# United International University
## Department of Computer Science and Engineering

CSE 2213: Discrete Mathematics  Midterm : Fall 2022

Total Marks: 30  Time: 1 hour 45 minutes

Any examinee found adopting unfair means will be expelled from the trimester / program as per UIU disciplinary rules.

---

*Answer all the questions. Numbers to the right of the questions denote their marks.*

1. (a) Let, proposition **p: Stokes is dependable**, proposition **q: Stokes is calculative**, proposition **r: Livingstone comes to party**, and proposition **s: Buttler is not out**. Now using the logical operators formulate the following compound propositions. **[1 x 3 = 3]**

   i. Either Stokes is dependable, or he is not dependable but calculative.

   ii. Stokes is either dependable or calculative, but Stokes is not calculative if Buttler is not out.

   iii. A necessary condition for Livingstone coming to the play is that, if Stokes is not calculative and Buttler is out then Stokes is not dependable.

   (b) Prove using different logical equivalence laws that $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a Tautology. **[3]**

2. (a) Look at the following predicates:

   *P(x): x is a student.*
   *Q(x,y): x and y are classmates.*
   *R(x,y): x and y are friends.*

   Represent the following sentences using these predicates, appropriate quantifiers, and logical connectives. The domain of all the variables is the set of all people. **[1 x 3 = 3]**

   i. There is a student who is a classmate of *Peter Parker*.

   ii. Everyone has a friend who is his/her classmate.

   iii. There are at least two students who are classmates, but not friends.

   (b) With justification, find out whether the following propositions are true or false. The domain of all the variables is the set of real numbers. **[1.5 x 2 = 3]**

   i. $\forall x \forall y \exists z \left(z = \frac{x}{y}\right)$

   ii. $\exists x \exists y \left(\frac{x}{y} = \frac{y}{x}\right)$

3. (a) Use *direct proof* to show that the product of two rational numbers is rational. **[3]**

   (b) Show using *proof by contradiction* that if n is an integer and $n^3 + 5$ is odd, then n is even. **[3]**

4. (a) Given a set X = {Tamim, Shakib, Mushfiq} , find the power set of X **[1]**

   (b) Given a set A = {DM, OOP, ICS} and set B = {Easy, Medium, Hard}, find the sets $A \times B$ and $B \times A$ and their cardinalities. **[2]**

   (c) Shade the following on a Venn diagram: **[3]**

   - $(A \cup B) - C$
   - $(A - C) \cap \overline{A}$
   - $(\overline{B} - C) \cap A$

5. (a) Is the function $f : R \rightarrow R$, $f(x) = \sqrt[3]{x}$ injective, surjective or bijective? Explain with proper numerical examples. **[2]**

   (b) A function f has domain = {A,B,C,D,E} and codomain = {Apple, Banana, Orange, Pineapple}. f(A) = Apple, f(B) = Banana, f(C) = Pineapple, f(D) = Apple, f(E) = Orange. What is the inverse of the function f? Give proper reasoning for your answer. **[1]**

   (c) Let *a* be the function from the set {x,y,z} to itself such that a(x) = y, a(y) = z, a(z) = x. Let *b* be the function from the set {x,y,z} to the set {Hi, Hello, Bye} such that b(x) = Bye, b(y) = Hi, b(z) = Hello. What is the composition of a and b? What is the composition of b and a? **[3]**