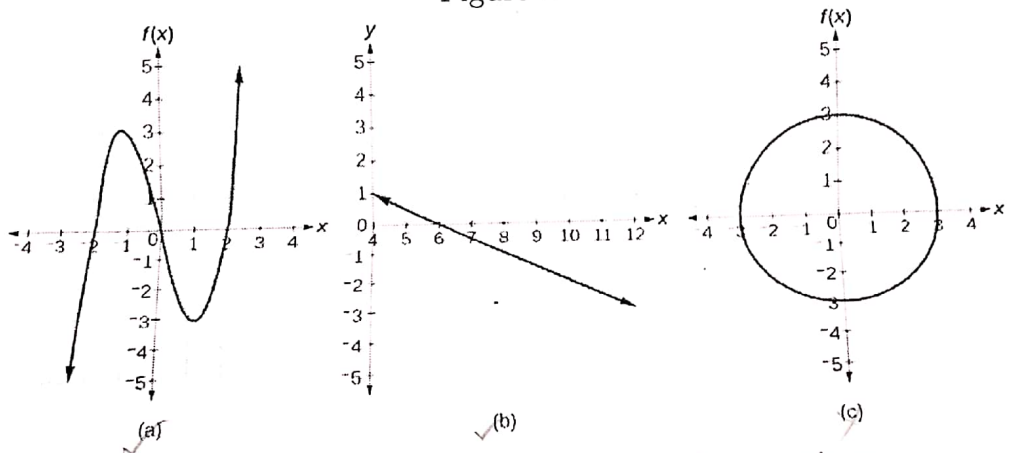
United International University
School of Science and Engineering
Midterm Exam, Fall 2022
Course Code : MATH 1151, Course Title: Fundamental Calculus
Total Marks :30 Time : 1 hour 45 minutes

Solve all questions.

1. (a) (2 points) Determine whether the equation defines y as a function of x.
   i. $(y+3)^3 + 1 = x$
   ii. $y^2 + (x-1)^2 = 2$

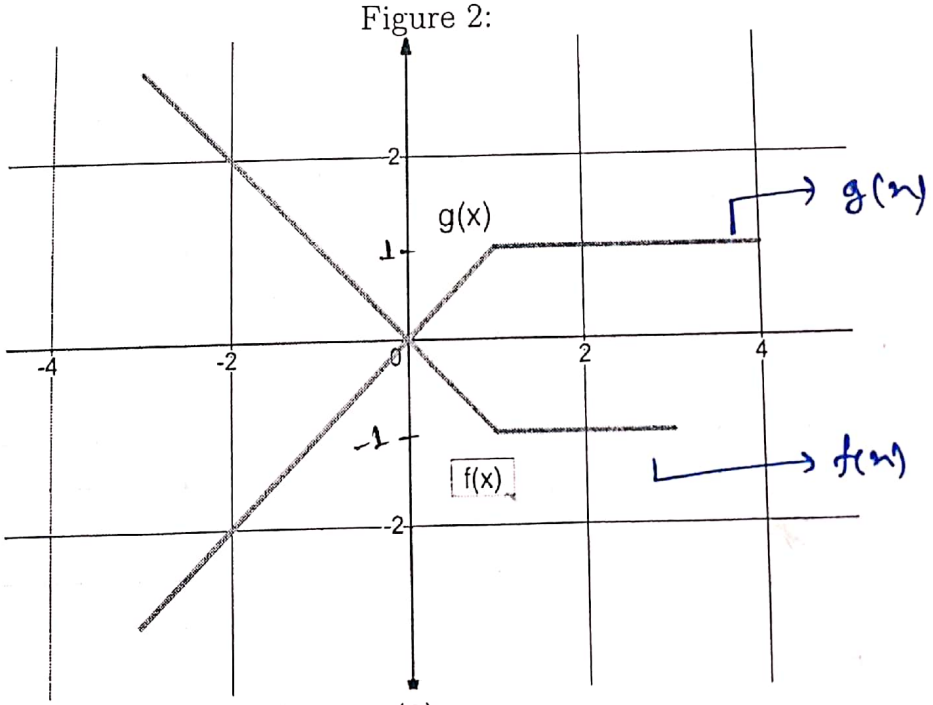
   (b) (2 points) Determine whether the curve is the graph (Figure 1) of a function of x. If it is, Find the domain and range of the functions.

Figure 1:



   (c) (3 points) The graph (Figure 2) of function f and g are given.

Figure 2:



   i. State the values of $f(0)$ and $g(3)$.
   ii. On what interval(s) is $f(x) \leq g(x)$?
   iii. Find the domain and the range of the function g

(d) (3 points) Sketch the graph along with the range of the function.

$$f(x) = \begin{cases} (x-1)^2 - 1 & \text{if } x \leq 1 \\ -1 & \text{if } x > 1 \end{cases}$$

2. (6 points) Draw the graph of following functions and find the domain and range for each of the given functions.

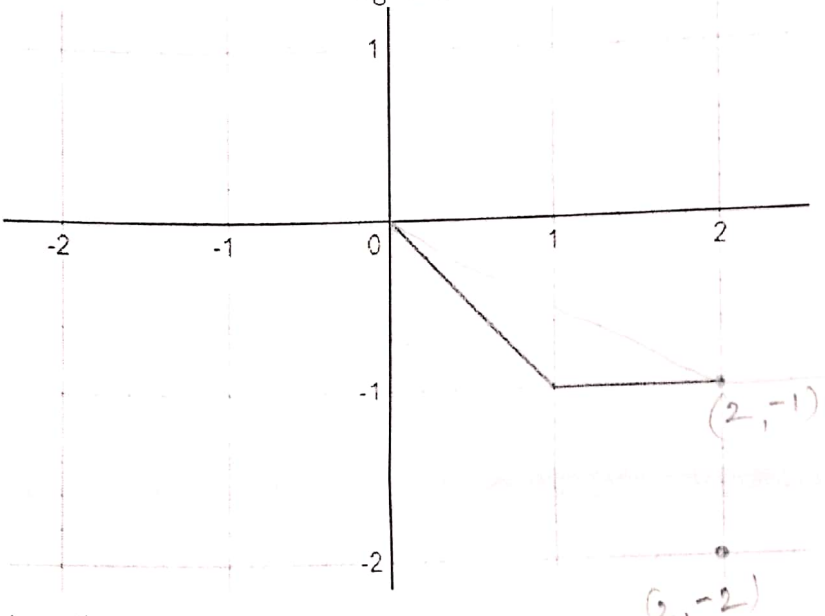
   (a) $y = -\sqrt{1-x} + 1$

   (b) $y = |x-2| - 1$

   (c) $y = 0.5 \sin 2x$

3. (4 points) Use the given graph (Figure 3) of $y = g(x)$ to sketch the following functions:

Figure 3:



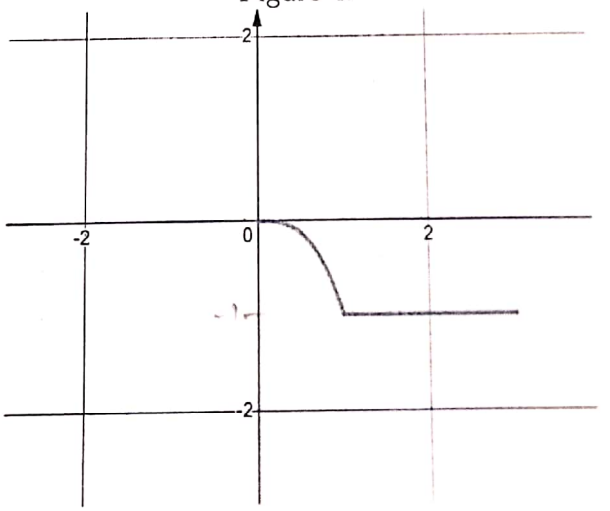
   (a) $y = g(x-2)$

   (b) $y = |g(x) - 1|$

   (c) $y = 2g(\frac{1}{2}x)$

4. (a) (2 points) The given graph (Figure 4) of the function is defined for $x \geq 0$. Complete the graph for $x < 0$ to make it (a) an odd function & (b) an even function.

Figure 4:



   (b) (1 point) Determine whether f is even, odd, or neither.

   i. $y = \sqrt[3]{x-1} - 2$

ii. $y = \frac{1}{x^2+2}$

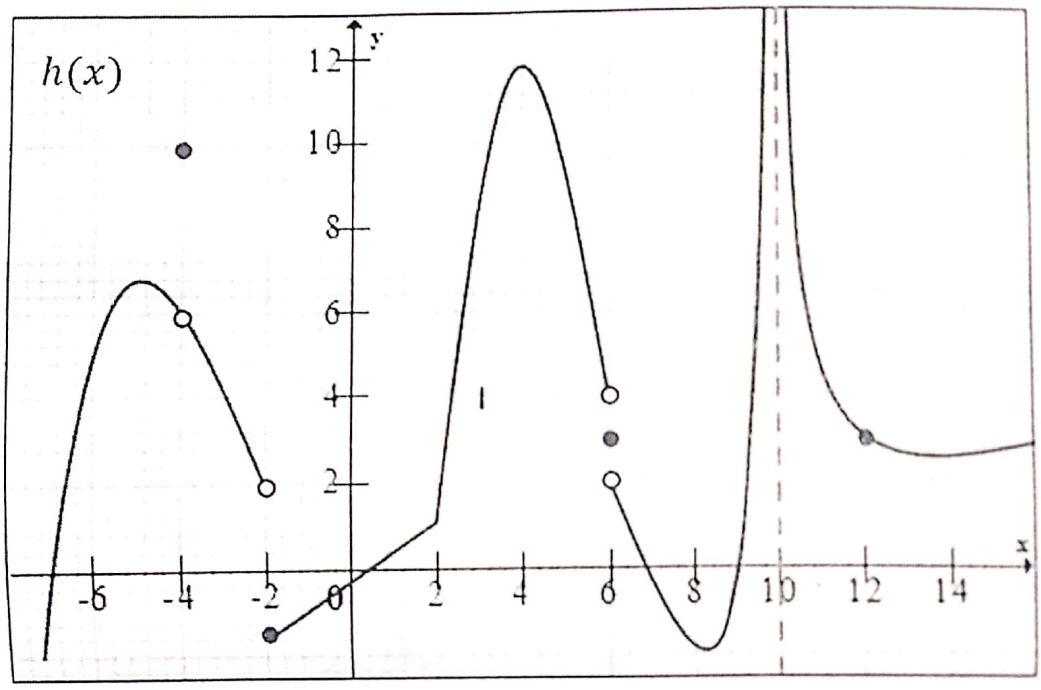
5. (4 points) Determine whether the following functions are one to one, or many to one. Find the inverse of each functions(if possible). Draw the graph of the each function and its inverse in the same diagram. Also, state the domain and range of the inverse function.

   (a) $f(x) = 4^{-x} - 1$

   (b) $f(x) = \sqrt{x-1},\ x \geq 1$

6. (3 points) Consider following graph of a function $y = f(x)$

Figure 5:



From the figure write the following answers:

(a) $\lim_{x \to 10} f(x)$

(b) check the continuity at $x = -4, 4$