



United International University

School of Science and Engineering

Final Examination Trimester: Summer-2023

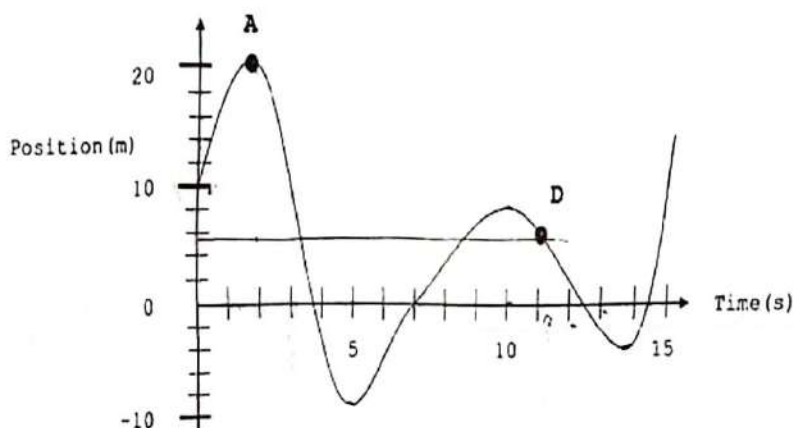
Course Title: Fundamental Calculus

Course Code: Math 1151 Marks: 40 Time: 2 Hours

Answer all the questions. Answer all parts of a question together.

1. (a) The following figure represents a position function of a particle at time t . [5]

- (i) Find the average velocity over the time from A to D.
(ii) Find the value(s) of t at which the instantaneous velocity is zero.
(iii) Roughly sketch the velocity graph of the particle.



- (b) Consider the function

$$f(x) = \begin{cases} x^2 - 1, & x \leq 2 \\ 2x - 1, & x > 2 \end{cases}$$

- (i) Sketch the graph of $f(x)$.
(ii) Determine whether the function $f(x)$ is continuous and differentiable at $x = 2$.

2. (a) Find the derivative of $f(x) = 5 + 2x - x^2$ with respect to x by using the formula $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$, and use it to find the equation of tangent line to $f(x)$ at $x = -1$. [3]

- (b) The following table defines the values of $f(x)$, $g(x)$, $f'(x)$ and $g'(x)$ at x . [3]

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	4	-2	7	-3
0	7	3	-1	-2

If $u(x) = f(x)g(x)$, and $v(x) = \frac{f(x)}{g(x)}$, then find $u'(-1)$, and $v'(0)$.

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(c) Use chain rule to **evaluate** the following derivatives: [4]

(i) If $x = \tan u$ and $u = t^3 - 2t \cos t + 5$, then find $\frac{dx}{dt}$.

(ii) If $y = \cot^3 \sqrt{2 - 3 \sin x}$, then find $\frac{dy}{dx}$.

3. (a) **Evaluate** the following integrals: [4]

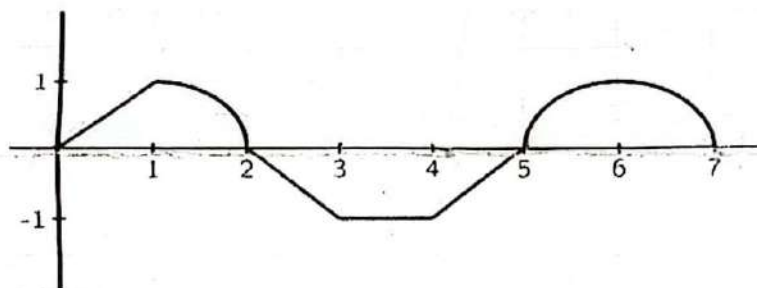
(i) $\int \frac{x^4 - x^2 + x^3 - 1}{x^3} dx$ (ii) $\int \frac{x^2}{\sqrt{1 - 4x^6}} dx$

(b) Use integration by parts to **evaluate** the following integrals: [6]

(i) $\int e^{-x} \sin 2x dx$ (ii) $\int x \tan^{-1} 2x dx$

4. (a) The graph of $f(x)$ is shown. Use the graph to **evaluate** the following integrals: [4]

(i) $\int_0^3 f(x) dx$ (ii) $\int_3^7 f(x) dx$



(b) **Find** the area of the shaded region by [6]

(i) integrating with respect to x .

(ii) integrating with respect to y .

