



United International University
School of Science and Engineering
Final Examination Trimester: Spring 2022
Course title: Coordinate Geometry and Vector Analysis
Course Code: Math 2201 Marks: 40 Time: 2 hours

Answer all the questions

1. a) Find parametric equations of the tangent line to the curve of intersection of the paraboloid $z = \sqrt{x^2 + y^2}$ and the plane $2x - 5y + 3z = 9$ at the point $(-1, -2, 3)$. [3]
- b) Evaluate $\int_C x^2 dy + y dx$; where C is the triangle with vertices $(0, 0)$, $(3, 0)$ and $(3, 9)$, oriented counterclockwise. [3]
- c) Find the value of the integral $\int_1^5 \int_2^4 \frac{xy}{\sqrt{x^2 + y^2 + 1}} dy dx$ [4]
2. a) Find the volume of the tetrahedron bounded by the coordinate planes and the plane $z = 6 - 8x - 2y$. [5]
- b) Evaluate
$$\int_{-4}^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_0^{\sqrt{16-x^2-y^2}} \sqrt{x^2 + y^2} dz dy dx$$
 [5]
3. a) Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 5$ and between the planes $z = 2$ and $x + z = 8$. [5]
- b) Use Green's theorem to evaluate $\oint \frac{xy}{1+x} dx - \ln(1+x) dy$, where C is the rectangle with vertices $(0, 0)$, $(4, 0)$, $(4, 2)$ and $(0, 2)$, oriented counterclockwise. [5]
4. a) Evaluate $\int_{(0,0)}^{(1,2)} (3x^2y^2 + 2x \cos y) dx + (2x^3y - x^2 \sin y) dy$ using the fundamental theorem of line integrals. [5]
- b) Find the directional derivative of $f(x, y) = \sqrt[3]{x+y}$ at $P(1, 0)$ in the direction of $b = 2i + j$. [3]
- c) Prove that $\text{div}(\text{curl } F) = 0$ [2]

20 +