United International **University**
School of Science and Engineering
Final Examination Trimester: Fall 2023
Course Title: Coordinate Geometry and Vector Analysis
Course Code: Math 2201 Marks: 40
Total Time: 2 hours

Answer all questions.

1. **a)** Consider, $\boldsymbol{F(x,y)} = e^x \boldsymbol{siny\ i} + e^x \boldsymbol{cosy\ j}$ **[5]**

 i) Show that $\boldsymbol{F}$ is a conservative vector field on the entire $\boldsymbol{xy}$ −plane.
 ii) Find the potential function$\boldsymbol{\phi(x,y)}$.
 iii) Find $\int_{(\mathbf{0},\mathbf{0})}^{\left(\mathbf{1},\frac{\pi}{2}\right)} \boldsymbol{F.dr}$ using **ii)**

 b) Using Green's theorem find the value of $\oint_{\boldsymbol{c}} \boldsymbol{F.dr}$
 Where $\boldsymbol{F}(x,y) = (25e^{3x} - y^3)\boldsymbol{i} + (5y^3 + x^3)\boldsymbol{j}$ and C is the closed circle with parametric equations $\boldsymbol{x = cost}, and\ \boldsymbol{y = sint}$. **[5]**

2. **a)** Evaluate $\int_{\boldsymbol{c}} (\boldsymbol{x+y})\boldsymbol{dx} + (\boldsymbol{-y-x})\boldsymbol{dy}$ along the rectangle with vertices $(0,\ 0), (0,2), (2,2)$ and $(2,0)$. **[5]**

 b) Evaluate the surface integral $\iint_{\boldsymbol{\sigma}} \boldsymbol{2xz\ ds}$; σ is the part of the plane **[5]**

 $\boldsymbol{x+y+z=2}$ that lies in the first octant.

3. **a)** Find the flux of the vector field $\boldsymbol{F}(x,y,z) = x\boldsymbol{i} + y\boldsymbol{j} + 2z\boldsymbol{k}$ across σ, where σ is the portion of the cone $\boldsymbol{z} = \sqrt{\boldsymbol{x^2+y^2}}$ between the planes $\boldsymbol{z=1}$ and $\boldsymbol{z=4}$, oriented upward unit normal. **[5]**

 Or

 Use the Divergence Theorem to find the outward flux of the vector field $\boldsymbol{F}(x,y,z) = x^3\boldsymbol{i} + y^3\boldsymbol{j} + z^3\boldsymbol{k}$ across the surface of the region that is enclosed by $\boldsymbol{z = 25 - x^2 - y^2}$ and the plane $\boldsymbol{z=0}$.

 b) Using double integral to find the area enclosed by the equations **[5]**

 $\boldsymbol{-x+y=2, x+y=2}$ and $\boldsymbol{y=0}$.

4. **a)** Use cylindrical coordinate to evaluate **[5]**
$$\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}}\int_{0}^{16-x^2-y^2}(\boldsymbol{x^2+y^2})\boldsymbol{dzdydx}.$$

 b) Find the volume of the sphere by using spherical coordinate system where the radius of sphere is 3. **[5]**

 Or

 Using triple integral find the volume of the solid bounded by the $\boldsymbol{x^2+y^2=25}$, $\boldsymbol{xy} - plane$ and $\boldsymbol{z=4}$.