United International **University**
School of Science and Engineering
Final Examination Trimester: Summer 2023
Course Title: Coordinate Geometry and Vector Analysis
Course Code: Math 2201 Marks: 40
Total Time: 2 hours

Answer all questions.

1. **a)** Let $\boldsymbol{F}(x,y) = 2xy^3\boldsymbol{i} + 3x^2y^2\boldsymbol{j}$ **[5]**

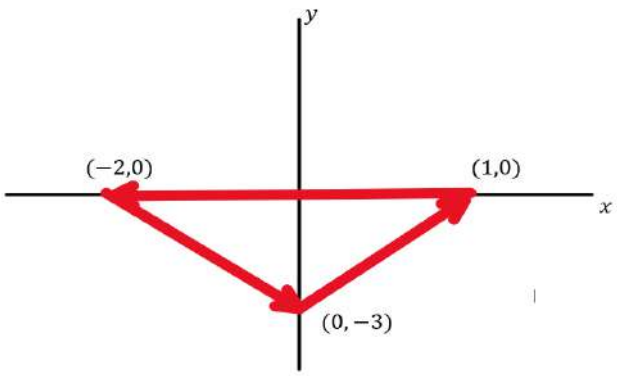
i. Show that $\boldsymbol{F}$ is a conservative vector field on the entire xy −plane.

ii. Find the potential function $\emptyset(x,y)$.

iii. Find $\int_{(2,-2)}^{(-1,0)} \boldsymbol{F}.\boldsymbol{dr}$ using (b)

b) Using Green's theorem find the value of $\oint_c \boldsymbol{F}.d\boldsymbol{r}$ **[5]**
Where $\boldsymbol{F}(x,y) = (e^x - y^3)\boldsymbol{i} + (cosy + x^3)\boldsymbol{j}$ and C is the closed curve bounded by the rectangular region with boundary line $x = 0$, $x = 2, y = -x$ and $y = 2$.

2. **a)** Evaluate $\int_c \boldsymbol{xydx - xdy}$ along the curve shown in the figure **[5]**



b) Use a double integral to find the area of the region R enclosed by **[5]**
$\boldsymbol{y = -x + 1, y = e^x, y = 0}$ and $\boldsymbol{x = 2.}$

3. **a)** Find the flux of the vector field $\boldsymbol{F}(x,y,z) = x\boldsymbol{i} - y\boldsymbol{j} + z\boldsymbol{k}$ across σ, **[5]**
where σ is the portion of the surface $\boldsymbol{z = 2 - x^2 - y^2}$ that lies above the xy − plane, and suppose that σ is oriented up.

Or

Use the Divergence Theorem to find the outward flux of the vector field $\boldsymbol{F}(x,y,z) = 7x^3\boldsymbol{i} + 7y^3\boldsymbol{j} + 7z^3\boldsymbol{k}$ across the surface of the region that is enclosed by $\boldsymbol{z = \sqrt{49 - x^2 - y^2}}$ and the plane $\boldsymbol{z = 0}$.

b) Evaluate the surface integral $\iint_\sigma (\boldsymbol{2x - 2y})\,\boldsymbol{ds}$; σ is the part of the plane $\boldsymbol{x + y + z = 2}$ that lies in the first octant. **[5]**

4. **a)** Use cylindrical coordinate to evaluate $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{9-x^2-y^2} \boldsymbol{y^2 dzdydx}$. **[5]**

b) Use a triple integral to find the volume of the solid bounded by the surface **[5]**
$\boldsymbol{z = x^2 sinxy}$ and the enclosed region in $\boldsymbol{xy}$ −plane by the equations
$\boldsymbol{x + y = 1, x = 0}$ *and* $\boldsymbol{y = 0}$.