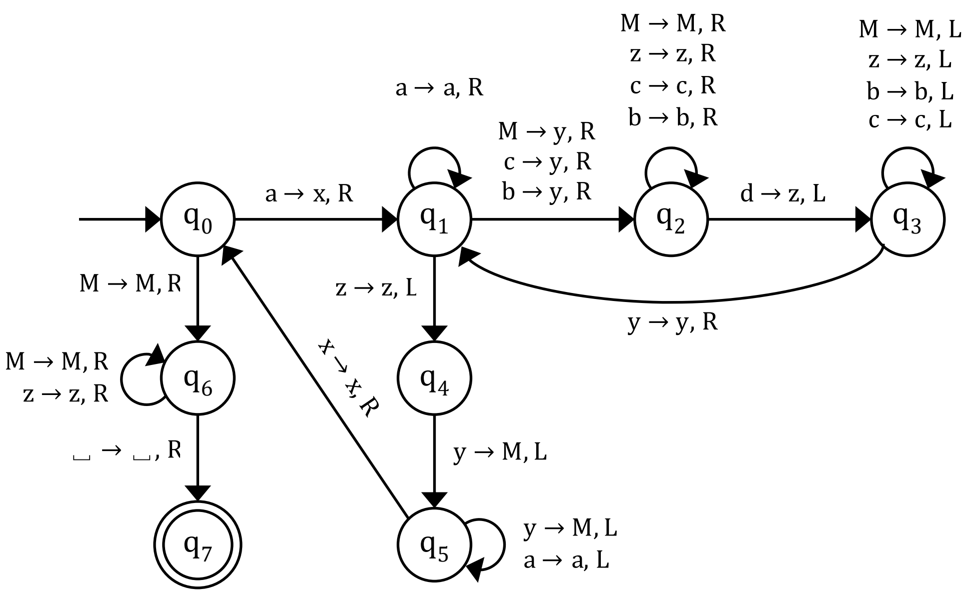
b) $L = \{\, W\#W \mid W \in \{0{,}1\} \,\}$ **| Input String: 010#010**

Solution:

$L = \{\, a^l b^m c^n d^k \mid \text{where } k = (m+n) * l \text{ and } l, m, n, k \geq 0 \,\}$

Now, here our idea is we will consider 'b' and 'c' as same symbol and after first traversal we will replace 'b' and 'c' both as a common symbol. (Here we replaced with 'M')

Turing Machine:



Tape Traversal:

[P.T.O]

$\underset{\uparrow}{a}\ a\ \ b\ \ c\ \ c\ \ d\ d\ d\ \ d\ d\ d\ \ \sqcup$

$x\ \underset{\uparrow}{a}\ \ b\ \ c\ \ c\ \ d\ d\ d\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ \underset{\uparrow}{b}\ \ c\ \ c\ \ d\ d\ d\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ \underset{\uparrow}{c}\ \ c\ \ d\ d\ d\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ c\ \ c\ \ \underset{\uparrow}{d}\ d\ d\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ c\ \ \underset{\uparrow}{c}\ \ z\ d\ d\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ \underset{\uparrow}{y}\ \ c\ \ c\ \ z\ d\ d\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ \underset{\uparrow}{c}\ \ c\ \ z\ d\ d\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ y\ \ \underset{\uparrow}{c}\ \ z\ d\ d\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ y\ \ c\ \ z\ \underset{\uparrow}{d}\ d\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ y\ \ c\ \ \underset{\uparrow}{z}\ z\ d\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ \underset{\uparrow}{y}\ \ c\ \ z\ z\ d\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ y\ \ \underset{\uparrow}{c}\ \ z\ z\ d\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ y\ \ y\ \ \underset{\uparrow}{z}\ z\ d\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ y\ \ y\ \ z\ z\ \underset{\uparrow}{d}\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ y\ \ y\ \ z\ \underset{\uparrow}{z}\ z\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ y\ \ \underset{\uparrow}{y}\ \ z\ z\ z\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ y\ \ y\ \ \underset{\uparrow}{z}\ z\ z\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ y\ \ \underset{\uparrow}{y}\ \ z\ z\ z\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ y\ \ \underset{\uparrow}{y}\ \ M\ \ z\ z\ z\ \ d\ d\ d\ \ \sqcup$

$x\ a\ \ \underset{\uparrow}{y}\ \ M\ \ M\ \ z\ z\ z\ \ d\ d\ d\ \ \sqcup$

$x\ \underset{\uparrow}{a}\ \ M\ \ M\ \ M\ \ z\ z\ z\ \ d\ d\ d\ \ \sqcup$

$\underset{\uparrow}{x}\ a\ \ M\ \ M\ \ M\ \ z\ z\ z\ \ d\ d\ d\ \ \sqcup$

$\underset{\uparrow}{x}\ a\ \ M\ \ M\ \ M\ \ z\ z\ z\ \ d\ d\ d\ \ \sqcup$

$x\ \underset{\uparrow}{a}\ \ M\ \ M\ \ M\ \ z\ z\ z\ \ d\ d\ d\ \ \sqcup$

$x\ x\ \ \underset{\uparrow}{M}\ \ M\ \ M\ \ z\ z\ z\ \ d\ d\ d\ \ \sqcup$

$x\ x\ \ y\ \ \underset{\uparrow}{M}\ \ M\ \ z\ z\ z\ \ d\ d\ d\ \ \sqcup$

$x\ x\ \ y\ \ M\ \ M\ \ z\ z\ z\ \ \underset{\uparrow}{d}\ d\ d\ \ \sqcup$

$x\ x\ \ y\ \ M\ \ M\ \ z\ z\ \underset{\uparrow}{z}\ \ z\ d\ d\ \ \sqcup$

$x\ x\ \ \underset{\uparrow}{y}\ \ M\ \ M\ \ z\ z\ z\ \ z\ d\ d\ \ \sqcup$

$x\ x\ \ y\ \ \underset{\uparrow}{M}\ \ M\ \ z\ z\ z\ \ z\ d\ d\ \ \sqcup$

$x\ x\ \ y\ \ y\ \ \underset{\uparrow}{M}\ \ z\ z\ z\ \ z\ d\ d\ \ \sqcup$

$x\ x\ \ y\ \ y\ \ M\ \ z\ z\ z\ \ z\ \underset{\uparrow}{d}\ d\ \ \sqcup$

$x\ x\ \ y\ \ y\ \ M\ \ z\ z\ z\ \ \underset{\uparrow}{z}\ z\ d\ \ \sqcup$

$x\ x\ \ y\ \ \underset{\uparrow}{y}\ \ M\ \ z\ z\ z\ \ z\ z\ d\ \ \sqcup$

$x\ x\ \ y\ \ y\ \ \underset{\uparrow}{M}\ \ z\ z\ z\ \ z\ z\ d\ \ \sqcup$

$x\ x\ \ y\ \ y\ \ y\ \ \underset{\uparrow}{z}\ z\ z\ \ z\ z\ d\ \ \sqcup$

$x\ x\ \ y\ \ y\ \ y\ \ z\ z\ z\ \ z\ z\ \underset{\uparrow}{d}\ \ \sqcup$

$x\ x\ \ y\ \ y\ \ y\ \ z\ z\ z\ \ z\ \underset{\uparrow}{z}\ z\ \ \sqcup$

$x\ x\ \ y\ \ y\ \ \underset{\uparrow}{y}\ \ z\ z\ z\ \ z\ z\ z\ \ \sqcup$

$x\ x\ \ y\ \ y\ \ y\ \ \underset{\uparrow}{z}\ z\ z\ \ z\ z\ z\ \ \sqcup$

$x\ x\ \ y\ \ y\ \ \underset{\uparrow}{y}\ \ z\ z\ z\ \ z\ z\ z\ \ \sqcup$

$x\ x\ \ y\ \ \underset{\uparrow}{y}\ \ M\ \ z\ z\ z\ \ z\ z\ z\ \ \sqcup$

$x\ x\ \ \underset{\uparrow}{y}\ \ M\ \ M\ \ z\ z\ z\ \ z\ z\ z\ \ \sqcup$

$x\ x\ \ \underset{\uparrow}{y}\ \ M\ \ M\ \ z\ z\ z\ \ z\ z\ z\ \ \sqcup$

$x\ \underset{\uparrow}{x}\ \ M\ \ M\ \ M\ \ z\ z\ z\ \ z\ z\ z\ \ \sqcup$

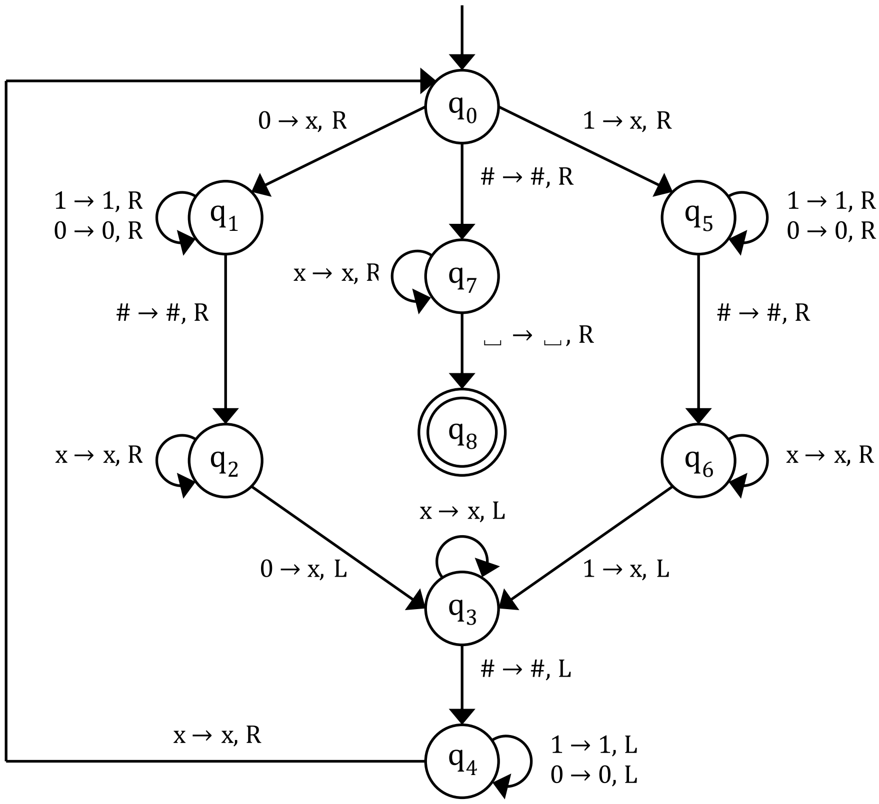
$x\ x\ \ \underset{\uparrow}{M}\ \ M\ \ M\ \ z\ z\ z\ \ z\ z\ z\ \ \sqcup$

$x\ x\ \ M\ \ M\ \ M\ \ z\ z\ z\ \ z\ z\ \underset{\uparrow}{z}\ \ \sqcup$

$x\ x\ \ M\ \ M\ \ M\ \ z\ z\ z\ \ z\ z\ z\ \ \underset{\uparrow}{\sqcup}$

b) $L = \{\, W\#W \mid W \in \{0{,}1\} \,\}$

Turing Machine:



Tape Traversal:

```
0 1 0 # 0 1 0 ␣        x x 0 # x x 0 ␣
↑                            ↑
x 1 0 # 0 1 0 ␣        x x 0 # x x 0 ␣
  ↑                      ↑
x 1 0 # 0 1 0 ␣        x x 0 # x x 0 ␣
      ↑                    ↑
x 1 0 # 0 1 0 ␣        x x x # x x 0 ␣
        ↑                    ↑
x 1 0 # x 1 0 ␣        x x x # x x 0 ␣
      ↑                            ↑
x 1 0 # x 1 0 ␣        x x x # x x x ␣
↑                                ↑
x 1 0 # x 1 0 ␣        x x x # x x x ␣
  ↑                          ↑
x x 0 # x 1 0 ␣        x x x # x x x ␣
    ↑                      ↑
x x 0 # x 1 0 ␣        x x x # x x x ␣
      ↑                      ↑
x x 0 # x 1 0 ␣        x x x # x x x ␣
          ↑                        ↑
x x 0 # x x 0 ␣        x x x # x x x ␣
        ↑                            ↑
```

Summer 2022

1. Consider the following context-free grammars (CFG). With the help of **Top-Down Parse Tree** decide whether the grammars are ambiguous or not:

a)

S	→	2BA | 1S | 2A	
B	→	1B3 | 1S3 | ε	**211211313**
A	→	A11 | 12AS3 | B | ε	

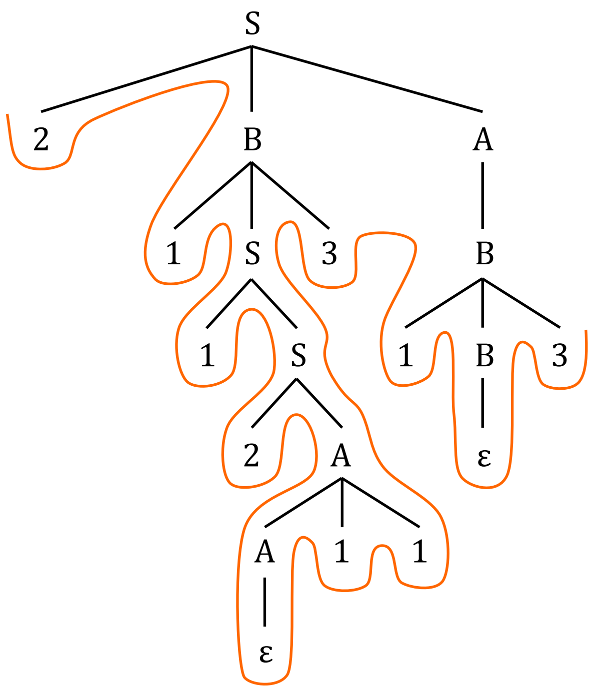
b)

B	→	11BS | 0S0B | ε	
S	→	AC01 | 0S | 1S | A1	**011010**
A	→	1 | B | CA | ε	
C	→	x | y | A	

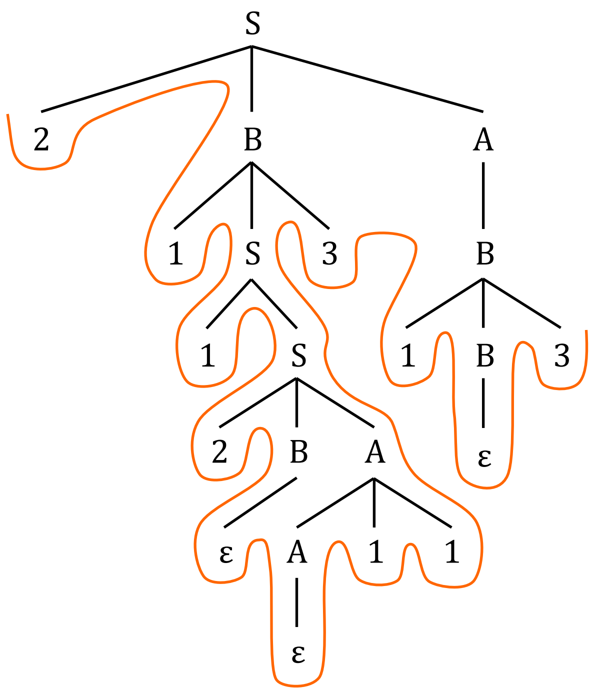
Solution:

a)

First Parse Tree:



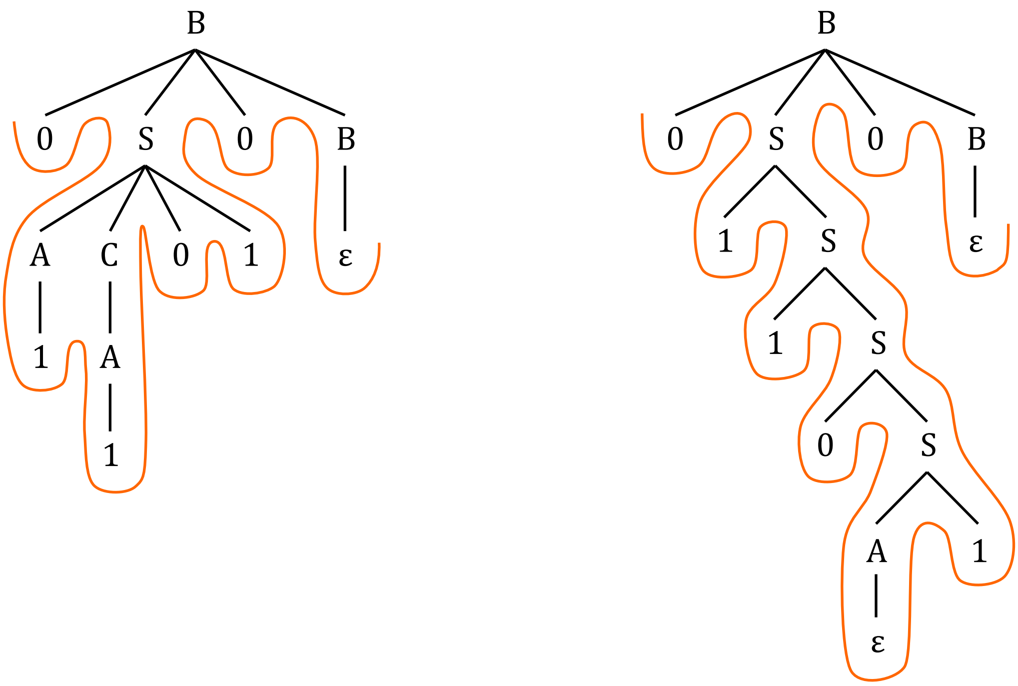
Second Parse Tree:



Since there are two different parse tree for same input string,
∴ The given CFG is ambiguous for the string '211211313'.

a) [P.T.O]

First Parse Tree: Second Parse Tree:



Since there are two different parse tree for same input string,
∴ The given CFG is ambiguous for the string '011010'.

2. Find a CFG that generates the following languages.

a) $L = \{\, a^m b^n c^{3n} d^{2m} \mid \text{where } m, n \geq 1 \,\}$

b) $L = \{\, x^i y^j z^k \mid \text{where } i = k \text{ or } j = k \text{ and } i, j, k \geq 0 \,\}$

c) $L = \{\, w \text{ is considered of } \{0{,}1\} \mid |w| \text{ is odd and smybol is } 0 \,\}$

Solution:

a) $L = \{\, a^m b^n c^{3n} d^{2m} \mid \text{where } m, n \geq 1 \,\}$

<u>CFG:</u>

$$\begin{aligned} S &\rightarrow aSdd \mid aAdd \\ A &\rightarrow bAccc \mid bccc \end{aligned}$$

b) $L = \{\, x^i y^j z^k \mid \text{where } i = k \text{ or } j = k \text{ and } i, j, k \geq 0 \,\}$

Here, $i = k \;\downarrow\; x^i y^j z^i$ or, $j = k \;\downarrow\; x^i y^j z^j$

<u>CFG:</u>

$$\begin{aligned} S &\rightarrow S_{i=k} \mid S_{j=k} \\ S_{i=k} &\rightarrow xS_{i=k}z \mid Y \mid \varepsilon \\ S_{j=k} &\rightarrow XA \\ Y &\rightarrow yY \mid \varepsilon \\ X &\rightarrow xX \mid \varepsilon \\ A &\rightarrow yAz \mid \varepsilon \end{aligned}$$

c) $L = \{ w \text{ is considered of } \{0,1\} \mid |w| \text{ is odd and smybol is } 0 \}$

CFG: $S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0$

3. Convert the following CFGs into the equivalent Chomsky Normal Form (CNF) [Show all the Steps]

a)

$A \rightarrow 1 \mid B \mid CA \mid \varepsilon$
$B \rightarrow 1BS \mid 0S0B \mid \varepsilon$
$C \rightarrow x \mid y \mid A$
$S \rightarrow 1A1 \mid 0S \mid S \mid A1$

b)

$W \rightarrow 2XY \mid 1W \mid 2Y$
$X \rightarrow 1X3 \mid 1W3 \mid \varepsilon$
$Y \rightarrow Y11 \mid 12YW3 \mid X \mid \varepsilon$

Solution:

a) Given CFG:

$A \rightarrow 1 \mid B \mid CA \mid \varepsilon$
$B \rightarrow 1BS \mid 0S0B \mid \varepsilon$
$C \rightarrow x \mid y \mid A$
$S \rightarrow 1A1 \mid 0S \mid S \mid A1$

Step 1: Adding new starting variable:

$A_0 \rightarrow A$
$A \rightarrow 1 \mid B \mid CA \mid \varepsilon$
$B \rightarrow 1BS \mid 0S0B \mid \varepsilon$
$C \rightarrow x \mid y \mid A$
$S \rightarrow 1A1 \mid 0S \mid S \mid A1$

Step 2: Removing null production:

Removing $A \rightarrow \varepsilon$:

$A_0 \rightarrow A \mid \varepsilon$
$A \rightarrow 1 \mid B \mid CA \mid C$
$B \rightarrow 1BS \mid 0S0B \mid \varepsilon$
$C \rightarrow x \mid y \mid A \mid \varepsilon$
$S \rightarrow 1A1 \mid 0S \mid S \mid A1 \mid 11 \mid 1$

Removing $B \rightarrow \varepsilon$:

$A_0 \rightarrow A$
$A \rightarrow 1 \mid B \mid CA \mid C$
$B \rightarrow 1BS \mid 0S0B \mid 1S \mid 0S0$
$C \rightarrow x \mid y \mid A \mid \varepsilon$
$S \rightarrow 1A1 \mid 0S \mid S \mid A1 \mid 11 \mid 1$

Removing $C \rightarrow \varepsilon$:

$A_0 \rightarrow A \mid \varepsilon$
$A \rightarrow 1 \mid B \mid CA \mid C \mid A$
$B \rightarrow 1BS \mid 0S0B \mid 1S \mid 0S0$
$C \rightarrow x \mid y \mid A$
$S \rightarrow 1A1 \mid 0S \mid S \mid A1 \mid 11 \mid 1$

Removing $A_0 \rightarrow \varepsilon$:

$A_0 \rightarrow A$
$A \rightarrow 1 \mid B \mid CA \mid C \mid A$
$B \rightarrow 1BS \mid 0S0B \mid 1S \mid 0S0$
$C \rightarrow x \mid y \mid A$
$S \rightarrow 1A1 \mid 0S \mid S \mid A1 \mid 11 \mid 1$

Step 3: Removing unit production:

Removing $A_0 \rightarrow A$, $A \rightarrow B$, $A \rightarrow C$, $A \rightarrow C$, $A \rightarrow A$, $C \rightarrow A$, $S \rightarrow S$:

$$
\begin{array}{lcl}
A_0 & \rightarrow & 1 \mid CA \mid 1BS \mid 0S0B \mid 1S \mid 0S0 \mid x \mid y \\
A & \rightarrow & 1 \mid CA \mid 1BS \mid 0S0B \mid 1S \mid 0S0 \mid x \mid y \\
B & \rightarrow & 1BS \mid 0S0B \mid 1S \mid 0S0 \\
C & \rightarrow & x \mid y \mid 1 \mid CA \mid 1BS \mid 0S0B \mid 1S \mid 0S0 \\
S & \rightarrow & 1A1 \mid 0S \mid A1 \mid 11 \mid 1
\end{array}
$$

Step 4: Reducing rules that have length>2:

Let, $X \rightarrow 1B$, $Y \rightarrow 0S$, $Z \rightarrow 0B$, $W \rightarrow 1A$

$$
\begin{array}{lcl}
A_0 & \rightarrow & 1 \mid CA \mid XS \mid YZ \mid 1S \mid Y0 \mid x \mid y \\
A & \rightarrow & 1 \mid CA \mid XS \mid YZ \mid 1S \mid Y0 \mid x \mid y \\
B & \rightarrow & XS \mid YZ \mid 1S \mid Y0 \\
C & \rightarrow & x \mid y \mid 1 \mid CA \mid XS \mid YZ \mid 1S \mid Y0 \\
S & \rightarrow & W1 \mid 0S \mid A1 \mid 11 \mid 1 \\
X & \rightarrow & 1B \\
Y & \rightarrow & 0S \\
Z & \rightarrow & 0B \\
W & \rightarrow & 1A
\end{array}
$$

Step 5: Bring the rules to CNF form:

Let, $P \rightarrow 1$, $Q \rightarrow 0$

$$
\begin{array}{lcl}
A_0 & \rightarrow & 1 \mid CA \mid XS \mid YZ \mid PS \mid YQ \mid x \mid y \\
A & \rightarrow & 1 \mid CA \mid XS \mid YZ \mid PS \mid YQ \mid x \mid y \\
B & \rightarrow & XS \mid YZ \mid PS \mid YQ \\
C & \rightarrow & x \mid y \mid 1 \mid CA \mid XS \mid YZ \mid PS \mid YQ \\
S & \rightarrow & WP \mid QS \mid AP \mid PP \mid 1 \\
X & \rightarrow & PB \\
Y & \rightarrow & QS \\
Z & \rightarrow & QB \\
W & \rightarrow & PA \\
P & \rightarrow & 1 \\
Q & \rightarrow & 0
\end{array}
$$

This is our final Chomsky Normal Form (CNF).

b) Given CFG:

$$
\begin{array}{lcl}
W & \rightarrow & 2XY \mid 1W \mid 2Y \\
X & \rightarrow & 1X3 \mid 1W3 \mid \varepsilon \\
Y & \rightarrow & Y11 \mid 12YW3 \mid X \mid \varepsilon
\end{array}
$$

Step 1: Adding new starting variable:

$$
\begin{array}{lcl}
S & \rightarrow & W \\
W & \rightarrow & 2XY \mid 1W \mid 2Y \\
X & \rightarrow & 1X3 \mid 1W3 \mid \varepsilon \\
Y & \rightarrow & Y11 \mid 12YW3 \mid X \mid \varepsilon
\end{array}
$$

Step 2: Removing null production:

Removing $X \rightarrow \varepsilon$:

$$\begin{aligned} S &\rightarrow W \\ W &\rightarrow 2XY \mid 1W \mid 2Y \\ X &\rightarrow 1X3 \mid 1W3 \mid \varepsilon \\ Y &\rightarrow Y11 \mid 12YW3 \mid X \mid \varepsilon \end{aligned}$$

Removing $Y \rightarrow \varepsilon$:

$$\begin{aligned} S &\rightarrow W \\ W &\rightarrow 2XY \mid 1W \mid 2Y \mid 2X \mid 2 \\ X &\rightarrow 1X3 \mid 1W3 \mid 13 \\ Y &\rightarrow Y11 \mid 12YW3 \mid X \mid 11 \mid 12W3 \end{aligned}$$

Step 3: Removing unit production:

Removing $S \rightarrow A$, $Y \rightarrow X$:

$$\begin{aligned} S &\rightarrow 2XY \mid 1W \mid 2Y \mid 2X \mid 2 \\ W &\rightarrow 2XY \mid 1W \mid 2Y \mid 2X \mid 2 \\ X &\rightarrow 1X3 \mid 1W3 \mid 13 \\ Y &\rightarrow Y11 \mid 12YW3 \mid 11 \mid 12W3 \mid 1X3 \mid 1W3 \mid 13 \end{aligned}$$

Step 4: Reducing rules that have length>2:

Let, $A \rightarrow 2X$, $B \rightarrow 1X$, $C \rightarrow 1W$, $D \rightarrow EY$, $E \rightarrow 12$, $F \rightarrow W3$, $G \rightarrow X3$, $H \rightarrow 11$

$$\begin{aligned} S &\rightarrow AY \mid 1W \mid 2Y \mid 2X \mid 2 \\ W &\rightarrow AY \mid 1W \mid 2Y \mid 2X \mid 2 \\ X &\rightarrow B3 \mid C3 \mid 13 \\ Y &\rightarrow YH \mid DF \mid 11 \mid EF \mid 1G \mid 1F \mid 13 \\ A &\rightarrow 2X \\ B &\rightarrow 1X \\ C &\rightarrow 1W \\ D &\rightarrow EY \\ E &\rightarrow 12 \\ F &\rightarrow W3 \\ G &\rightarrow X3 \\ H &\rightarrow 11 \end{aligned}$$

Step 5: Bring the rules to CNF form:

Let, $P \rightarrow 1$, $Q \rightarrow 2$, $R \rightarrow 3$

$$\begin{aligned} S &\rightarrow AY \mid PW \mid QY \mid QX \mid 2 \\ W &\rightarrow AY \mid PW \mid QY \mid QX \mid 2 \\ X &\rightarrow BR \mid CR \mid PR \\ Y &\rightarrow YH \mid DF \mid PP \mid EF \mid PG \mid PF \mid PR \\ A &\rightarrow QX \\ B &\rightarrow PX \\ C &\rightarrow PW \\ D &\rightarrow EY \\ E &\rightarrow PQ \\ F &\rightarrow WR \\ G &\rightarrow XR \\ H &\rightarrow PP \end{aligned}$$

$$\begin{aligned} P &\rightarrow 1 \\ Q &\rightarrow 2 \\ R &\rightarrow 3 \end{aligned}$$

This is our final Chomsky Normal Form (CNF).

4. a) Draw Push Down Automata (PDA) for the Language
$L = \{\, a^m b^n c^k \mid \text{where } k = m - n \text{ and } m \geq 1 \text{ and } n \geq 1 \,\}$

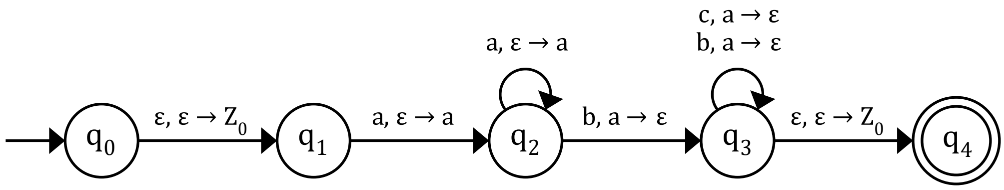
b) Draw Push Down Automata (PDA) for the Language
$L = \{\, W \text{ which is an Odd Palindrome where } W \in \{0, 1\} \,\}$

Solution:

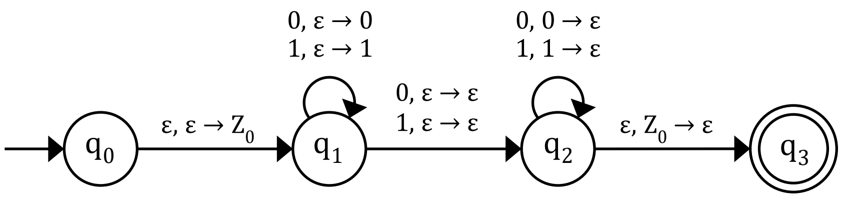
a) $L = \{\, a^m b^n c^k \mid \text{where } k = m - n \text{ and } m \geq 1 \text{ and } n \geq 1 \,\}$

Here,

$$k = m - n$$
$$\therefore m = k + n$$
$$\downarrow$$
$$a^m b^n c^k \rightarrow a^k a^n b^n c^k$$



b) $L = \{\, W \text{ which is an Odd Palindrome where } W \in \{0, 1\} \,\}$



5. Draw Turing Machine for the following Languages and Show the Tape Traversal for the Given input

a) $L = \{\, a^m b^n c^k \mid \text{where } m = \frac{k}{n} \text{ and } m, n, k \geq 1 \,\}$ **| Input String:** $\boldsymbol{aabbbcccccc}$

b) $L = \{\, W\#W^R \mid W \in \{x, y\} \text{ and } W^R \text{ is the reverse string of } W \,\}$ **| Input String:** $\boldsymbol{xyy\#yyx}$

Solution:

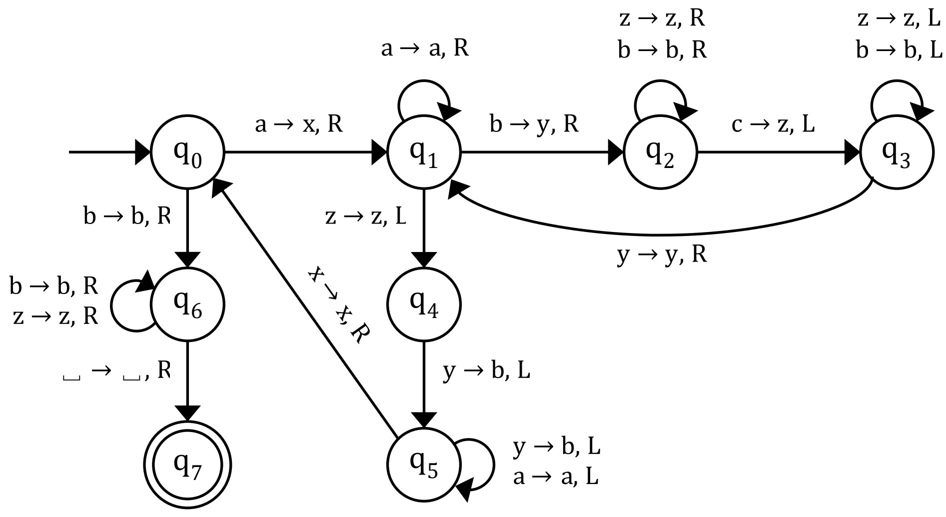
a) $L = \{\, a^m b^n c^k \mid \text{where } m = \frac{k}{n} \text{ and } m, n, k \geq 1 \,\}$

Here,

$$m = \frac{k}{n}$$
$$\therefore k = m \times n$$
$$\downarrow$$
$$a^m b^n c^k \rightarrow a^m b^n c^{m \times n}$$

Turing Machine:

[P.T.O]



Tape Traversal:

```
a a  b b b  c c c  c c c  ␣
↑
x a  b b b  c c c  c c c  ␣
  ↑
x a  b b b  c c c  c c c  ␣
     ↑
x a  y b b  c c c  c c c  ␣
       ↑
x a  y b b  c c c  c c c  ␣
            ↑
x a  y b b  z c c  c c c  ␣
         ↑
x a  y b b  z c c  c c c  ␣
     ↑
x a  y b b  z c c  c c c  ␣
       ↑
x a  y y b  z c c  c c c  ␣
         ↑
x a  y y b  z c c  c c c  ␣
              ↑
x a  y y b  z z c  c c c  ␣
            ↑
x a  y y b  z z c  c c c  ␣
       ↑
x a  y y b  z z c  c c c  ␣
         ↑
x a  y y y  z z c  c c c  ␣
            ↑
x a  y y y  z z c  c c c  ␣
                ↑
x a  y y y  z z z  c c c  ␣
              ↑
x a  y y y  z z z  c c c  ␣
         ↑
```

```
x a  y y y  z z z  c c c  ␣
            ↑
x a  y y y  z z z  c c c  ␣
         ↑
x a  y y b  z z z  c c c  ␣
       ↑
x a  y b b  z z z  c c c  ␣
     ↑
x a  b b b  z z z  c c c  ␣
  ↑
x a  b b b  z z z  c c c  ␣
↑
x a  b b b  z z z  c c c  ␣
  ↑
x x  b b b  z z z  c c c  ␣
     ↑
x x  y b b  z z z  c c c  ␣
       ↑
x x  y b b  z z z  c c c  ␣
                   ↑
x x  y b b  z z z  z c c  ␣
                ↑
x x  y b b  z z z  z c c  ␣
     ↑
x x  y b b  z z z  z c c  ␣
       ↑
x x  y y b  z z z  z c c  ␣
         ↑
x x  y y b  z z z  z c c  ␣
                     ↑
x x  y y b  z z z  z z c  ␣
                   ↑
x x  y y b  z z z  z z c  ␣
       ↑
```

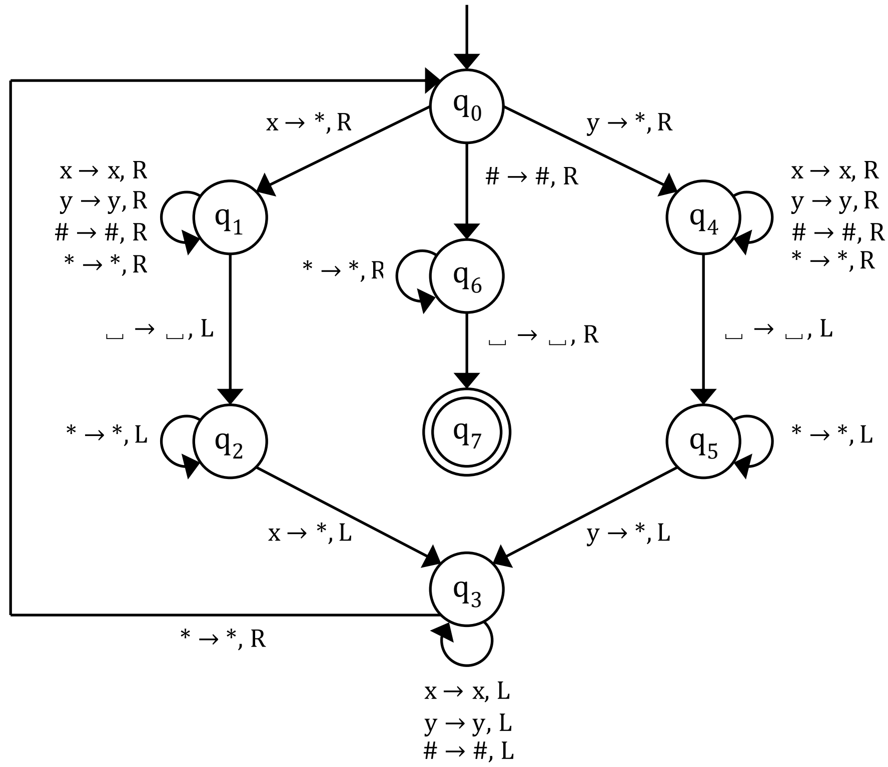
[P.T.O]

```
x x y y b z z z z z c ␣
        ↑

x x y y y z z z z z c ␣
          ↑

x x y y y z z z z z c ␣
                    ↑

x x y y y z z z z z z ␣
                  ↑

x x y y y z z z z z z ␣
        ↑

x x y y y z z z z z z ␣
          ↑

x x y y y z z z z z z ␣
        ↑
```

```
x x y y b z z z z z z ␣
      ↑

x x y b b z z z z z z ␣
    ↑

x x b b b z z z z z z ␣
  ↑

x x b b b z z z z z z ␣
    ↑

x x b b b z z z z z z ␣
                    ↑

x x b b b z z z z z z ␣
                      ↑
```

b) $L = \{\ W\#W^R \mid W \in \{x, y\}$ and W^R is the reverse string of $W\ \}$

Turing Machine:



Tape Traversal:

```
x y y # y y x ␣
↑

* y y # y y x ␣
  ↑

* y y # y y x ␣
              ↑

* y y # y y x ␣
            ↑

* y y # y y * ␣
          ↑
```

```
* y y # y y * ␣
↑

* y y # y y * ␣
  ↑

* * y # y y * ␣
    ↑

* * y # y y * ␣
              ↑

* * y # y y * ␣
          ↑
```

```
* * y # y * * ␣
        ↑

* * y # y * * ␣
  ↑

* * y # y * * ␣
    ↑

* * * # y * * ␣
      ↑

* * * # y * * ␣
              ↑
```

```
* * * # y * * ␣
        ↑

* * * # * * * ␣
      ↑

* * * # * * * ␣
    ↑

* * * # * * * ␣
      ↑

* * * # * * * ␣
              ↑
```