



United International University
School of Science and Engineering
 Final Examination Trimester: Summer-2023
 Course Title: Calculus and Linear Algebra
 Course Code: MATH 2183 Marks: 40 Time: 2 Hour

Answer all the questions.

1.	<p>a) Solve the following system by Gauss-Jordan elimination method</p> $\begin{aligned} x - y - z + 2p &= 2 \\ 2x + y - 2z + p &= 0 \\ -x + 2y - z - 4p &= -1 \end{aligned}$ <p>b) State (do not solve) how many solutions does the following set of equations have?</p> $\begin{aligned} x + y &= 1 \\ 3x + 3y &= -5 \end{aligned}$ <p>c) Solve the homogeneous system of linear equations</p> $\begin{aligned} 3x - 2y - z + 3w &= 0 \\ x - y + 2z - 2w &= 0 \end{aligned}$	<p>[6]</p> <p>[2]</p> <p>[2]</p>
2.	<p>a) Given that,</p> $\begin{aligned} x - 2y + z &= 1 \\ x - 3y - z &= 0 \\ 2x + y - 2z &= 2 \end{aligned}$ <p>i. Write the above system of linear equations in the form $AX = B$, where A, X and B are matrices.</p> <p>ii. Find the inverse of A and hence solve the above system of linear equations.</p> <p>b) Find eigenvalues and eigenvectors of the Matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 6 \end{bmatrix}$. Also sketch the eigenspace in xy-coordinates.</p>	<p>[5]</p> <p>[5]</p>
3.	<p>a)</p> <p>Given $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 2 & 1 & 1 & 2 \\ 3 & 0 & 0 & 0 \end{bmatrix}$, $B = [1 \ 2 \ 1 \ 3]$</p> <p>i. Find $\det(A)$ and $\det(B)$</p> <p>ii. Evaluate AB and BA.</p> <p>iii. Find $3A - 3$.</p> <p>iv. Find x, Such that $\text{tr}(A) = x^2 + 3$</p> <p>b) Solve $(3x^2y + 2x)dx + (x^3 + 2y)dy = 0$</p>	<p>[5]</p> <p>[5]</p>
4.	<p>a) Solve the following second order ordinary differential equations</p> <p>i) $\frac{d^2y}{dx^2} + 16\frac{dy}{dx} + 64y = 0$; $y(0) = 1$, $y'(0) = 0$.</p> <p>ii) $y'' + y' + 16y = 0$</p> <p>b) Solve $y'' + y' + y = 2e^x - \cos 2x - \ln 5 + e^{2x} \sin x$.</p>	<p>[5]</p> <p>[5]</p>

