



United International University
School of Science and Engineering

Final Exam, Trimester : Spring 2024

Course Title: Calculus and Linear Algebra

Marks : 40

Course Code: MATH 2183

Time: 2 Hours

You have to answer all the questions.

1. a) Consider the following system of linear equation

$$\begin{aligned}x + 2y - 2z &= 3 \\ 3x - y + z &= 1 \\ -x + 5y - 5z &= 5\end{aligned}$$

Solve the above system by elementary row operations. (5)

- b) Make a statement about the number of solutions of the following system. Confirm your conclusion algebraically. (3)

$$\begin{aligned}x + y &= 4 \\ 3x + 3y &= 6\end{aligned}$$

- c) Determine the value(s) of "k" for which the system has no solutions, unique solution, or infinitely many solutions. (2)

$$\begin{aligned}x + ky &= 6 \\ 2x + 6y &= 10\end{aligned}$$

2. Consider the following matrices

$$A = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 1 & 2 \\ 0 & -2 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \quad b = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} \quad P = \begin{bmatrix} 4 & 1 & 0 & 3 \end{bmatrix} \quad Q = \begin{bmatrix} 1 \\ 2 \\ 3 \\ -1 \end{bmatrix}$$

- a) Find A^{-1} by using matrix inversion algorithm. Also, find X such that $AX = b$. (5)

- b) Find $P(B)$ for $P(x) = x^2 - x + 1$. Is $P(B)$ a solution of $P(x)$? (3)

- c) Find $tr(P+Q)$ and also evaluate $\det(PQ)$. (2)

3. Find the eigenvalues and eigenvectors of matrix $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$. Also verify that $AX = \lambda X$. (5)

- a) Solve the following higher order differential equations:

i) $y'' + 9y = 0$; $y(\frac{\pi}{3}) = 2$, $y'(\frac{\pi}{3}) = 6$ (4)

ii) $y'' - 6y' + 9y = 2e^{3x} + 5e^{-2x} \cos x - 3$ (6)

- b) Solve: $y(x+y)dx + x^2dy = 0$ (5)