United International **University**
School of Science and Engineering
Mid- term Examination Trimester: Summer-2023
Course Title: Calculus and Linear Algebra
Course Code: MATH 2183 Marks: 30 Time: 1 Hour 45 Minutes

Answer all the questions.

1. **a)** Consider the function $\boldsymbol{f(x) = x^4 - 8x^2 + 5}$,find **[5]**

(i) The intervals on which $\boldsymbol{f(x)}$ is increasing and decreasing.

(ii) The intervals on which $\boldsymbol{f(x)}$ is concave up and down.

b) Find the maximum and minimum value(s) of the function using both first and second derivative test. **[3]**

$\boldsymbol{y = 8x^3 - 24x.}$

c) Draw a graph of polynomial function $\boldsymbol{y = f(x)}$ with degree **4** by using the following information **[2]**

Increasing Interval	Decreasing Interval	y −intercepts	x −intercepts	Critical Points
$(-\infty, -5]$	$[-5,2]$	$(0,4)$	$(-7,0), (6,0)$	$(-5, 6)$
$[2, 10]$	$[10, +\infty)$		$(1, 0), (15,0)$	$(2, -8)$
				$(10,1)$

2.

a) Consider the function$\boldsymbol{f(x, y) = \sin(2x^2y^2 + 6x^3y) + 10,}$ **[5]**
Show that $\boldsymbol{f_{xy}(x, y) = f_{yx}(x, y)}$.

b) Using chain rule find $\frac{\partial T}{\partial x}$, where **[3]**

$$\boldsymbol{T(p, q, r) = p^3qr + \sqrt{pqr},}$$

$$\boldsymbol{p = sin(xy)\,, \quad q = ln(2x^2 + 2y^3)\,, \quad r = y - x}$$

c) Use implicit differentiation to find $\frac{dy}{dx}$, where **[2]**

$$\boldsymbol{x^3 + 5x^2y = 5}$$

3. **a)** Find the solution of the given differential equations **[6]**

i) $\boldsymbol{t^2y' + 4ty = t^2,} \qquad \boldsymbol{y(1) = \frac{1}{2}}$

ii) $\boldsymbol{y' = \frac{2xy^3}{5+x^2},} \qquad \boldsymbol{y(0) = 1}$

b) The initial temperature of a pizza when it is removed from an oven was measured as **350**°F. Five minutes later the temperature of the pizza was recorded as **150**°F. How long will the pizza take to cool down to the room temperature of **85**°F **?** **[4]**