United International University

School of Science and Engineering

Mid- term Examination Trimester: Spring-2023

Course Title: Linear Algebra, Ordinary &Partial Differential Equations / Calculus and Linear Algebra

Course Code: Math 183/Math-2183 Marks: 30 Time: 1 Hour 45 Mins

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Q1 **[5+5 =10]**

(i) The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. The curve passes through the point (3,5)

(a) Find the equation of the curve.

(b) Find the x coordinate of the stationary point.

(c) State the values of x for which y is increasing.

(ii) The total external surface area of a solid cylinder is $192\pi \text{cm}^2$. The cylinder has a radius of r cm and a height of h cm.

(a) Express h in terms of r and hence show that the volume V cm^3, of the cylinder is given by $V = \pi(96r - r^3)$.

(b) Find the stationary value and determine whether it a maximum or a minimum.

Q2 **[3+2+5 =10]**

(i) Using chain rule find $\frac{\partial w}{\partial x}$, where

$$W = u^3 v + \sqrt{v}, \quad u = cosx + xy \;\; and \;\; v = (x^2 + y)$$

(ii) Given that $x^3 + 2xy - y^2 + 3x + 2y + 7 = 0$, find $\frac{dy}{dx}$.

(iii) The variables x and y are related by the function $f(x, y) = 3x^2 + xy - 9x + 2y^2 + 10y + 1$. Evaluate $f_x', f_{xx}', f_{xy}, f_y'$ and f_{yy} and hence state the nature of the turning point.

Q3 **[2+6+2 = 10]**

(i) Show that $y = e^{-2t}$ is the solution of the differential equation $y'' - 4y = 0$

(ii) A liquid is heated so that its temperature is x (in degree centigrade) after t seconds. It is given that the rate of increase of x is proportional to (100 - x). The initial temperature of the liquid is 25^0C.

(a) Form a differential equation relating x, t and a constant of proportionality, k to model this information.

(b) Solve the differential equation and obtain an expression for x in terms t and k.

(c) After 180 seconds the temperature of the liquid is 85^0C. find the value of k and hence find the temperature of the liquid after 200 seconds.

(iii) Solve the following differential equation.

$$t\frac{dx}{dt} + x = 3, \quad x(1) = 6$$