United International University

School of Science and Engineering

Mid Term Examination Trimester: Spring-2019

Course Title: Linear Algebra, Ordinary &Partial Differential Equations

Course Code: Math 183 Marks: 30 Time: 105 min

There are 4 questions. Answer 1,4 and any one from the rest.

1. [CO1] a) Find the values of a for which the system below has unique, infinite or no solution. [3]

$$x + (a^2 - 1)y = a$$
$$(a^2 - 4)y = a - 1$$

b) Solve the system of homogenous linear equations. Show the result in vector form. Is there any non-trivial solution? [3]

$$-x_1 + 2x_2 - x_3 + x_4 = 0$$
$$3x_1 - 2x_2 + x_3 - x_4 = 0$$
$$x_1 - 3x_2 + x_3 - 2x_4 = 0$$

c) Solve the following system by row reduction method [4]

$$x + y + z = 1$$
$$-x + 2y - 2z = -1$$
$$x + y - 2z = 2$$

2. [CO2] a) Find the inverse of $A = \begin{bmatrix} 3 & -2 & 0 \\ 1 & 0 & 1 \\ -2 & 3 & 0 \end{bmatrix}$ by applying inversion algorithm (row reduction). [4]

b) Find eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$. Also sketch the eigenspace in xy −coordinates. [5]

c) Determine whether the matrix $A = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$ has nontrivial fixed points. [1]

3. [CO2] a) Given $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{7}{2} & -\frac{3}{2} \\ -2 & 1 \end{bmatrix}$ [4]

i) Show $(AB)^T = B^T A^T$ ii) Find A^{-3}

b) Find $P(A)$ for $p(x) = x^2 - 5x + 5$ where $A = \begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$. [3]

c) Find cofactor matrix of $A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$. [3

4. [CO3] a) Find the solution of the given differential equations [

i) $ty' + 2y = t^2 - t + 1, \quad y(1) = \frac{1}{2}$

ii) $y' = \frac{3x^2 - e^x}{2y - 5}, \quad y(0) = 1$

b)Determine the values of r for which the differential equation $t^2y'' - 6ty' + 10y = 0$ has solutions of the form t^r.