



MID-TERM QUESTION SOLUTIONS

THEORY OF COMPUTATION

CSE 2233

SOLUTION BY

NURUL ALAM ADOR

UPDATED TILL SPRING 2024

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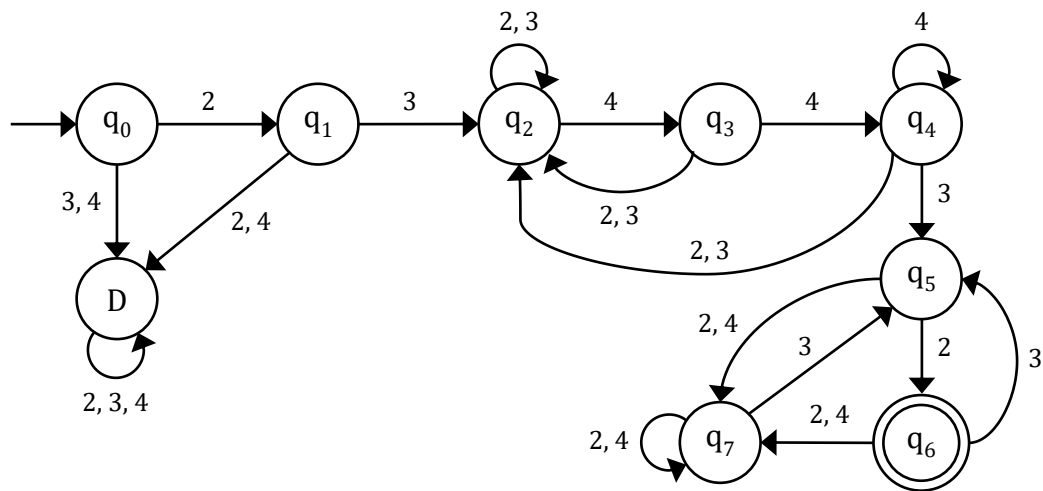
1. Design DFAs that accepts the following languages:

- a) $L = \{w \mid w \text{ starts with "23", contains "443" as a substring and ends with "32"}\}$
Where $\Sigma = \{2,3,4\}$
- b) $L = \{w \mid w \text{ ends with either "01" or "10"}\}$ Where, $\Sigma = \{0,1\}$
- c) $L = \{w \mid w \text{ contains an odd number of b's, and ends with 'ac'}\}$ Where, $\Sigma = \{a,b,c\}$

Solution:

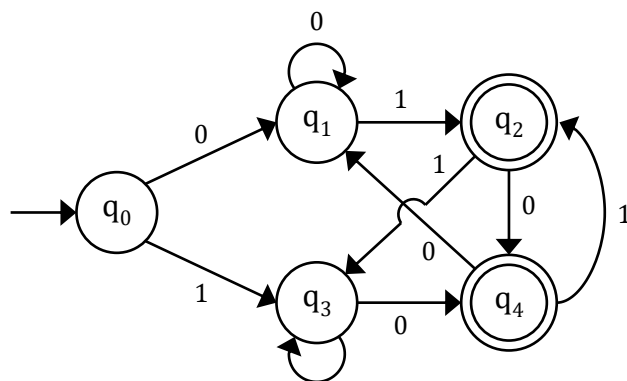
- a) $L = \{w \mid w \text{ starts with "23", contains "443" as a substring and ends with "32"}\} \mid \Sigma = \{2,3,4\}$

The DFA has been designed below:



- b) $L = \{w \mid w \text{ ends with either "01" or "10"}\} \mid \Sigma = \{0,1\}$

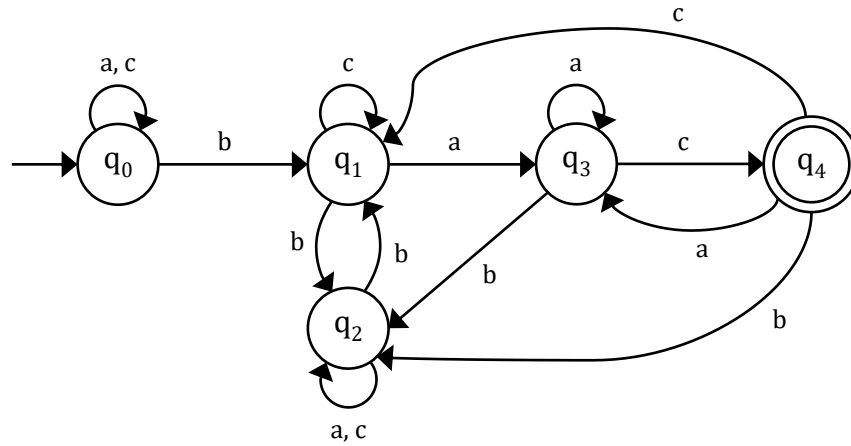
The DFA has been designed below:



- c) $L = \{w \mid w \text{ contains an odd number of b's, and ends with 'ac'}\} \mid \Sigma = \{a,b,c\}$

The DFA has been designed below:

[P.T.O]



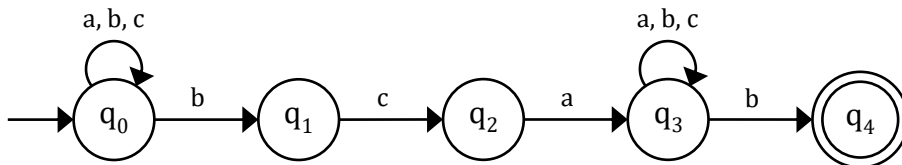
2. Design NFAs that accepts the following languages:

- a) $L = \{ w \mid w \text{ ends with 'b' and contains 'bca' } \} \mid \Sigma = \{a, b, c\}$
- b) $L = \{ w \mid w \text{ starts and ends with different symbols when the total length is a multiple of 2 } \mid \Sigma = \{0,1\}$
- c) $L = \{ w \mid w \text{ starts with 'xy' and contains 'xxy' or 'yyz' or 'zzx' and ends with 'yz' } \mid \Sigma = \{x, y, z\}$

Solution:

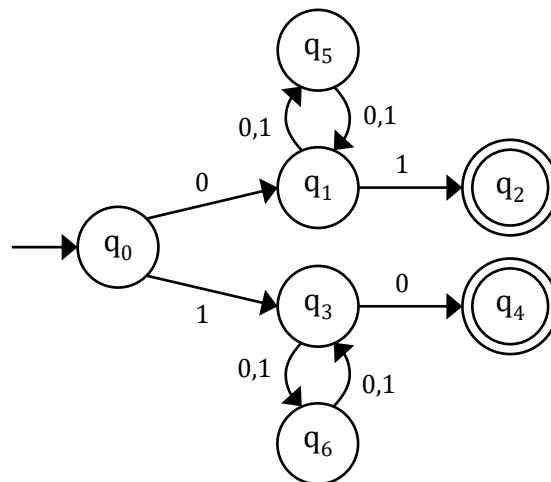
- a) $L = \{ w \mid w \text{ starts with 'a' or 'b' and contains 'cca' and ends with 'b' or 'c' } \mid \Sigma = \{a, b, c\}$

The NFA has been designed below:



- b) $L = \{ w \mid w \text{ starts and ends with different symbols when the total length is a multiple of 2 } \mid \Sigma = \{0,1\}$

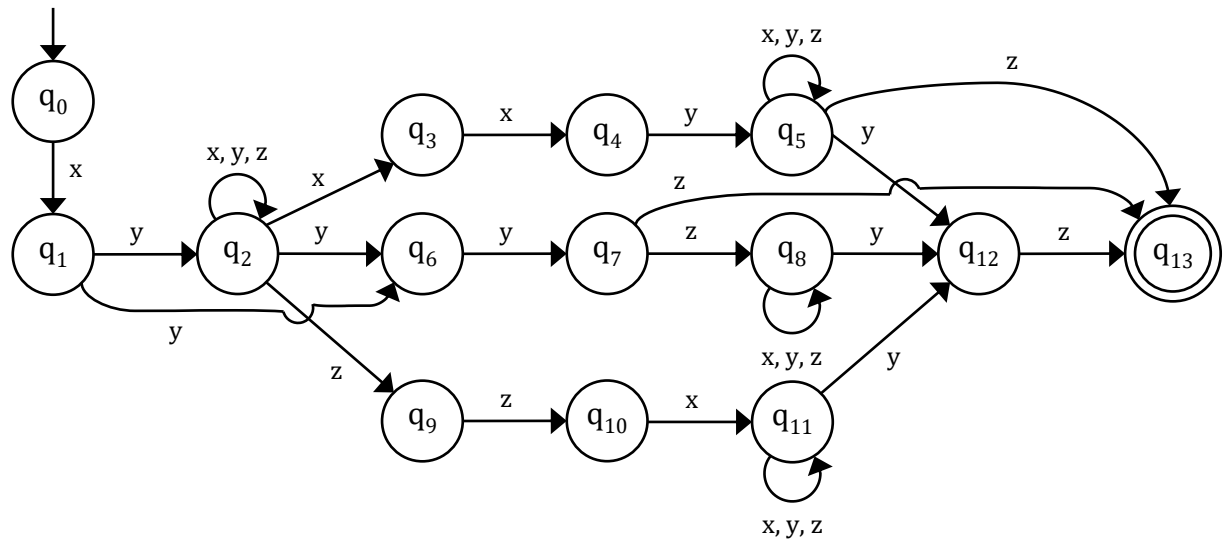
The NFA has been designed below:



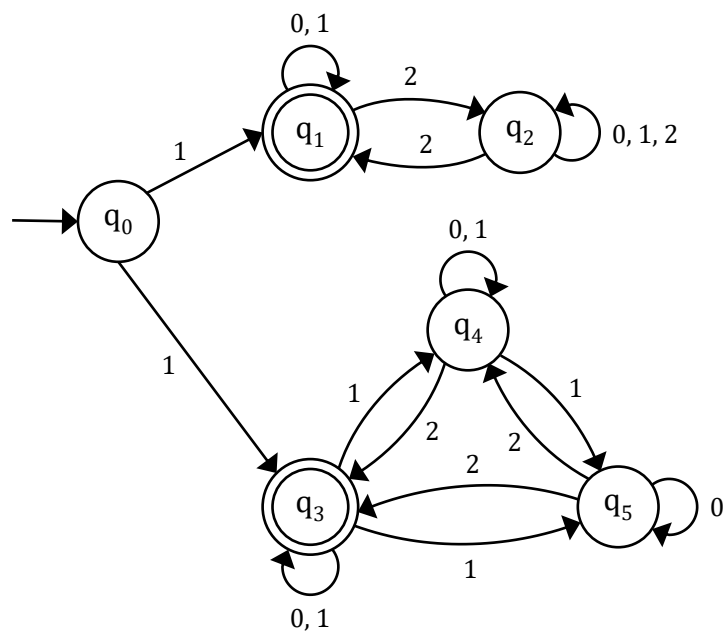
- c) $L = \{ w \mid w \text{ starts with 'xy' and contains 'xxy' or 'yyz' or 'zzx' and ends with 'yz' } \}$

$\Sigma = \{x, y, z\}$

The NFA has been designed below:



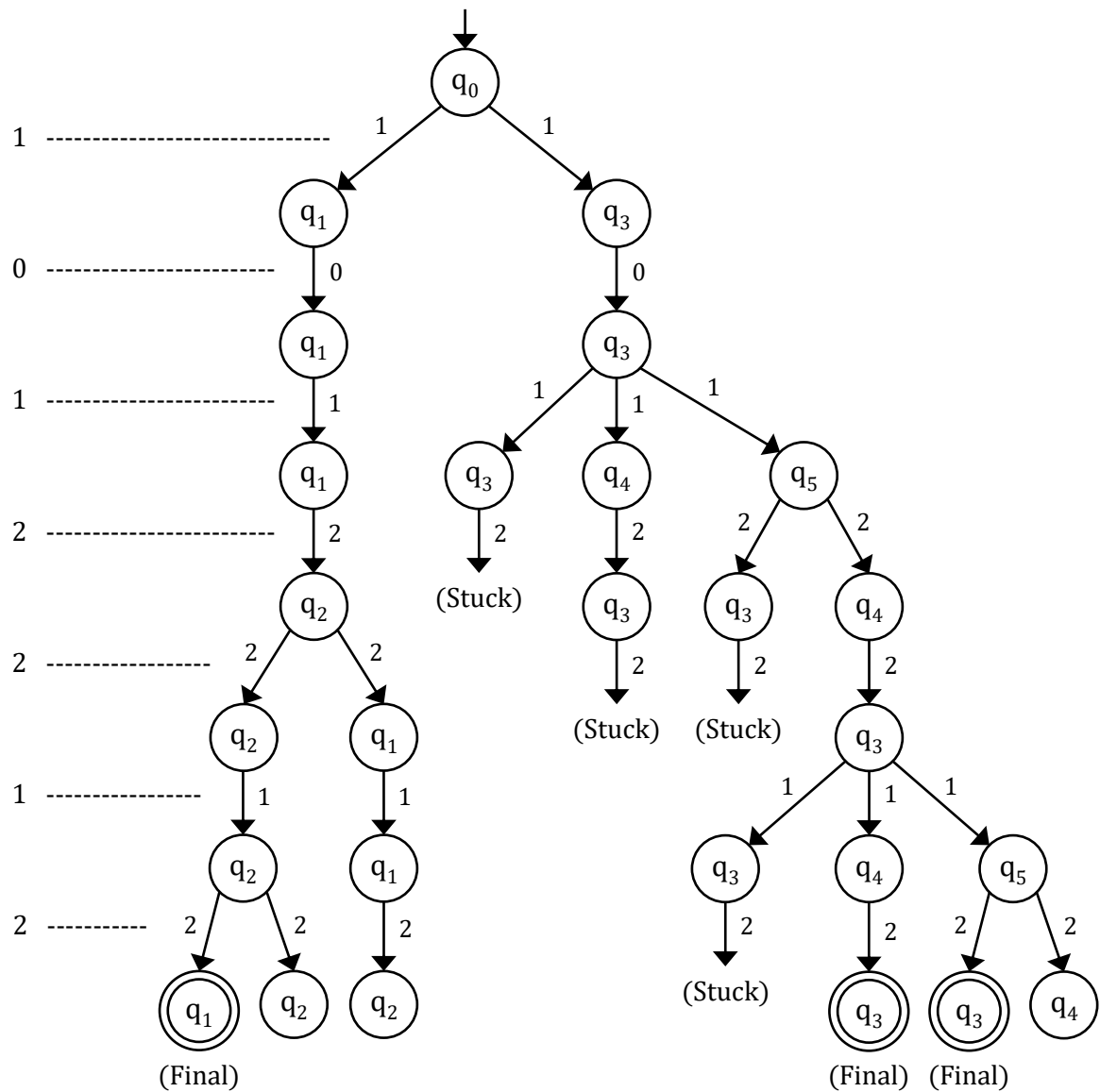
3. Consider the following NFA, and show with help of NFA-tree whether the string “1012212” is accepted.



Solution:

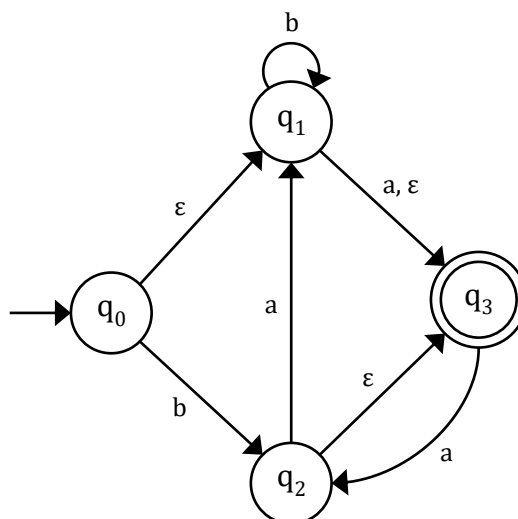
NFA Tree:

[P.T.O]



With help of NFA-tree, we can see the string "1012212" reach the final state.
 \therefore The string "1012212" is accepted.

4. Convert the following ϵ -NFA over the alphabet $\Sigma = \{a, b\}$ to an equivalent DFA



Solution:

Transition Table of the given NFA:

	a	b
$\rightarrow q_0$	\emptyset	q_2
q_1	q_3	q_1
q_2	q_1	\emptyset
$* q_3$	q_2	\emptyset

ϵ -Closure of all state of the given ϵ -NFA:

$$\epsilon\text{-Closure}(q_0) = \{q_0, q_1, q_3\}$$

$$\epsilon\text{-Closure}(q_1) = \{q_1, q_3\}$$

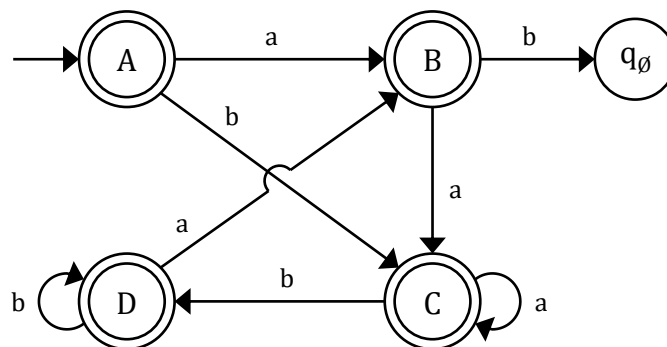
$$\epsilon\text{-Closure}(q_2) = \{q_2, q_3\}$$

$$\epsilon\text{-Closure}(q_3) = \{q_3\}$$

Transition Table for the Equivalent DFA:

	a	b
$\rightarrow * A = \{q_0, q_1, q_3\}$	$\{q_2, q_3\} = B$	$\{q_1, q_2, q_3\} = C$
$* B = \{q_2, q_3\}$	$\{q_1, q_2, q_3\} = C$	$\{\emptyset\} = q_\emptyset$
$* C = \{q_1, q_2, q_3\}$	$\{q_1, q_2, q_3\} = C$	$\{q_1, q_3\} = D$
$* D = \{q_1, q_3\}$	$\{q_2, q_3\} = B$	$\{q_1, q_3\} = D$
$q_\emptyset = \{\emptyset\}$	$\{\emptyset\} = q_\emptyset$	$\{\emptyset\} = q_\emptyset$

Equivalent DFA Diagram:



b) Design Regular Expression for the following languages where $\Sigma = \{a, b\}$:

a) W | W contains not more than one occurrence of the substring ' aa '

b) W | W does not end with ' ab '

c) W | W starts with **b** and ends with **a**

Solution:

a) W | W contains not more than one occurrence of the substring ' aa '

Regular Expression: $(b | ab)^* (aa | a) (b | ba)^* | (b | ab)^* (b | ba)^*$

b) W | W does not end with 'ab'

Regular Expression: $(a | b)^* (aa | bb | ba) | a | b$

c) W | W starts with b and ends with a

Regular Expression: $a (a | b)^* b$

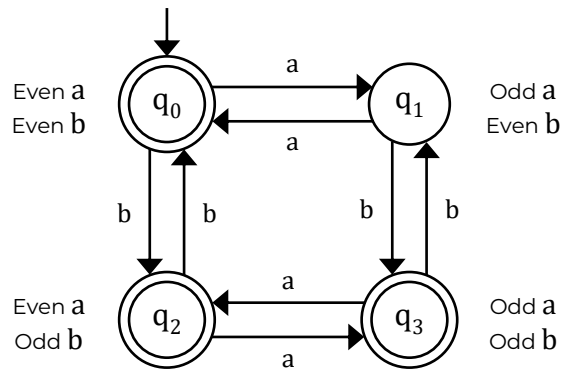
1. Design DFAs that accept the following languages:

- a) $L =$ accepts any string that has an even number of 'a' or odd number of 'b' over alphabet $\{a, b\}$
- b) $L =$ accepts any string which starts with an odd number of 'r' and ends with 'brb' over the alphabet $\{b, r\}$
- c) $L =$ accepts any string where the last two symbols are different over the alphabet $\{a, b\}$

Solution:

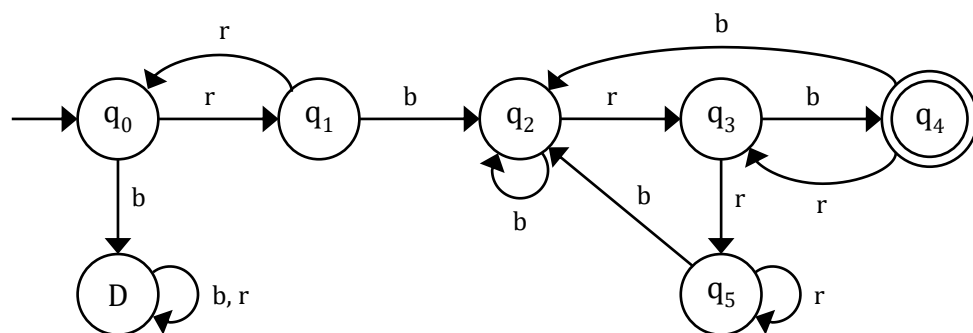
- a) $L =$ accepts any string that has an even number of 'a' or odd number of 'b' over alphabet $\{a, b\}$

The DFA has been designed below:



- b) $L =$ accepts any string which starts with an odd number of 'r' and ends with 'brb' over the alphabet $\{b, r\}$

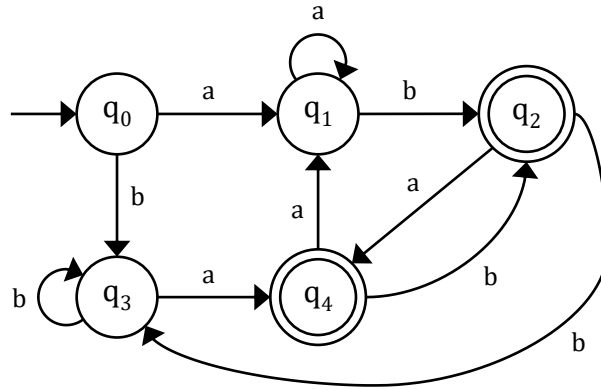
The DFA has been designed below:



- c) $L =$ accepts any string where the last two symbols are different over the alphabet $\{a, b\}$

The DFA has been designed below:

[P.T.O]



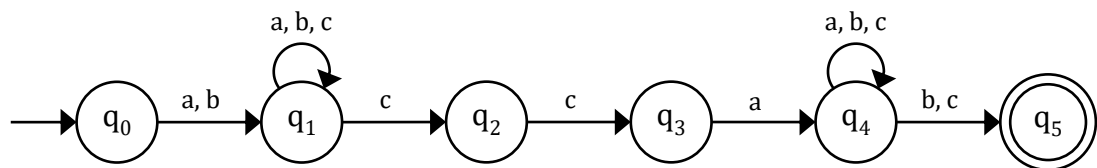
2. Design NFAs that accept the following languages:

- a) $L = \{ w \mid w \text{ starts with 'a' or 'b' and contains 'cca' and ends with 'b' or 'c' } \mid \Sigma = \{a, b, c\}$
- b) $L = \{ w \mid w \text{ starts and ends with different symbols with total length of at least 2 } \mid \Sigma = \{0,1\}$
- c) $L = \{ w \mid w \text{ contains 'xyz' or 'yzx' or 'zxx' and ends with 'yz' } \mid \Sigma = \{x, y, z\}$

Solution:

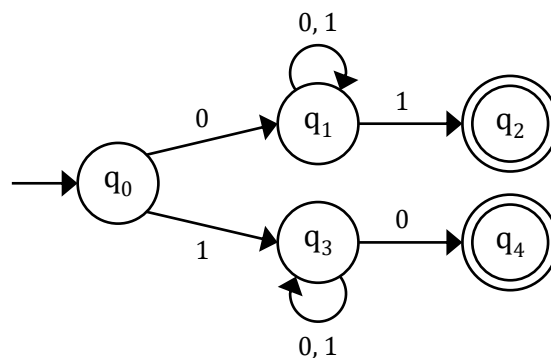
- a) $L = \{ w \mid w \text{ starts with 'a' or 'b' and contains 'cca' and ends with 'b' or 'c' } \mid \Sigma = \{a, b, c\}$

The NFA has been designed below:



- b) $L = \{ w \mid w \text{ starts and ends with different symbols with total length of at least 2 } \mid \Sigma = \{0,1\}$

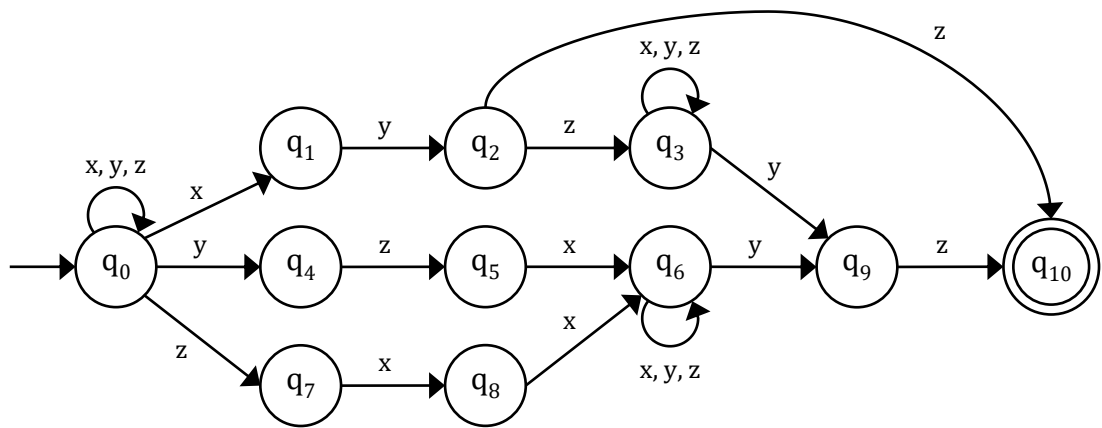
The NFA has been designed below:



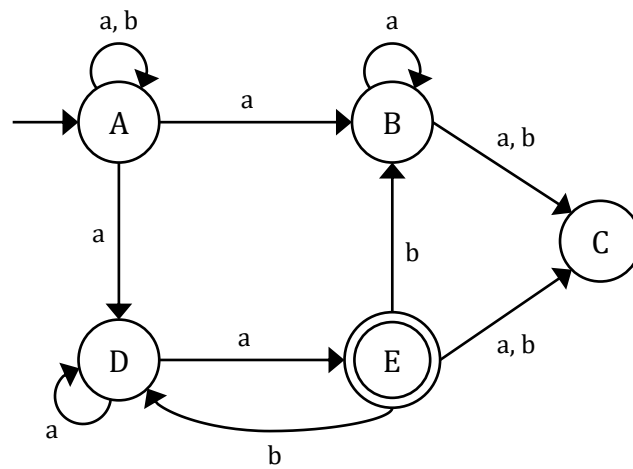
- c) $L = \{ w \mid w \text{ contains 'xyz' or 'yzx' or 'zxx' and ends with 'yz' } \mid \Sigma = \{x, y, z\}$

The NFA has been designed below:

[P.T.O]



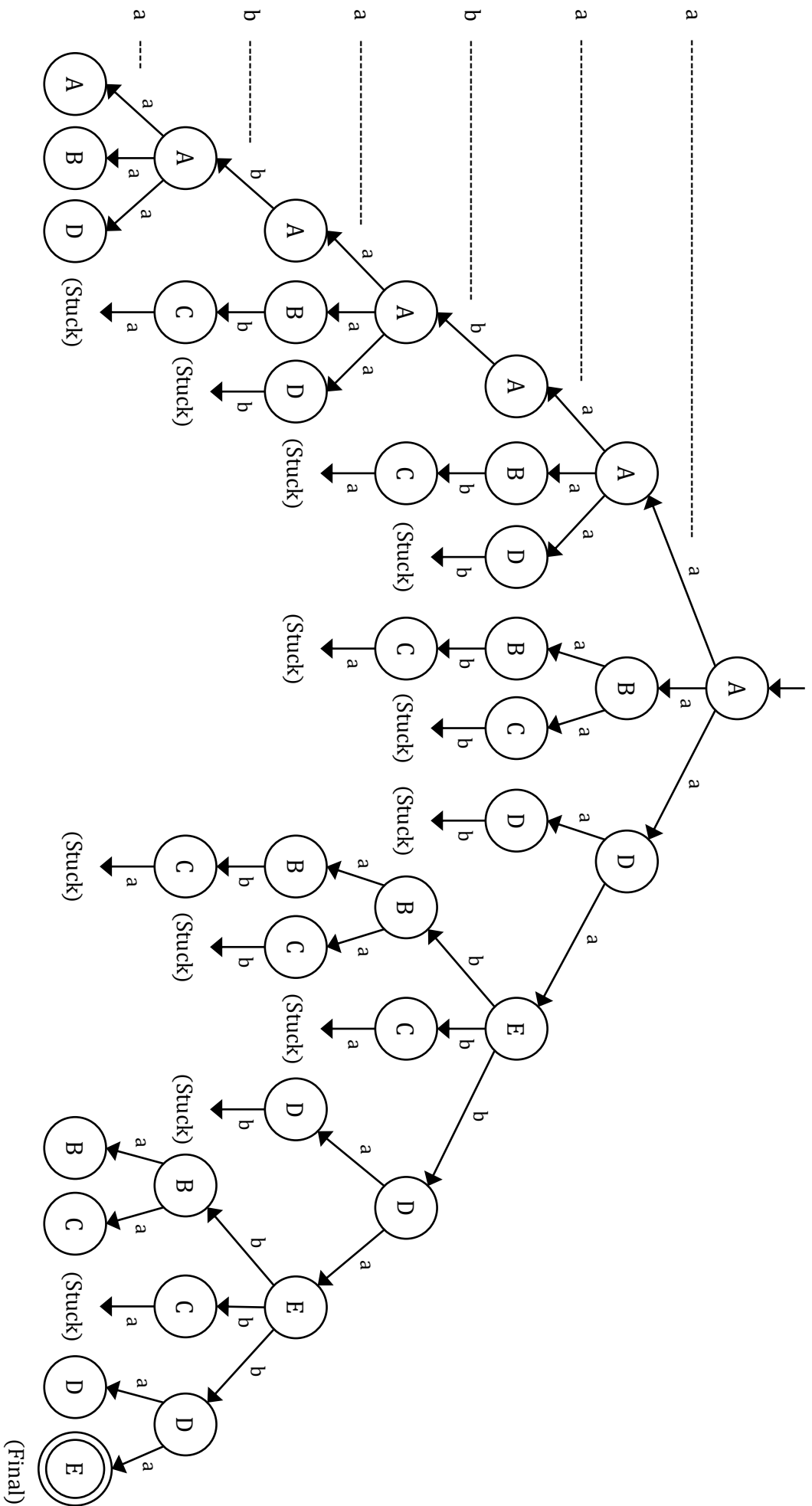
3. Consider the following NFA, and show with help of NFA-tree whether the string “aababa” is accepted.



Solution:

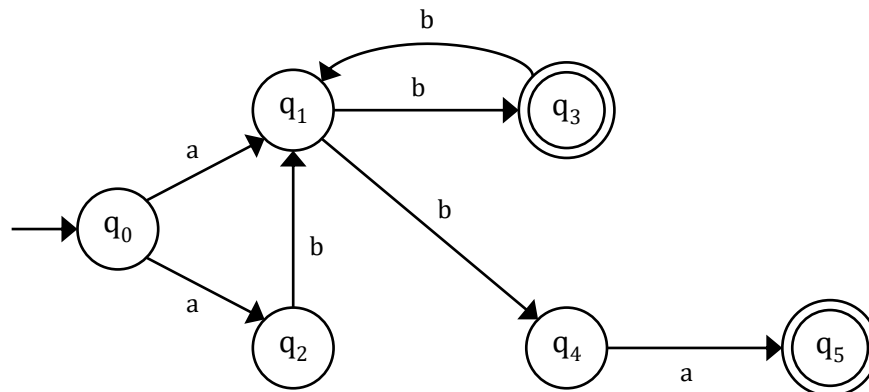
NFA Tree:

[P.T.O]



With help of NFA-tree, we can see the string “aababa” reach the final state.
 \therefore The string “aababa” is accepted.

4. Convert the following NFA over the alphabet $\Sigma = \{a, b\}$ to an equivalent DFA



Solution:

Transition Table of the given NFA:

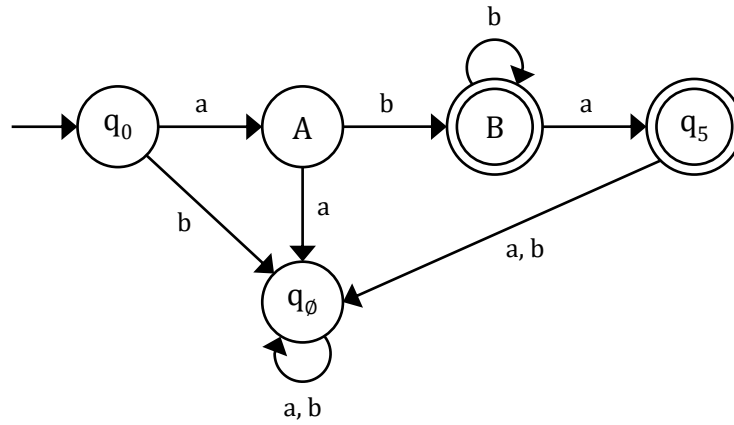
	a	b
→ q₀	q ₁ , q ₂	∅
q₁	∅	q ₃ , q ₄
q₂	∅	q ₁
* q₃	∅	q ₁
q₄	q ₅	∅
* q₅	∅	∅

Transition Table for the Equivalent DFA:

	a	b
→ q₀	{ q ₁ , q ₂ } = A	{ ∅ } = q _∅
A = { q₁, q₂ }	{ ∅ } = q _∅	{ q ₁ , q ₃ , q ₄ } = B
* B = { q₁, q₃, q₄ }	q ₅	{ q ₁ , q ₃ , q ₄ } = B
* q₅	{ ∅ } = q _∅	{ ∅ } = q _∅
q_∅ = { ∅ }	{ ∅ } = q _∅	{ ∅ } = q _∅

Equivalent DFA Diagram:

[P.T.O]



5. a) Convert the following regular expressions to finite automata:

i) $(ab)^* + (a + ab)^* b^* (a + b)^*$

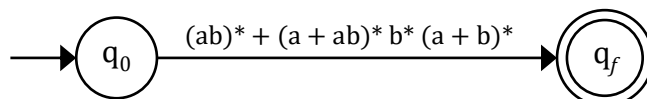
ii) $[a + ba(a + b)]^* a (ba)^* b^*$

Solution:

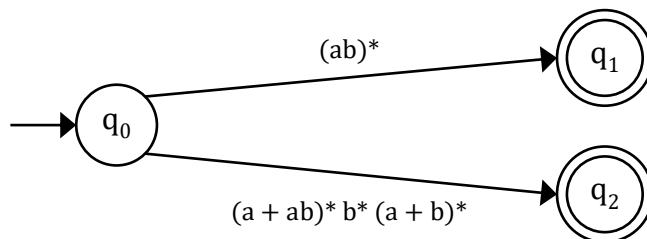
i) $(ab)^* + (a + ab)^* b^* (a + b)^*$

The equivalent finite automata for the following regular expression has been constructed below:

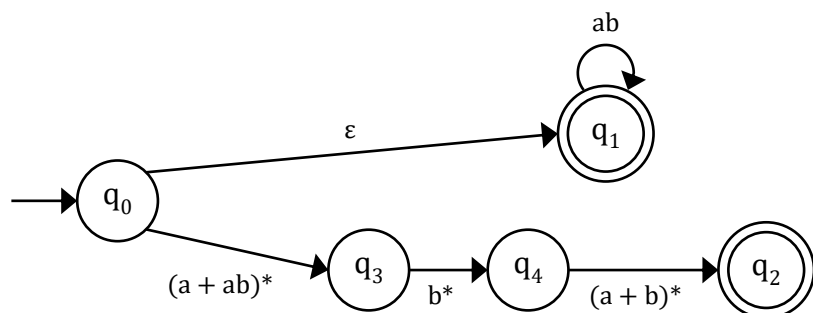
Step 1:



Step 2:

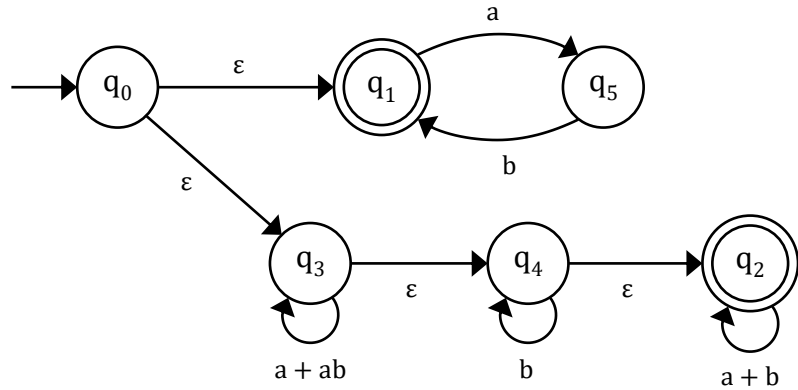


Step 3:

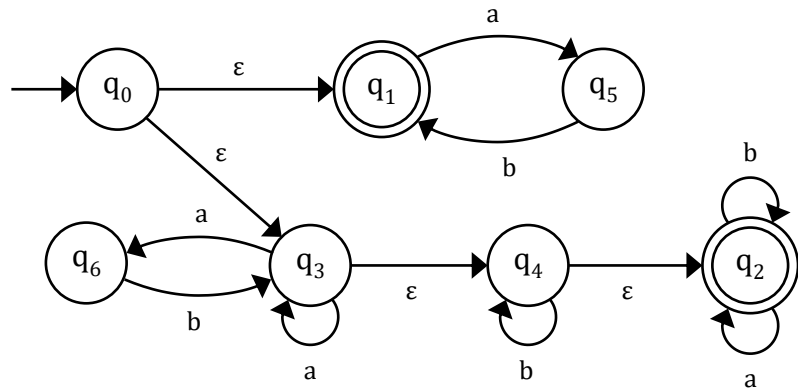


[P.T.O]

Step 4:



Step 5:

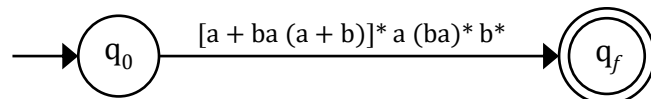


This is our final finite automata for following regular expression.

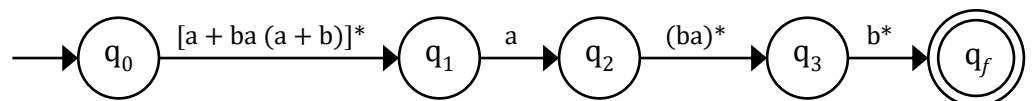
ii) $[a + ba(a + b)]^* a (ba)^* b^*$

The equivalent finite automata for the following regular expression has been constructed below:

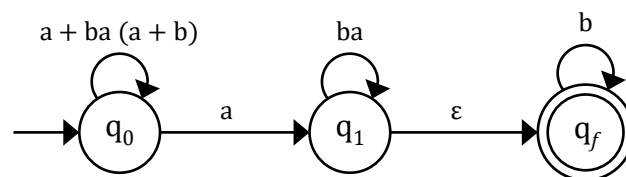
Step 1:



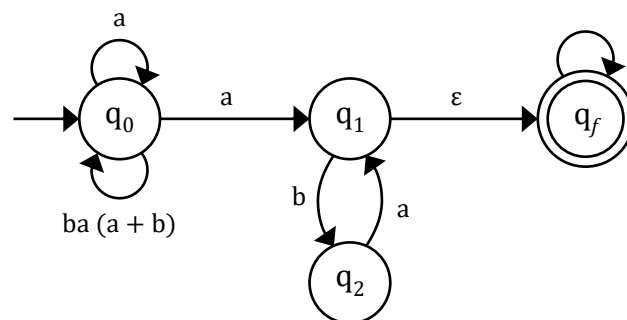
Step 2:



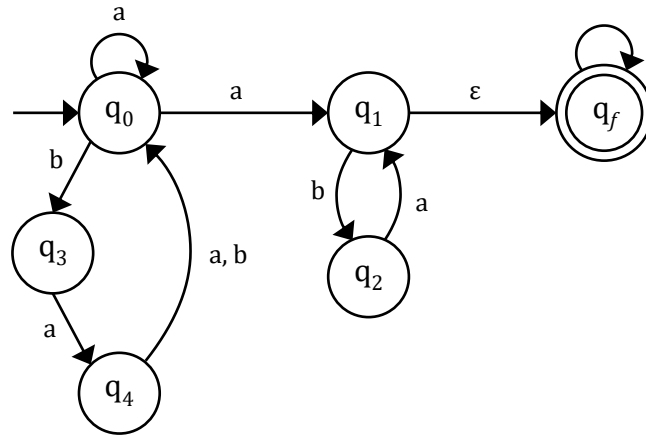
Step 3:



Step 4:



Step 5:



This is our final finite automata for following regular expression.

b) Convert the following regular expressions to finite automata:

a) $L = \{ \text{strings such that the 4th symbol from the right is b over the alphabet } \{a, b\} \}$

b) $L = \{ \text{strings such that they start and end with 'a' over the alphabet } \{a, b, c\} \}$

Solution:

a) $L = \{ \text{strings such that the 4th symbol from the right is b over the alphabet } \{a, b\} \}$

Regular Expression: $(a \mid b)^* a (a \mid b) (a \mid b) (a \mid b)$

b) $L = \{ \text{strings such that they start and end with 'a' over the alphabet } \{a, b, c\} \}$

Regular Expression: $a (a \mid b \mid c)^* a \mid a$

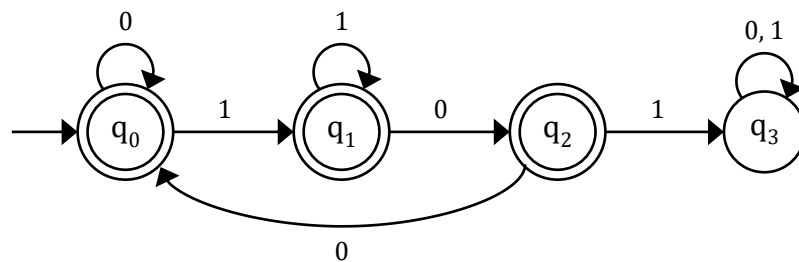
1. Design **DFA's** that accepts the following languages:

- a) $L = \{ w \mid w \text{ does not contain '101' } \mid \Sigma = \{0,1\}$
- b) $L = \{ w \mid w \text{ starts with an even number of 'a', contains 'ba' and ends with 'baa' } \mid \Sigma = \{a, b\}$
- c) $L = \{ w \mid w \text{ is a palindrome with a max length of 3 } \mid \Sigma = \{0,1\}$
- d) $L = \{ a^i b^j \mid i \geq 0, j \geq 0, i + j \text{ is an odd number } \mid \Sigma = \{a, b\}$

Solution:

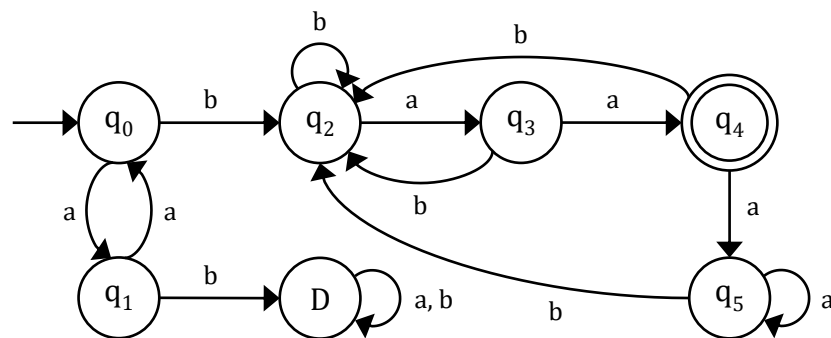
- a) $L = \{ w \mid w \text{ does not contain '101' } \mid \Sigma = \{0,1\}$

The DFA has been designed below:



- b) $L = \{ w \mid w \text{ starts with an even number of 'a', contains 'ba' and ends with 'baa' } \mid \Sigma = \{a, b\}$

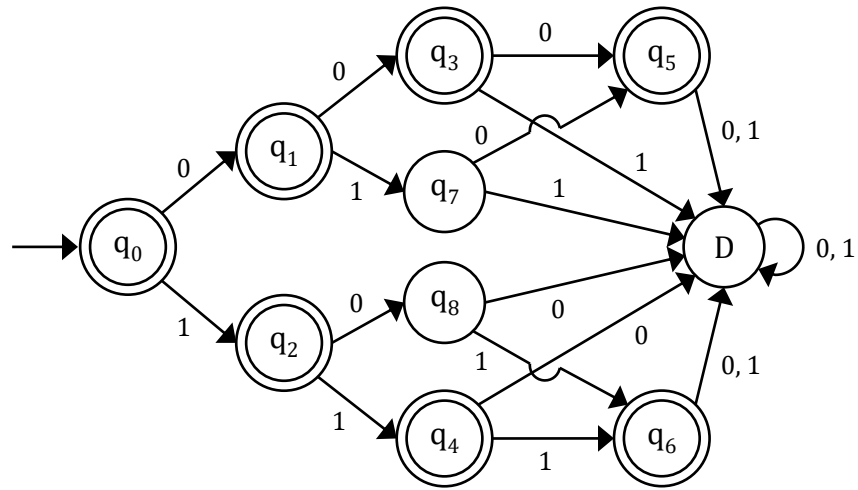
The DFA has been designed below:



- c) $L = \{ w \mid w \text{ is a palindrome with a max length of 3 } \mid \Sigma = \{0,1\}$

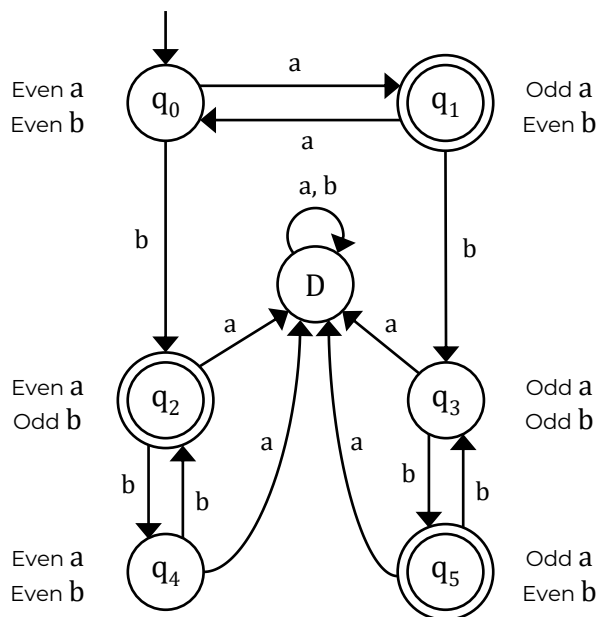
The DFA has been designed below:

[P.T.O]



- d) $L = \{ a^i b^j \mid i \geq 0, j \geq 0, i + j \text{ is an odd number} \} \mid \Sigma = \{a, b\}$

The DFA has been designed below:



We Know,

Even + Even = Even
Odd + Odd = Even
Even + Odd = Odd

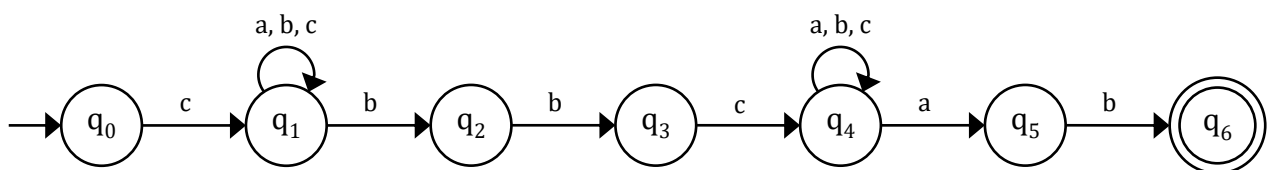
2. Design NFA's that accepts the following languages:

- a) $L = \{ w \mid w \text{ doesn't start with 'a' or 'b' and contains 'bbc' and ends with 'ab'} \} \mid \Sigma = \{a, b, c\}$
b) $L = \{ w \mid w \text{ starts with '10' or '21' and contains '220' and ends with '112'} \} \mid \Sigma = \{0,1,2\}$
c) $L = \{ w \mid w \text{ starts and ends with either 'xzy' or 'xy'} \} \mid \Sigma = \{x, y, z\}$

Solution:

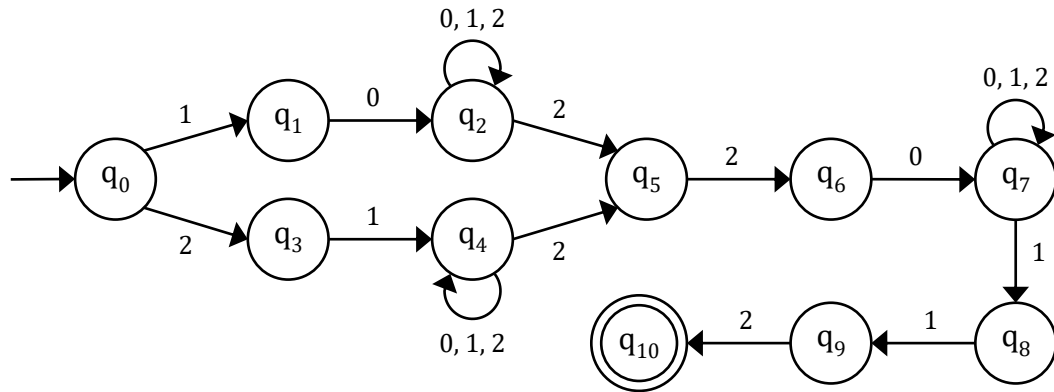
- a) $L = \{ w \mid w \text{ doesn't start with 'a' or 'b' and contains 'bbc' and ends with 'ab'} \} \mid \Sigma = \{a, b, c\}$

The NFA has been designed below:



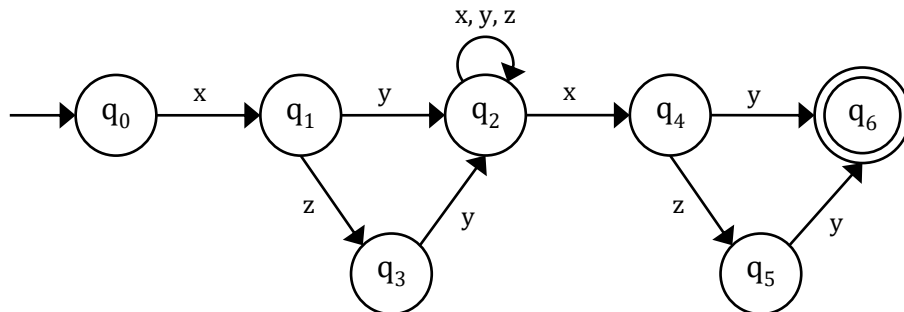
- b) $L = \{ w \mid w \text{ starts with '10' or '21' and contains '220' and ends with '112' } \mid \Sigma = \{0,1,2\}$

The NFA has been designed below:

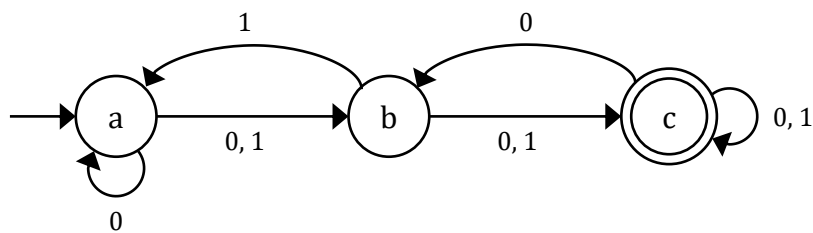


- c) $L = \{ w \mid w \text{ starts and ends with either 'xzy' or 'xy' } \mid \Sigma = \{x,y,z\}$

The NFA has been designed below:



3. Consider the following NFA, and show with help of **NFA-tree** whether the string “0100110” is accepted.



Solution:

NFA Tree:

[P.T.O]

With help of NFA-tree, we can see the string “0100110” reach the final state.

∴ The string “0100110” is accepted.

4. Convert the following NFA over the alphabet $\Sigma = \{1,2,3\}$ to an equivalent DFA including the diagram.

	1	2	3
→ a	{ a, b, d }	{ a, c }	{ d }
b	∅	{ a, d }	{ a, e }
* c	{ a, b, c, d, e }	∅	{ b, c }
d	{ d, e }	{ d }	{ a, e }
* e	{ b, e }	∅	∅

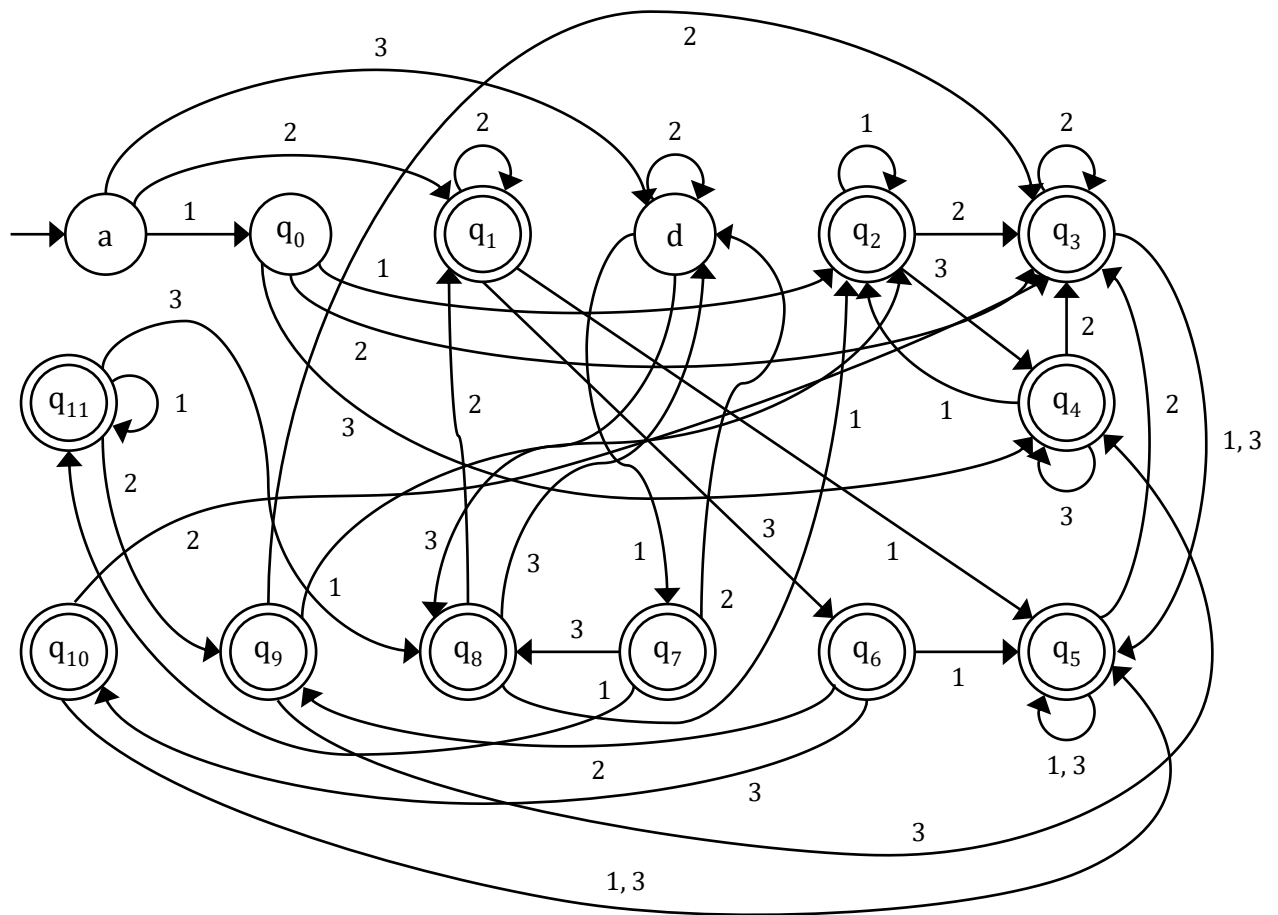
Solution:

Transition Table for the Equivalent DFA:

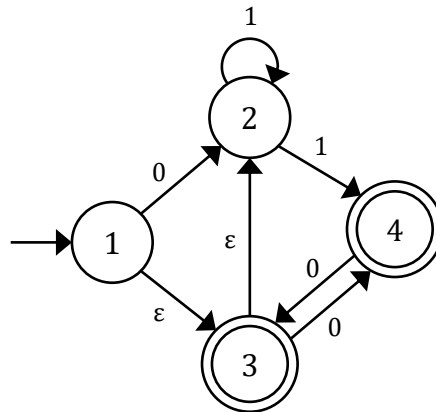
	1	2	3
→ a	{ a, b, d } = q ₀	{ a, c } = q ₁	d
q ₀ = { a, b, d }	{ a, b, d, e } = q ₂	{ a, c, d } = q ₃	{ a, d, e } = q ₄
* q ₁ = { a, c }	{ a, b, c, d, e } = q ₅	{ a, c } = q ₁	{ b, c, d } = q ₆
d	{ d, e } = q ₇	d	{ a, e } = q ₈
* q ₂ = { a, b, d, e }	{ a, b, d, e } = q ₂	{ a, c, d } = q ₃	{ a, d, e } = q ₄
* q ₃ = { a, c, d }	{ a, b, c, d, e } = q ₅	{ a, c, d } = q ₃	{ a, b, c, d, e } = q ₅
* q ₄ = { a, d, e }	{ a, b, d, e } = q ₂	{ a, c, d } = q ₃	{ a, d, e } = q ₄
* q ₅ = { a, b, c, d, e }	{ a, b, c, d, e } = q ₅	{ a, c, d } = q ₃	{ a, b, c, d, e } = q ₅
* q ₆ = { b, c, d }	{ a, b, c, d, e } = q ₅	{ a, d } = q ₉	{ a, b, c, e } = q ₁₀
* q ₇ = { d, e }	{ b, d, e } = q ₁₁	d	{ a, e } = q ₈
* q ₈ = { a, e }	{ a, b, d, e } = q ₂	{ a, c } = q ₁	d
q ₉ = { a, d }	{ a, b, d, e } = q ₂	{ a, c, d } = q ₃	{ a, d, e } = q ₄
* q ₁₀ = { a, b, c, e }	{ a, b, c, d, e } = q ₅	{ a, c, d } = q ₃	{ a, b, c, d, e } = q ₅
* q ₁₁ = { b, d, e }	{ b, d, e } = q ₁₁	{ a, d } = q ₉	{ a, e } = q ₈

Equivalent DFA Diagram:

[P.T.O]



5. Convert the following ϵ -NFA over the alphabet $\Sigma = \{0,1\}$ to an equivalent DFA.



Solution:

Transition Table of the given ϵ -NFA:

	0	1
$\rightarrow 1$	2	\emptyset
2	\emptyset	2, 4
* 3	4	\emptyset
* 4	3	\emptyset

ϵ -Closure of all state of the given ϵ -NFA:

$$\varepsilon\text{-Closure}(1) = \{1, 2, 3\}$$

$$\varepsilon\text{-Closure}(2) = \{2\}$$

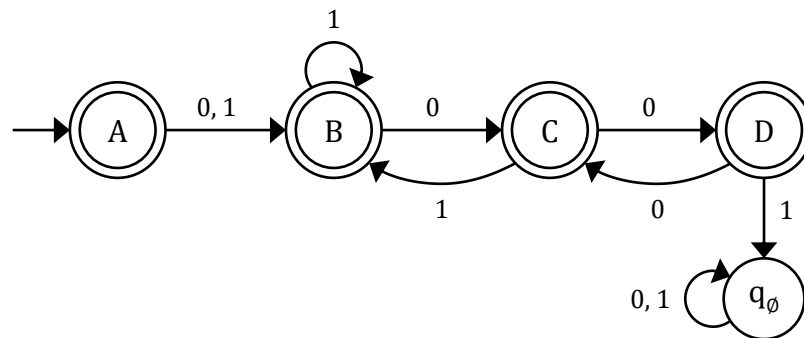
$$\varepsilon\text{-Closure}(3) = \{3, 2\}$$

$$\varepsilon\text{-Closure}(4) = \{4\}$$

Transition Table for the Equivalent DFA:

	0	1
$\rightarrow * A = \{1, 2, 3\}$	$\{2, 4\} = B$	$\{2, 4\} = B$
$* B = \{2, 4\}$	$\{2, 3\} = C$	$\{2, 4\} = B$
$* C = \{2, 3\}$	$\{4\} = D$	$\{2, 4\} = B$
$* D = \{4\}$	$\{2, 3\} = C$	$\{\emptyset\} = q_\emptyset$
$q_\emptyset = \{\emptyset\}$	$\{\emptyset\} = q_\emptyset$	$\{\emptyset\} = q_\emptyset$

Equivalent DFA Diagram:



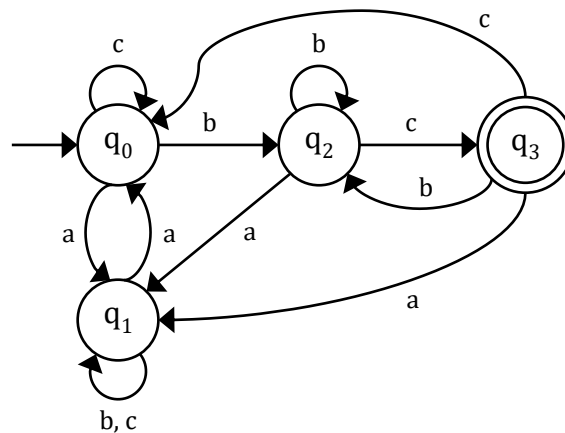
1. Design DFAs that accepts the following languages:

- a) $L = \text{contains even number of 'a' and ends with 'bc'}$ | $\Sigma = \{a, b, c\}$
- b) $L = \text{does not contain 'mnm'}$ | $\Sigma = \{m, n, w\}$
- c) $L = \text{starts with 'gh' and contains 'kgh' and ends with 'gh'}$ | $\Sigma = \{g, h, k\}$

Solution:

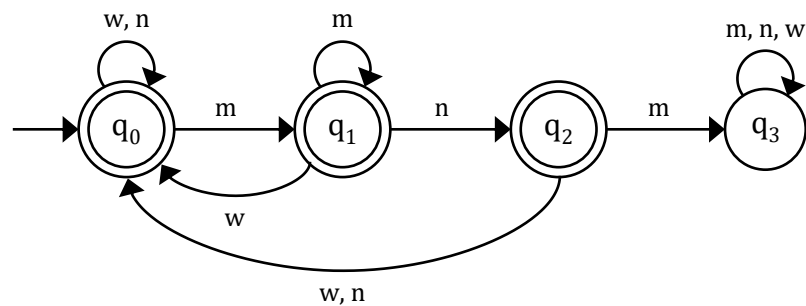
- a) $L = \text{contains even number of 'a' and ends with 'bc'}$ | $\Sigma = \{a, b, c\}$

The DFA has been designed below:



- b) $L = \text{does not contain 'mnm'}$ | $\Sigma = \{m, n, w\}$

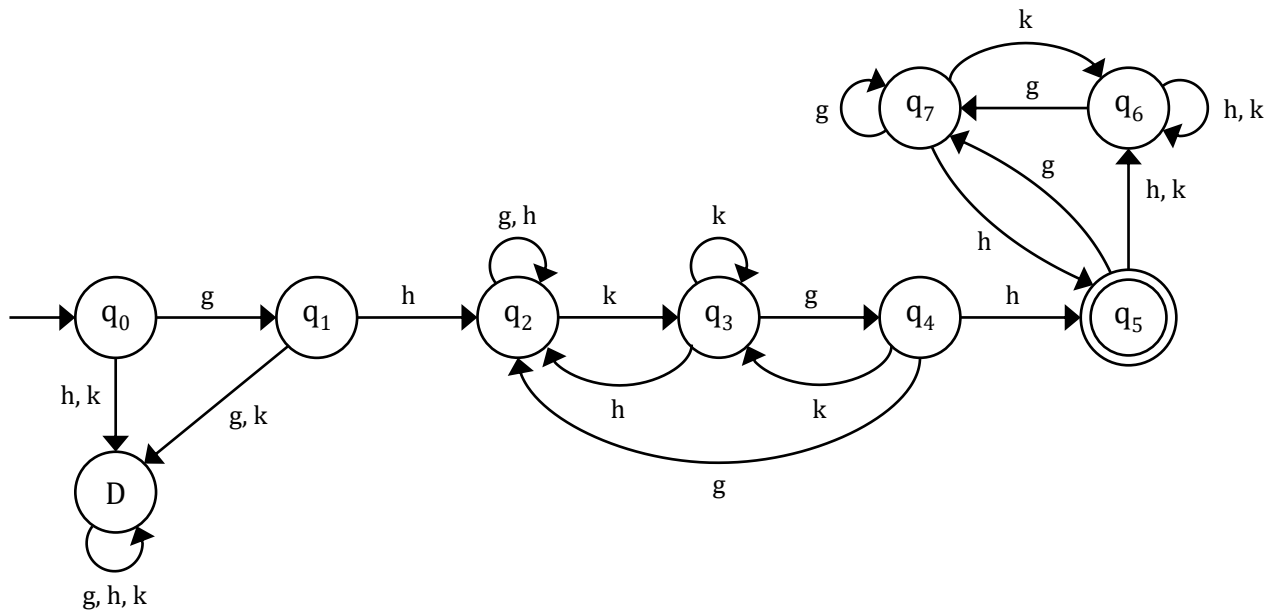
The DFA has been designed below:



- c) $L = \text{starts with 'gh' and contains 'kgh' and ends with 'gh'}$ | $\Sigma = \{g, h, k\}$

The DFA has been designed below:

[P.T.O]



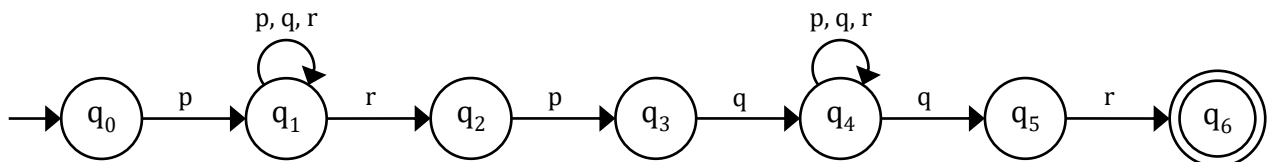
2. Design NFAs that accepts the following languages:

- a) $L = \text{starts with 'p', and contains 'rqp', and ends with 'qr' } | \Sigma = \{p, q, r\}$
- b) $L = \text{starts with '11' or '21' and contains '210' and ends with '101' } | \Sigma = \{0, 1, 2\}$
- c) $L = \text{starts with 'xyz' and contains 'yyz' or 'zyx' and ends with 'zy' } | \Sigma = \{x, y, z\}$

Solution:

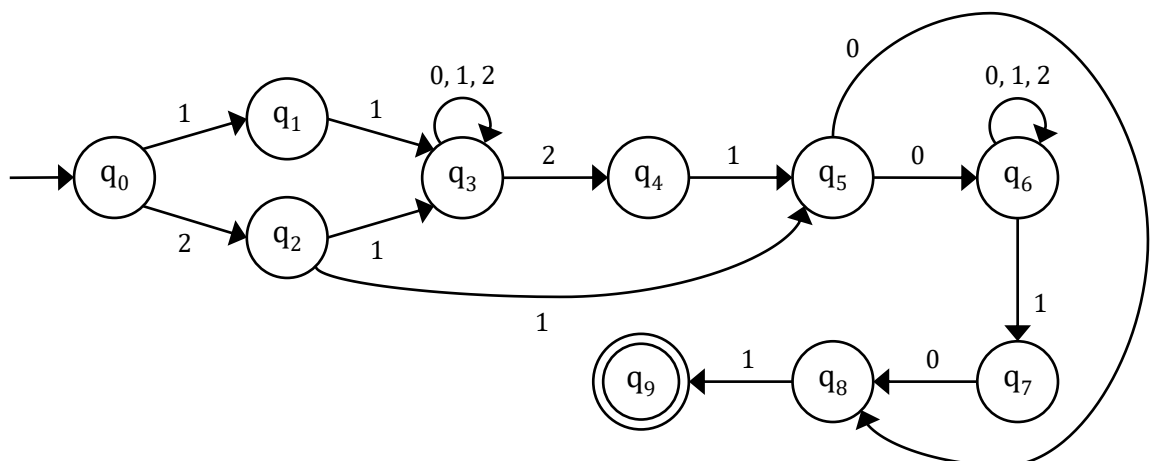
- a) $L = \text{starts with 'p', and contains 'rqp', and ends with 'qr' } | \Sigma = \{p, q, r\}$

The NFA has been designed below:



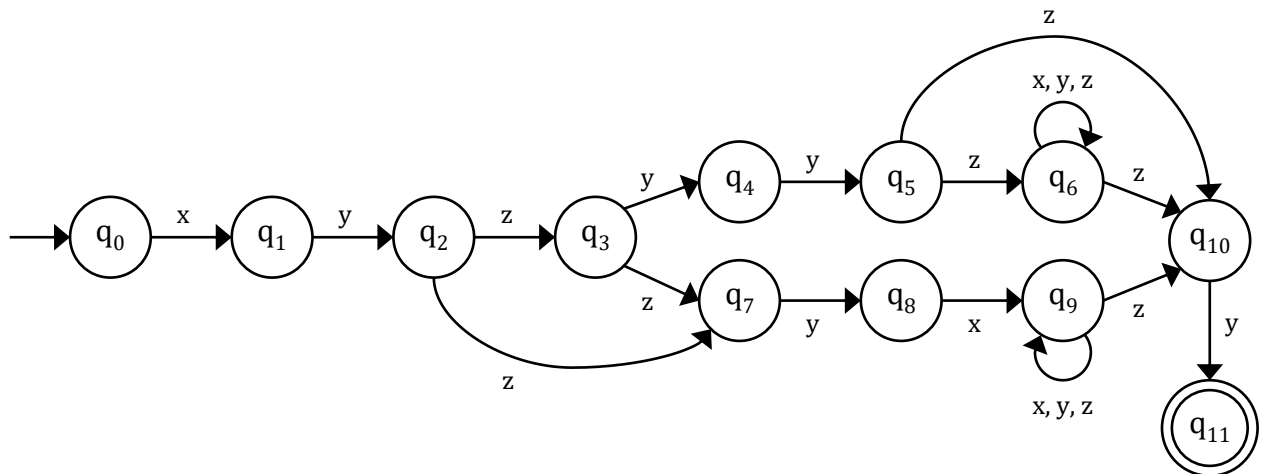
- b) $L = \text{starts with '11' or '21' and contains '210' and ends with '101' } | \Sigma = \{0, 1, 2\}$

The NFA has been designed below:

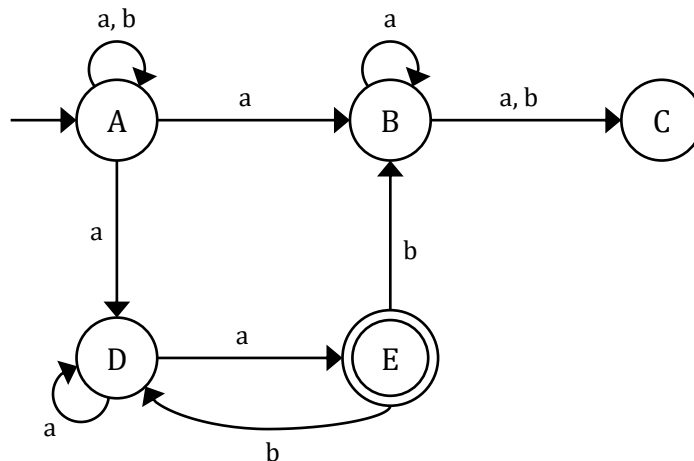


- c) $L = \text{starts with 'xyz' and contains 'yyz' or 'zyx' and ends with 'zy'} \mid \Sigma = \{x, y, z\}$

The NFA has been designed below:



3. Consider the following NFA, and show with help of NFA-tree whether the string "aabaab" is accepted or not.

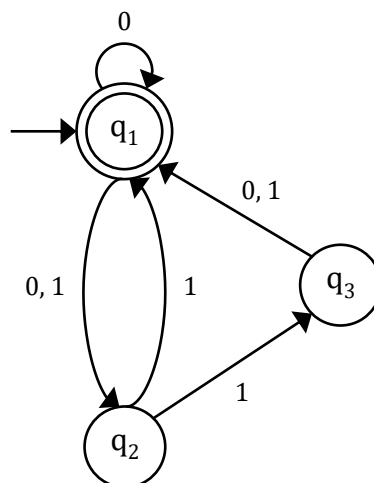


Solution:

NFA Tree:

[P.T.O]

4. Convert the following **NFA** over the alphabet $\Sigma = \{0,1\}$ to an equivalent **DFA**.



Solution:

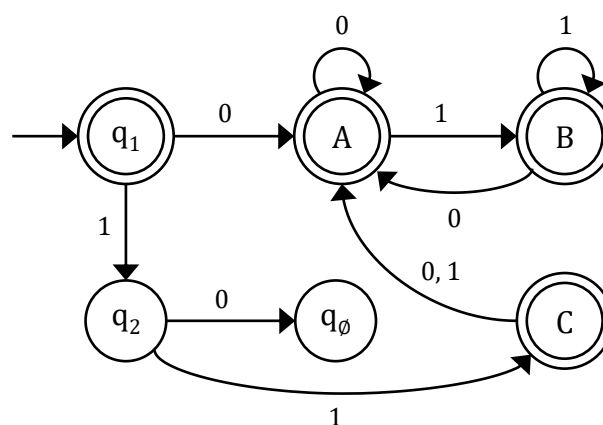
Transition Table of the given NFA:

	0	1
$\rightarrow * q_1$	q_1, q_2	q_2
q_2	\emptyset	q_1, q_3
q_3	q_1	q_1

Transition Table for the Equivalent DFA:

	0	1
$\rightarrow * q_1$	$\{q_1, q_2\} = A$	q_2
$* A = \{q_1, q_2\}$	$\{q_1, q_2\} = A$	$\{q_1, q_2, q_3\} = B$
q_2	$\{\emptyset\} = q_\emptyset$	$\{q_1, q_3\} = C$
$* B = \{q_1, q_2, q_3\}$	$\{q_1, q_2\} = A$	$\{q_1, q_2, q_3\} = B$
$* C = \{q_1, q_3\}$	$\{q_1, q_2\} = A$	$\{q_1, q_2\} = A$

Equivalent DFA Diagram:



5. Design Regular Expression for the following languages where $\Sigma = \{a, b\}$

- a) All strings w having even length strings and starting with **a** or odd length strings starting with **b**.
- b) All strings w which begins and ends with **b**.
- c) All strings w where every **a** is followed by at least one **b**.

Solution:

- a) All strings w having even length strings and starting with **a** or odd length strings starting with **b**.

Regular Expression: $a(a | b)((a | b)(a | b))^* | b((a | b)(a | b))^*$

- b) All strings w which begins and ends with **b**.

Regular Expression: $b(a | b)^*b | b$

- c) All strings w where every **a** is followed by at least one **b**.

Regular Expression: $b^*(ab^*)^*$

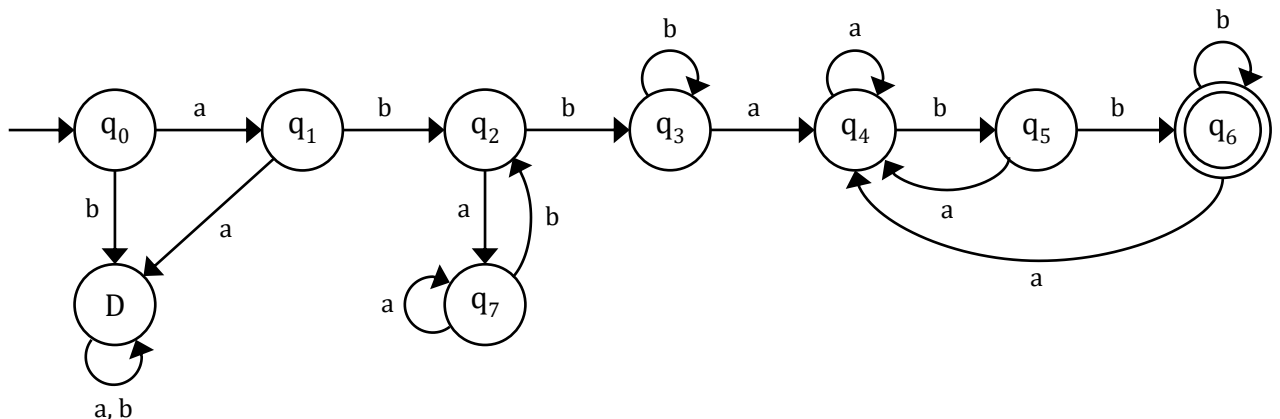
1. Design DFAs that accepts the following languages:

- a)** $L = \{ w \mid w \text{ starts with 'ab' and contains 'bba' and ends with 'bb' } \}$
 $\mid \Sigma = \{a, b\}$
- b)** $L = \{ w \mid w \text{ contains the set of all strings that has length exactly 3 and its third symbol is from the left side is 'a' } \}$
 $\mid \Sigma = \{a, b\}$
- c)** $L = \{ w \mid w \text{ contains the set of all strings that has neither '00' nor '11' as substring } \}$
 $\mid \Sigma = \{0,1,2\}$
- d)** $L = \{ w \mid w \text{ contains the set of all strings whose length always returns remainder 2 when divided by 4 } \}$
 $\mid \Sigma = \{0,1\}$

Solution:

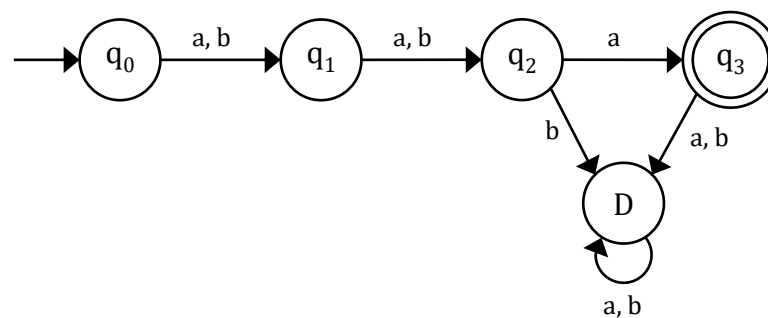
- a)** $L = \{ w \mid w \text{ starts with 'ab' and contains 'bba' and ends with 'bb' } \} \mid \Sigma = \{a, b\}$

The DFA has been designed below:



- b)** $L = \{ w \mid w \text{ contains the set of all strings that has length exactly 3 and its third symbol is from the left side is 'a' } \} \mid \Sigma = \{a, b\}$

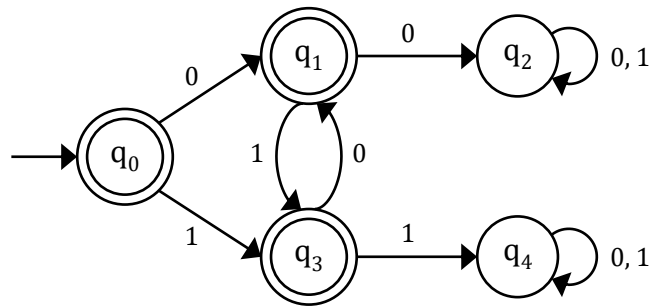
The DFA has been designed below:



- c)** $L = \{ w \mid w \text{ contains the set of all strings that has neither '00' nor '11' as substring } \}$
 $\mid \Sigma = \{0,1,2\}$

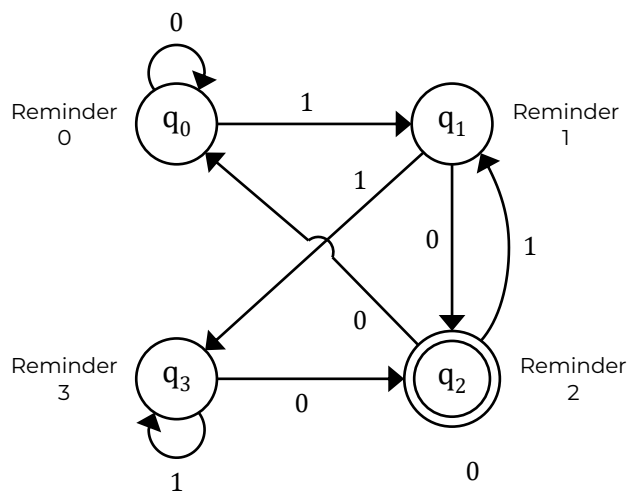
[P.T.O]

The DFA has been designed below:



- d)** $L = \{ w \mid w \text{ contains the set of all strings whose length always returns remainder 2 when divided by 4} \mid \Sigma = \{0,1,2\}$

The DFA has been designed below:



Decimal	Binary	Reminder
0	0	0
1	1	1
2	10	2
3	11	3
4	100	0
5	101	1
6	110	2
7	111	3
8	1000	0

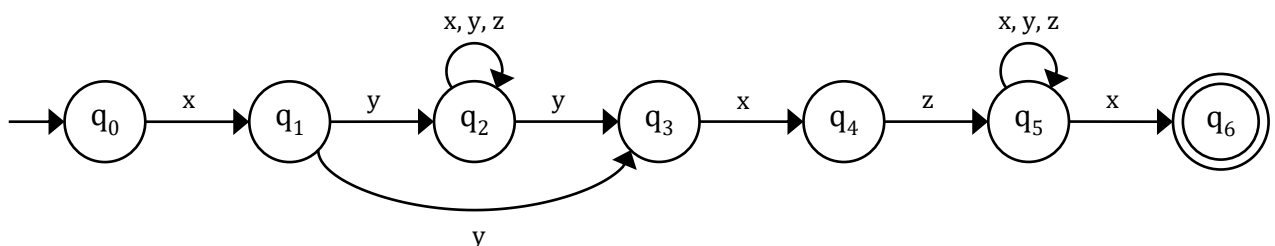
2. Design NFAs that accepts the following languages:

- a)** $L = \text{ends with 'x' and contains 'yxz' and starts with 'xy'}$
 $\mid \Sigma = \{x, y, z\}$
- b)** $L = \text{starts with 'pq' or 'qr' and contains 'pqp' or 'qrr' and ends with 'qqr'}$
 $\mid \Sigma = \{p, q, r\}$
- c)** $L = \text{starts with '211' and contains '112' or '321' and ends with '1'}$
 $\mid \Sigma = \{1,2,3\}$

Solution:

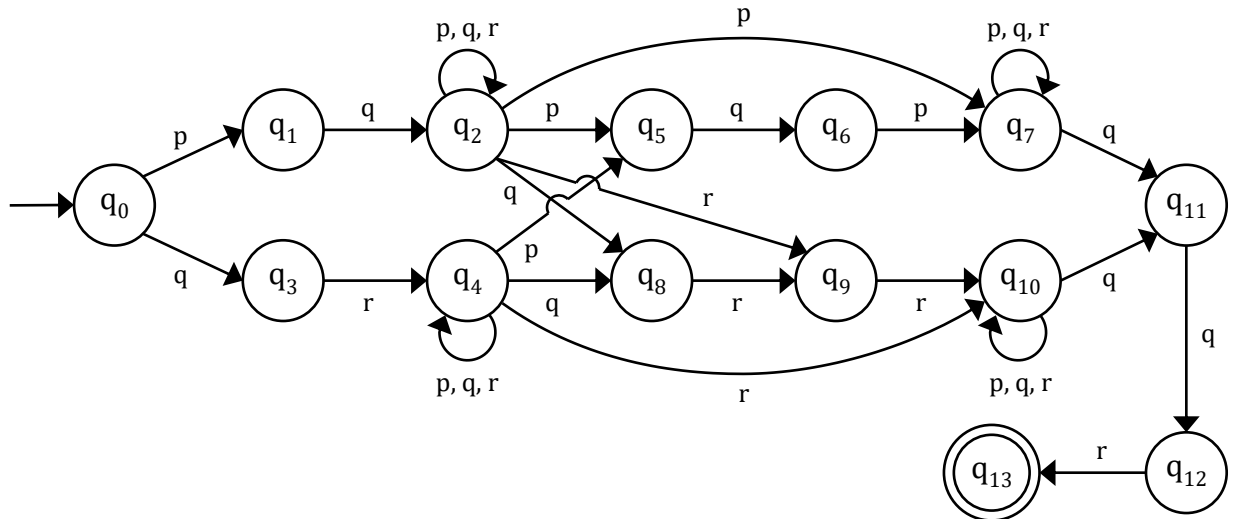
- a)** $L = \text{ends with 'x' and contains 'yxz' and starts with 'xy'}$ $\mid \Sigma = \{x, y, z\}$

The NFA has been designed below:



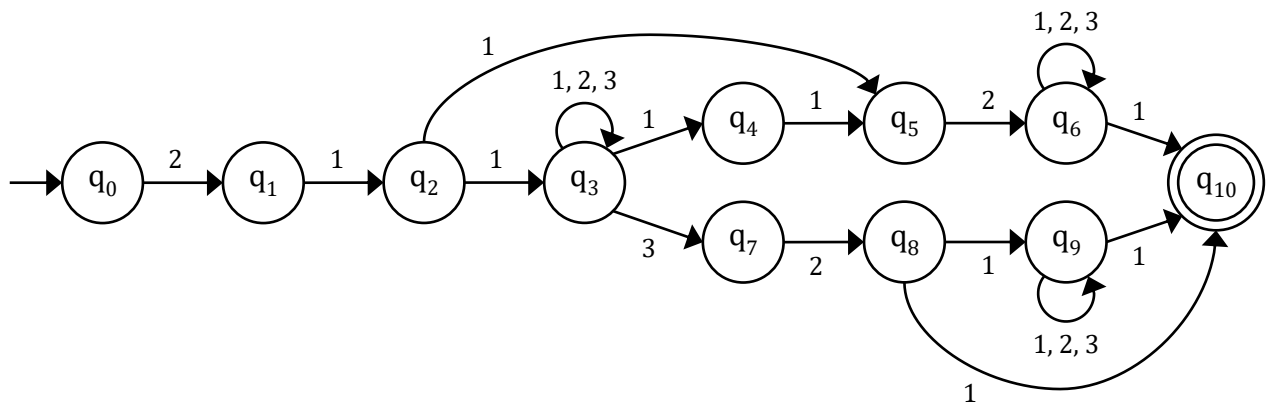
- b)** $L = \text{starts with 'pq' or 'qr' and contains 'pqp' or 'qrr' and ends with 'qqr'} \mid \Sigma = \{p, q, r\}$

The NFA has been designed below:

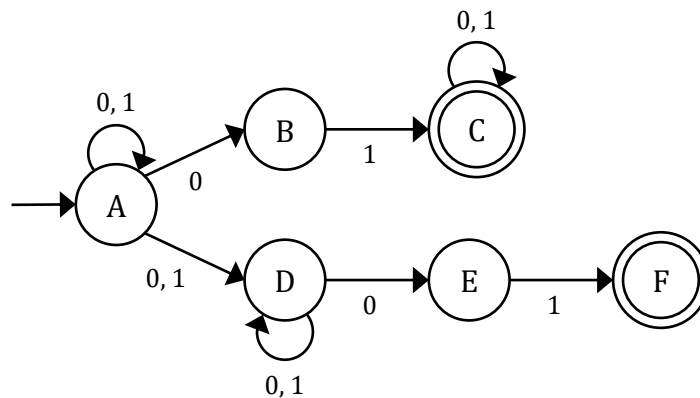


- c)** $L = \text{starts with '211' and contains '112' or '321' and ends with '1'} \mid \Sigma = \{1, 2, 3\}$

The NFA has been designed below:



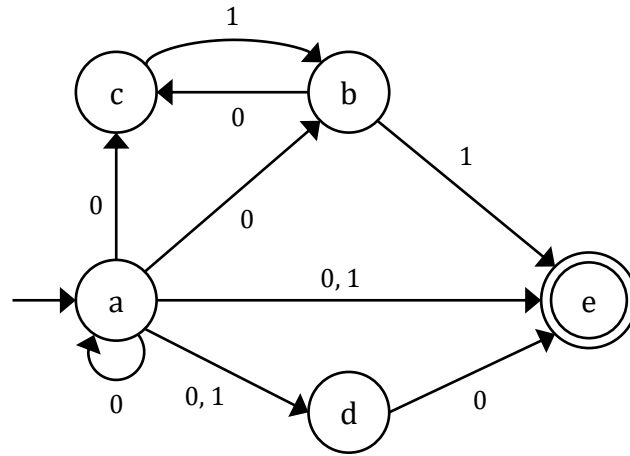
- 3.** Consider the following NFA, and show with help of NFA-tree whether the string "11010" is accepted or not.



Solution:

NFA Tree:

[P.T.O]



Solution:

Transition Table of the given NFA:

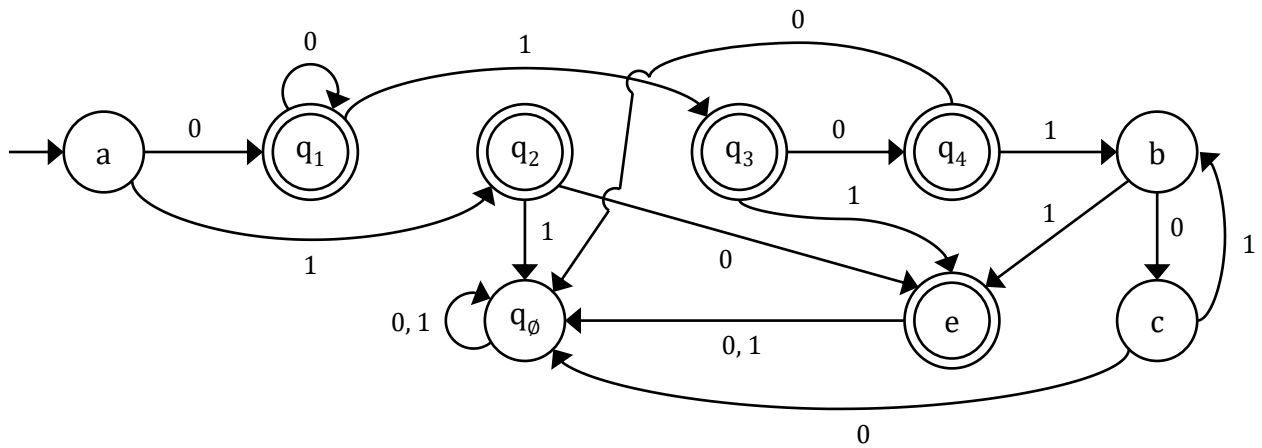
	0	1
→ a	a, b, c, d, e	d, e
b	c	e
c	∅	b
d	e	∅
* e	∅	∅

Transition Table for the Equivalent DFA:

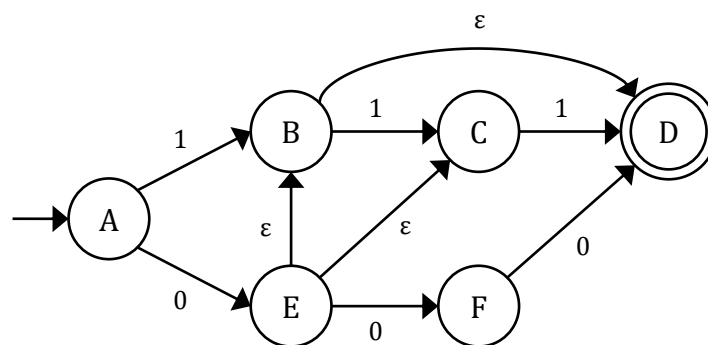
	0	1
→ a	{ a, b, c, d, e } = q ₁	{ d, e } = q ₂
* q ₁ = { a, b, c, d, e }	{ a, b, c, d, e } = q ₁	{ b, d, e } = q ₃
* q ₂ = { d, e }	e	{ ∅ } = q _∅
* q ₃ = { b, d, e }	{ c, e } = q ₄	e
* e	{ ∅ } = q _∅	{ ∅ } = q _∅
* q ₄ = { c, e }	{ ∅ } = q _∅	b
b	c	e
c	{ ∅ } = q _∅	b
* q _∅ = { ∅ }	{ ∅ } = q _∅	{ ∅ } = q _∅

Equivalent DFA Diagram:

[P.T.O]



5. Convert the following ϵ -NFA over the alphabet $\Sigma = \{0,1\}$ to an equivalent DFA.



Solution:

Transition Table of the given ϵ -NFA:

	0	1
$\rightarrow A$	E	B
B	\emptyset	C
C	\emptyset	D
* D	\emptyset	\emptyset
E	F	\emptyset
F	D	\emptyset

ϵ -Closure of all state of the given ϵ -NFA:

ϵ -Closure (A) = { A }

ϵ -Closure (B) = { B, D }

ϵ -Closure (C) = { C }

ϵ -Closure (D) = { D }

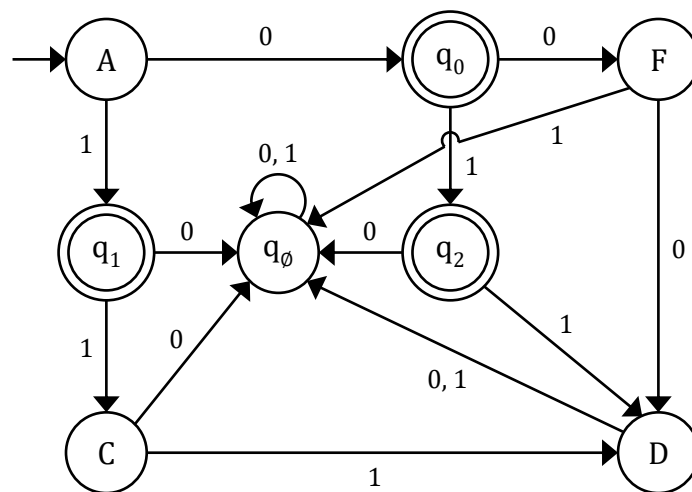
ϵ -Closure (E) = { E, B, C, D }

ϵ -Closure (F) = { F }

Transition Table for the Equivalent DFA:

	0	1
→ A	{ B, C, D, E } = q_0	{ B, D } = q_1
* $q_0 = \{ B, C, D, E \}$	F	{ C, D } = q_2
* $q_1 = \{ B, D \}$	{ \emptyset } = q_\emptyset	C
F	D	{ \emptyset } = q_\emptyset
* $q_2 = \{ C, D \}$	{ \emptyset } = q_\emptyset	D
C	{ \emptyset } = q_\emptyset	D
* D	{ \emptyset } = q_\emptyset	{ \emptyset } = q_\emptyset
$q_\emptyset = \{ \emptyset \}$	{ \emptyset } = q_\emptyset	{ \emptyset } = q_\emptyset

Equivalent DFA Diagram:



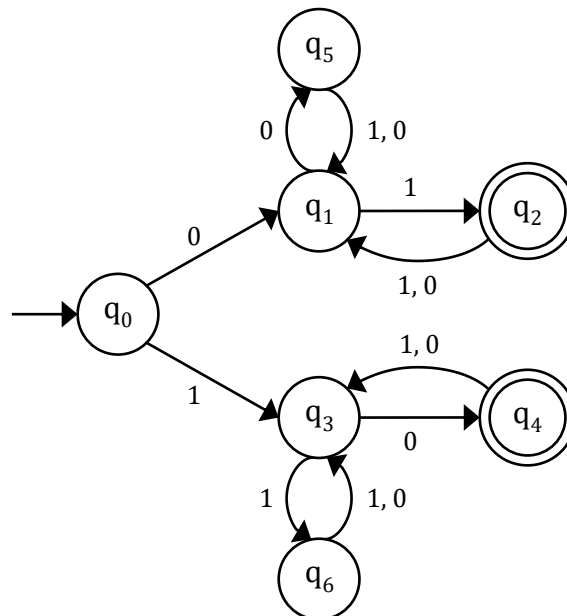
1. Design DFAs that accepts the following languages:

- a) $L = \{ w \mid w \text{ starts and ends with different symbols and the length of } w \text{ is even} \} \mid \Sigma = \{0,1\}$
- b) $L = \{ w \mid w \text{ contains at least two 'a's and at most one 'b' } \} \mid \Sigma = \{a, b\}$
- c) $L = \{ w \mid w \text{ contains even number of 0's or odd number of 2's.} \} \text{ over } \Sigma = \{0,1,2\}$
- d) $L = \{ w \mid w \text{ contains all the binary number which is divisible by 3 or ends with '011' } \} \mid \Sigma = \{0,1\}$

Solution:

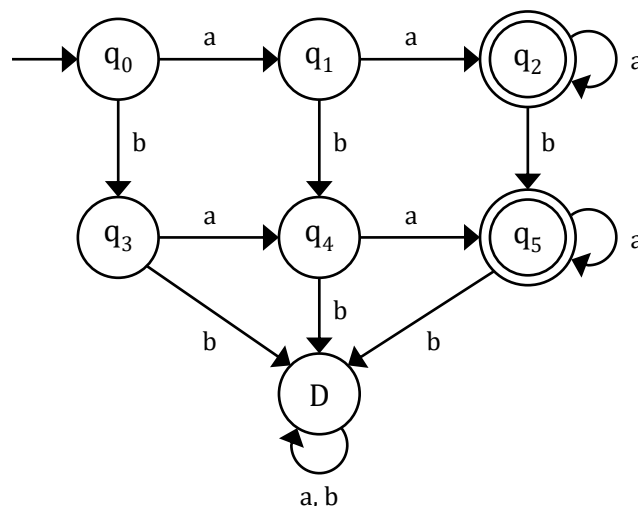
- a) $L = \{ w \mid w \text{ starts and ends with different symbols and the length of } w \text{ is even} \} \mid \Sigma = \{0,1\}$

The DFA has been designed below:



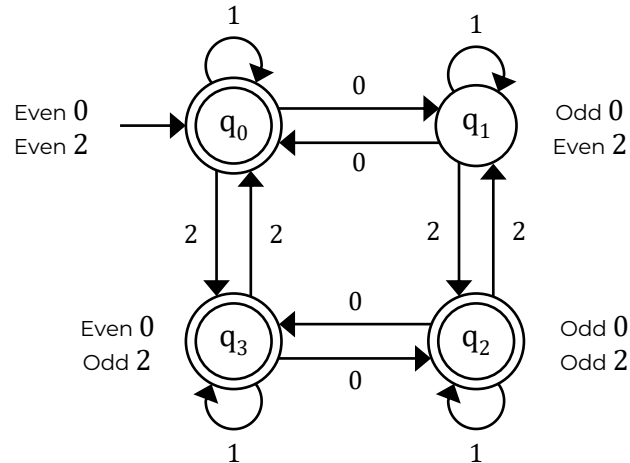
- b) $L = \{ w \mid w \text{ contains at least two 'a's and at most one 'b' } \} \mid \Sigma = \{a, b\}$

The DFA has been designed below:



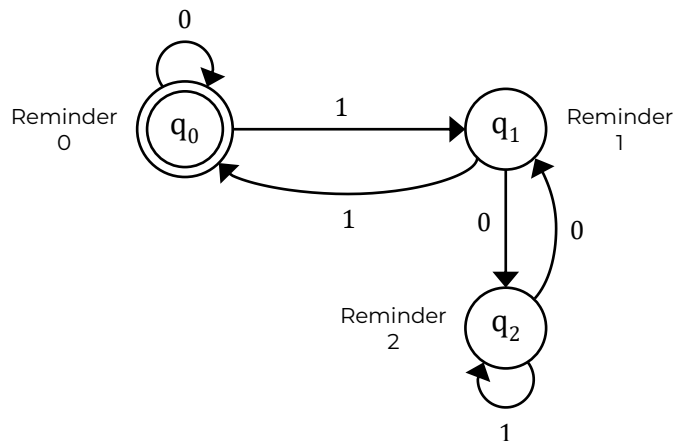
- c) $L = \{ w \mid w \text{ contains even number of 0's or odd number of 2's.} \}$ over $\Sigma = \{0,1,2\}$

The DFA has been designed below:



- d) $L = \{ w \mid w \text{ contains all the binary number which is divisible by 3 or ends with '011' } \}$

The DFA has been designed below:



Decimal	Binary	Reminder
0	0	0
1	1	1
2	10	2
3	11	0
4	100	1
5	101	2
6	110	0

2. Design NFAs that accepts the following languages:

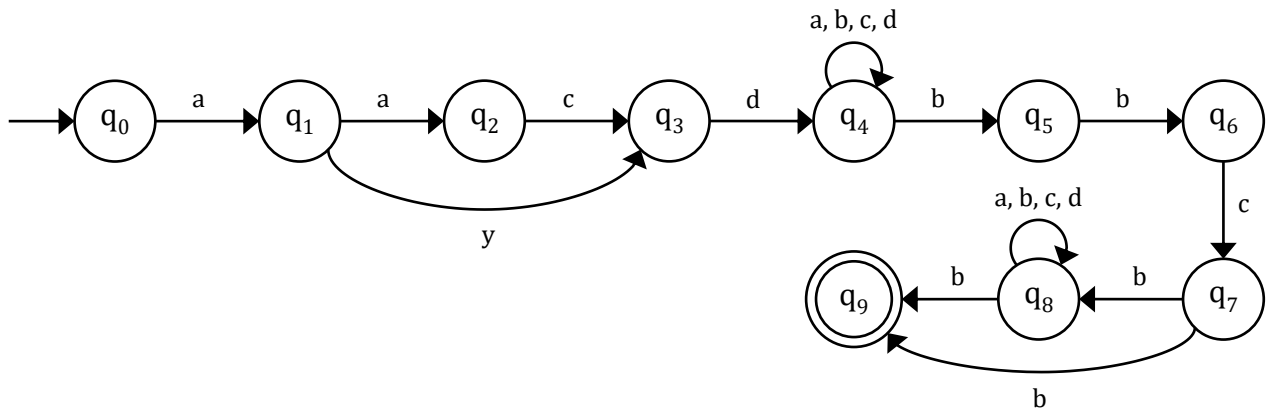
- a) $L = \text{ends with 'b' and contains 'bbcb' and starts with 'acd' } \mid \Sigma = \{a, b, c, d\}$
b) $L = \text{contains 'bba' or 'abb' or 'acc' and starts with 'ab' or 'bc' } \mid \Sigma = \{a, b, c\}$
c) $L = \text{starts with '121' and contains '212' or '312' and ends with '2' } \mid \Sigma = \{1,2,3\}$

Solution:

- a) $L = \text{ends with 'b' and contains 'bbcb' and starts with 'acd' } \mid \Sigma = \{a, b, c, d\}$

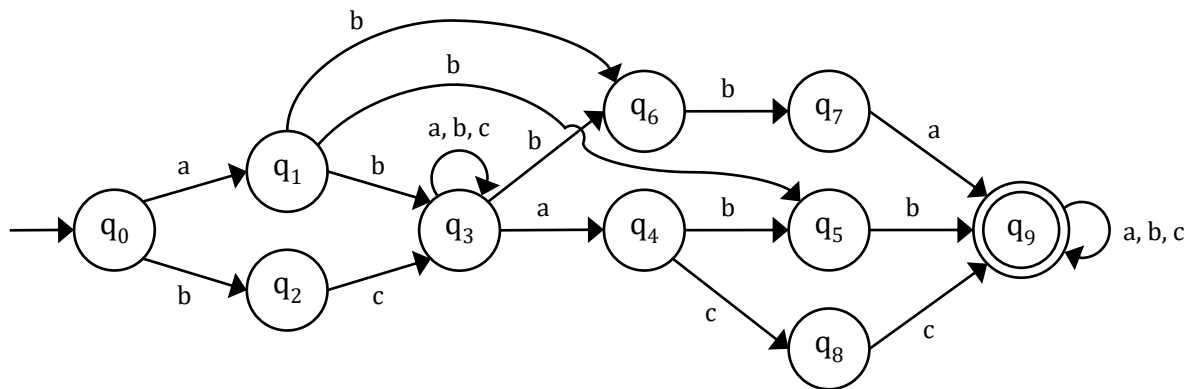
The NFA has been designed below:

[P.T.O]



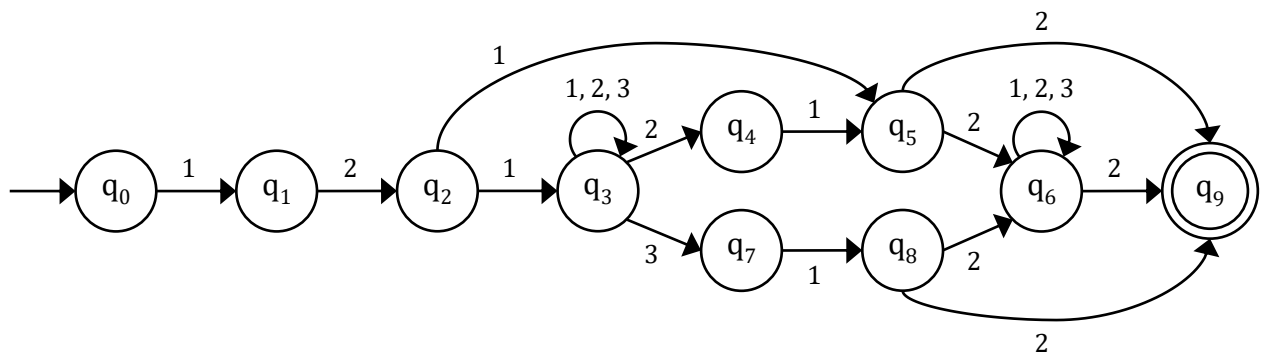
b) $L = \text{contains 'bba' or 'abb' or 'acc' and starts with 'ab' or 'bc' } \mid \Sigma = \{a, b, c\}$

The NFA has been designed below:

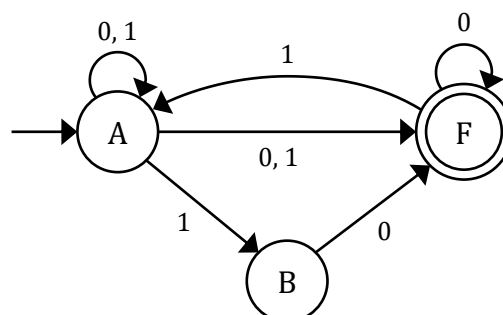


c) $L = \text{starts with '121' and contains '212' or '312' and ends with '2' } \mid \Sigma = \{1, 2, 3\}$

The NFA has been designed below:

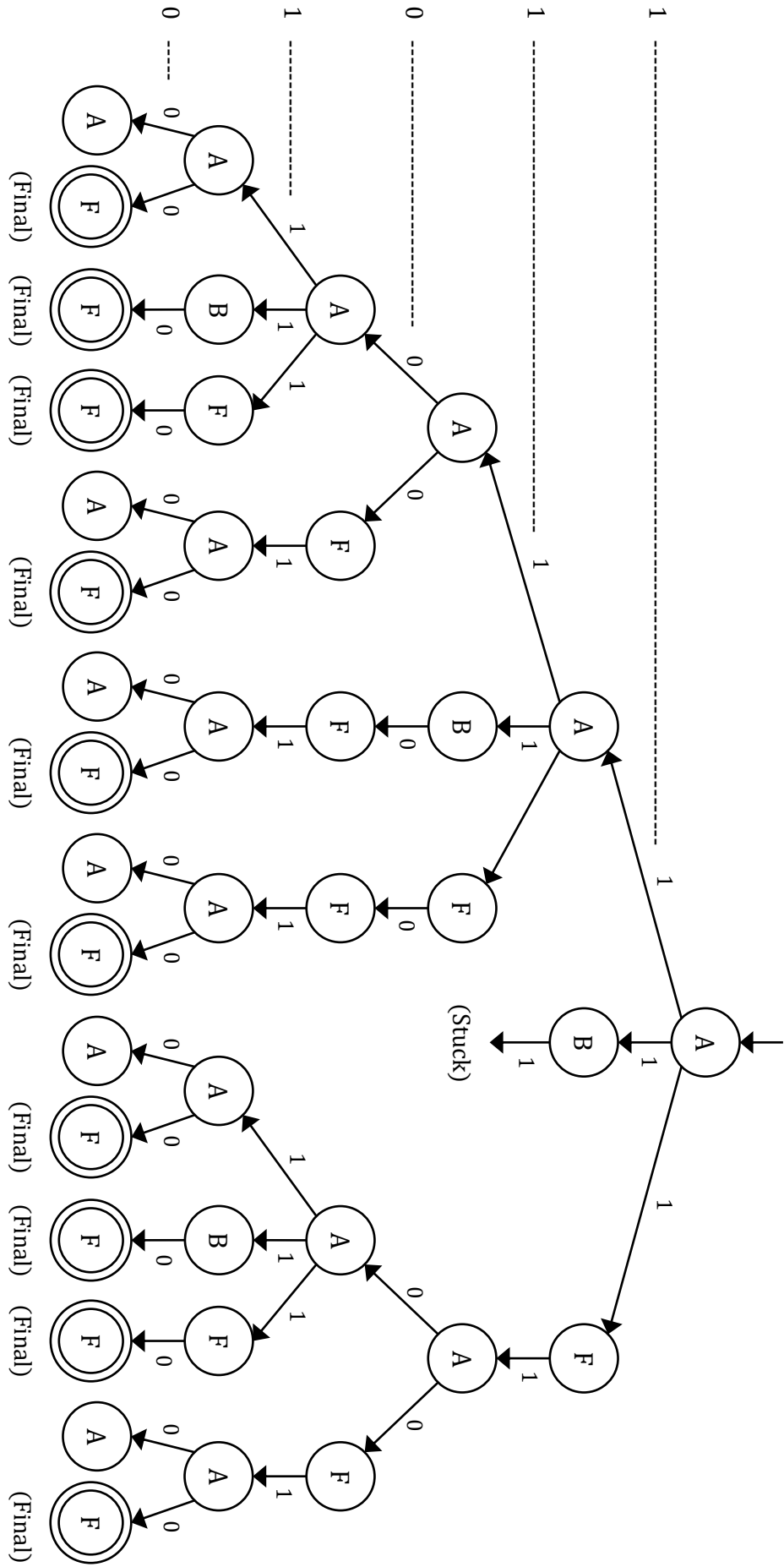


3. Consider the following NFA, and show with help of NFA-tree whether the string “11010” is accepted or not.



Solution:

NFA Tree:



With help of NFA-tree, we can see the string "11010" reach the final state.
∴ The string "11010" is accepted.