Solve all questions.

1. (6 points) The position function of a moving particle is given by $s(t) = t^2 - 1$, where s is in meters and t is in seconds.

   (a) Find the average velocity of the particle over the time interval [1 2].

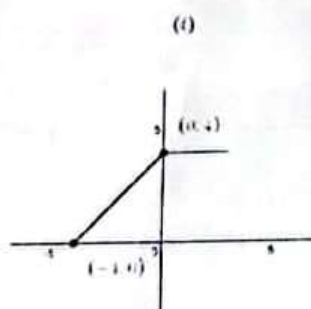   (b) Find the particle's instantaneous velocity at time $t = 1$ s.

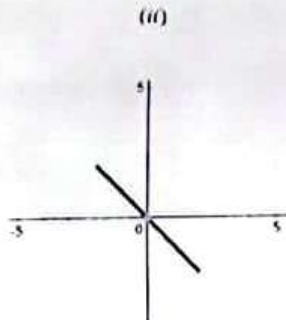   (c) Draw the position function $s(t)$, and velocity function $v(t)$ in the same plot.

2. (4 points) Match each of the following graphs($i - iv$) with its derivative graph($A - D$).

Figure 1:
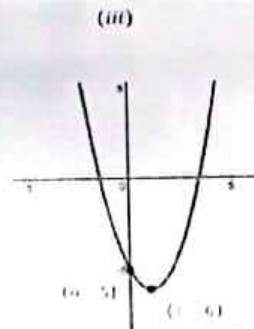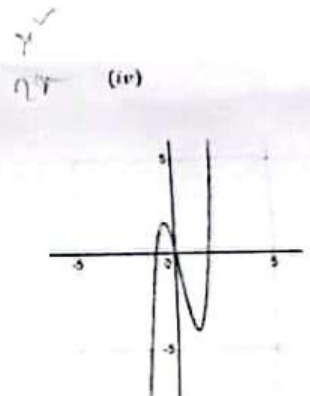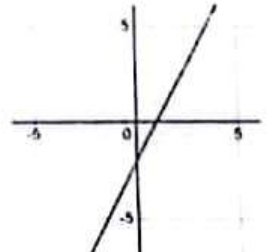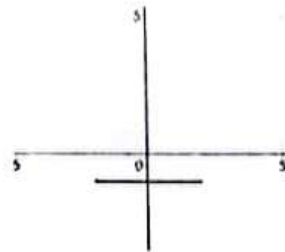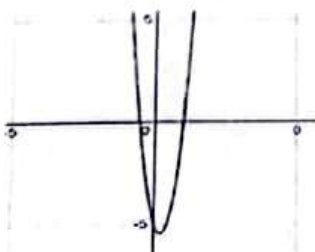

3. (4 points) Consider $f(x) = x^3$ and $g(x) = x - 3$ are two functions.

   (a) Find $y = f(g(x))$ and y' using the chain rule.

   (b) Find the equation of the tangent line to the graph of the function at $x = 3$

   (c) Draw the graph of the function $y = f(g(x))$ with the tangent line at $x = 3$.

4. (4 points) Consider the function

$$f(x) = \begin{cases} 2 & \text{if } 2 \le x \le 5 \\ 2x - 2 & \text{if } x < 2. \end{cases}$$

(a) Determine whether $f(x)$ is differentiable at $x = 2$. Is the function continuous at $x = 2$? If not, why?

(b) Sketch the graph of $f(x)$ and its derivative.

5. (4 points) Consider the function over the given interval.

$f(x) = 2x + 3; \ [-1, x]$

(a) Use an area formula from geometry to find the area function $A(x)$ that gives the area between the graph of the function $f(x)$ and the interval $[-1, x]$.

(b) Confirm that $A'(x) = f(x)$.

(c) Also, verify your answer by direct integration.

6. (6 points) (a) Draw the graphs of $y = x^2 + 2$, $y = x - 1$. $x = 0$ and $x = 1$,

(b) Find the area of the region enclosed by the graphs in part (a).

(c) Draw the graphs of $x = y^2$ and $y = x - 6$.

(d) Find the area of the region enclosed by the graphs in part (c).

7. (8 points) Evaluate the following integrals (answer any 4):

(a) $\int 3x^4 \sqrt{5 + x^5}\, dx$

(b) $\int x\sqrt{x - 3}\, dx$

(c) $\int_1^e x^2 \ln x\, dx$

(d) $\int \sin^4 x \cos x\, dx$

(e) $\int \frac{1}{1 + 4x^2}\, dx$

(f) $\int e^x \sin(3x)\, dx$

(g) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\, dx$

8. (4 points) Consider the function

$$f(x) = \begin{cases} |x + 5| & \text{if } x \le 0 \\ -x + 5 & \text{if } x > 0. \end{cases}$$

Use $f(x)$ to evaluate the following integrals:

(a) $\int_6^{10} f(x)\, dx$

(b) $\int_{-5}^4 f(x)\, dx$