School of Science and Engineering
Final Examination Trimester: Summer-2023
Course Title: Calculus and Linear Algebra
Course Code: MATH 2183 Marks: 40 Time: 2 Hour

Answer all the questions.

1. a) Solve the following system by Gauss-Jordan elimination method [6]

$$x - y - z + 2p = 2$$
$$2x + y - 2z + p = 0$$
$$-x + 2y - z - 4p = -1$$

b) State (do not solve) how many solutions does the following set of equations have? [2]

$$x + y = 1$$
$$3x + 3y = -5$$

c) Solve the homogeneous system of linear equations [2]

$$3x - 2y - z + 3w = 0$$
$$x - y + 2z - 2w = 0$$

2. a) Given that, [5]

$$x - 2y + z = 1$$
$$x - 3y - z = 0$$
$$2x + y - 2z = 2$$

i. Write the above system of linear equations in the form $\boldsymbol{AX} = \boldsymbol{B}$, where A, X and B are matrices.

ii. Find the inverse of A and hence solve the above system of linear equations.

b) Find eigenvalues and eigenvectors of the Matrix $\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 0 & 6 \end{bmatrix}$. Also sketch the eigenspace in xy −coordinates. [5]

3. a) [5]

Given $\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 2 & 1 & 1 & 2 \\ 3 & 0 & 0 & 0 \end{bmatrix}, \boldsymbol{B} = [1 \quad 2 \quad 1 \quad 3]$

i. Find $\det(\boldsymbol{A})$ and $\boldsymbol{det(B)}$

ii. Evaluate $\boldsymbol{AB}$ and $\boldsymbol{BA}$.

iii. Find $3\boldsymbol{A} - 3$.

iv. Find x, Such that $\boldsymbol{tr(A)} = x^2 + 3$

b) Solve $(3x^2y + 2x)dx + (x^3 + 2y)dy = 0$ [5]

4. a) Solve the following second order ordinary differential equations [5]

i) $\frac{d^2y}{dx^2} + 16\frac{dy}{dx} + 64y = 0\,; \quad y(0) = 1,\ y'(0) = 0.$

ii) $y'' + y' + 16y = 0$

b) Solve $y'' + y' + y = 2e^x - cos2x - ln5 + e^{2x}sinx.$ [5]