United International **University**

School of Science and Engineering

Mid Term Examination Trimester: Fall-2023

Course Title: Fundamental Calculus

Course Code: Math 1151 Marks: 30 Time: 1 Hour 45 Mins

Answer <u>all</u> the questions. Answer all parts of a question together.

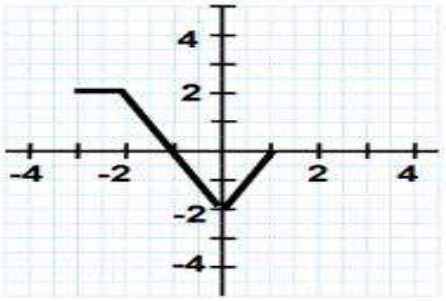
1. (a) **Draw** the graph of the following functions and **find** their domain and range. **[5]**

(i) $y = 2 - 2x - x^2$ **(ii)** $y = 1 - \sin 2x$

(b) The graph of $f(x)$ is given. Use it to **sketch** the graph of the following functions. **[5]**

(i) $2 + f(1 - x)$

(ii) $1 - 2\left|f\left(\frac{x}{2}\right)\right|$



2. (a) **Evaluate** $f(-5)$, $f(-1)$, and $f(3)$ for the piecewise defined function. Then **sketch** the graph of the function. **[3]**

$$f(x) = \begin{cases} 2; & x < -3 \\ \sqrt{9 - x^2}; & -3 \le x < 3 \\ x - 2; & x \ge 3 \end{cases}$$

(b) Use the table to evaluate the following expressions: **[3]**

x	-3	-1	2	5
$f(x)$	7	2	-1	-1
$g(x)$	9	-3	-8	2

(i) $(f \circ g)(-1)$ **(ii)** $(g \circ g)(5)$ **(iii)** $(g \circ f)(2)$

(c) The graph of $f(x)$ is given. **[4]**

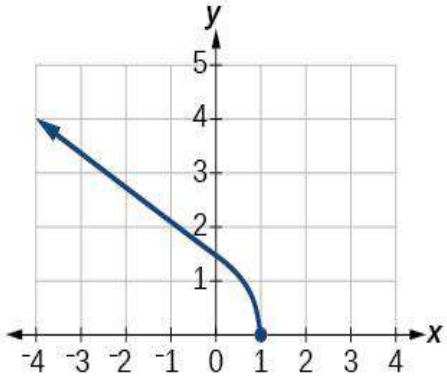
(i) **Determine** whether $f(x)$ is one to one function, or not.

(ii) **Complete** the following table.

x	0	2	4
$f^{-1}(x)$			

(iii) **Sketch** the graph of $f^{-1}(x)$ along with $f(x)$.

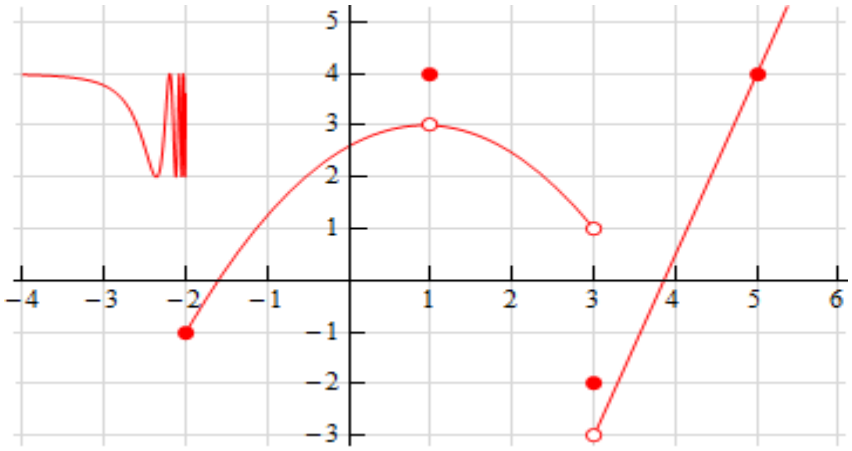
(iv) **What** is the domain and range of $f^{-1}(x)$?



Please Turn Over

3. **(a)** **Find** the inverse of $f(x) = 2 + 3^{-x}$, **draw** the graph of $f(x)$ and **its inverse** in the same diagram. Also, **state** the domain and range of the inverse function. **[3]**

(b) The graph of the function $y = f(x)$ is given. **[5]**



From the figure **write** the answers of the following questions:

(i) $\lim_{x\to -2^-} f(x)$ and $\lim_{x\to 3^+} f(x)$.

(ii) $\lim_{x\to 1} f(x)$.

(iii) $f(1)$ and **state** the horizontal asymptote(s) of $f(x)$.

(iv) **Check** and **explain** the continuity of $f(x)$ at $x = -2$ and 5.

(c) **Find** value of the constant k, if possible, that will make the function $f(x)$ continuous everywhere. **[2]**

$$f(x) = \begin{cases} x^3 + 2k; & x > -1 \\ x + 5; & x \le -1 \end{cases}$$

BEST OF LUCK!!!