United International **University**
School of Science and Engineering
Mid Term Exam Trimester: Summer 2023
Course Title: Coordinate Geometry and Vector Analysis
Course Code: Math 2201 Marks: 30
Total Time: 1 hour and 45 minutes

Answer all questions.

1. Rotate the coordinate axes to remove the xy-term, then identify the type of conic and sketch its graph. Find its vertex, focus and directrix in xy *coordinate*. **[10]**

$$x^2 + 2\sqrt{3}\,xy + 3y^2 + 16\sqrt{3}\,x - 16y - 96 = 0$$

2. **a)** Find the distance between the given skew lines **[4]**

L_1: $x = 1 + 7t,\quad y = 3 + t,\quad z = 5 - 3t$

L_2: $x = 4 - t,\quad y = 6,\quad z = 7 + 2t$

b) Find the equation of the plane passing through the points $p_1(0, -2, 3)$, $p_2(3, 0, -2)$ and $p_3(-2,\ 3, 0)$. **[3]**

c) Let L_1 and L_2 be the lines **[3]**

L_1: $x = 1 + 2t,\quad y = 3 - t,\quad z = 3t$

L_2: $x = 4 + 3t,\quad y = 2 - 5t,\quad z = -1 + 2t$

i) Are the lines parallel?

ii) Do the lines intersect?

3. **a)** Find the area of the triangle with vertices $P_1(1, 3, 0)$ $P_2(-2, 0, 1)$ and $P_3(0, 5, -6)$. **[3]**

b) Find the vector component (orthogonal projection) of **[4]**
$p = < -1, -2, 0 >$ along $q = < 0, 0, 1 >$ and orthogonal to q.

c) Determine the angle that is made by $< -\sqrt{3},\ 4,\ 5 >$ with the positive ***x***-axis and also with the positive **z**-axis. **[3]**