United International University
*School of Science and Engineering*
Final Examination Trimester: Spring 2024
Course Title: Coordinate Geometry and Vector Analysis
Course Code: Math 2201 Marks: 40
**Total Time: 2 hours**

***Answer all questions.***

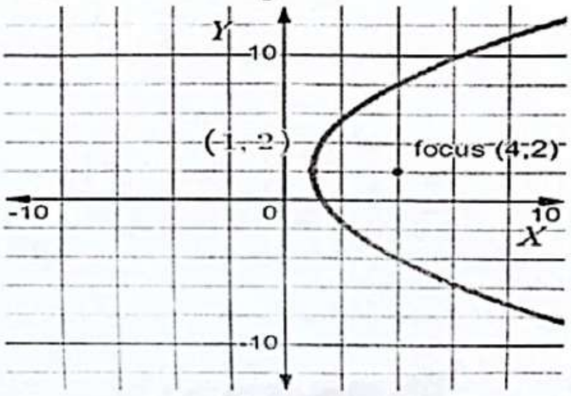
1. a) Identify and sketch the graph of the Conic. [4]

$$16x^2 - y^2 - 32x - 6y = 57$$

b) Rotate the coordinate axes to remove the $xy$-term, then identify the type of Conic. [4]

$$x^2 - 10\sqrt{3}xy + 11y^2 + 64 = 0$$

c) Write the equation of the following curve. [2]




2. a) Consider, $F(x, y) = 2xe^y\, i + x^2e^y\, j$ [4]
   i) Show that $F$ is a conservative vector field on the entire $xy$ −plane.
   ii) Find the potential function $\phi(x, y)$.
   iii) Find $\int_{(0,0)}^{(3,2)} F.dr$ using ii).

b) Find the work done by the force field $F(x, y) = (e^x - y^3)i + (\cos y + x^3)j$ on the particle that travels once around the unit circle $x^2 + y^2 = 1$. [4]

c) Determine the constant $a$ so that the vector $V(x, y, z) = (x + 3y)i + (y - 2z)j + (x + az)k$ is divergence free. [2]

3. a) Evaluate the line integral along the curve $C$ $\int_C (x + 2y)dx + (x - y)dy$ where $C: x = 2\cos t\ \ y = 4\sin t\ \left(0 \le t \le \frac{\pi}{4}\right)$. [5]

b) Use the Divergence Theorem to find the outward flux of the vector field $F(x, y, z) = x^3 i + y^3 j + zk$ across the surface of the region that is enclosed by $x^2 + y^2 = 16$ and the plane $z = 0$ and $z = 3$. [5]

4. a) Use spherical coordinate systems to evaluate: [5]

$$\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} \sqrt{x^2 + y^2 + z^2}\, dzdxdy$$

b) Find the flux of the vector field $F(x, y, z) = xi + yj + 3zk$ across $\sigma$, where $\sigma$ is the portion of the surface $z = 9 - x^2 - y^2$ that lies above the $xy$ − plane and suppose that $\sigma$ is oriented up. [5]

**15.4.1 THEOREM (Green's Theorem)** *Let $R$ be a simply connected plane region whose boundary is a simple, closed, piecewise smooth curve $C$ oriented counterclockwise. If $f(x, y)$ and $g(x, y)$ are continuous and have continuous first partial derivatives on some open set containing $R$, then*

$$\int_C f(x, y)\,dx + g(x, y)\,dy = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA \qquad (1)$$

*Let $\sigma$ be a surface with equation $z = g(x, y)$ and let $R$ be its projection on the xy-plane. If $g$ has continuous first partial derivatives on $R$ and $f(x, y, z)$ is continuous on $\sigma$, then*

$$\iint_\sigma f(x, y, z)\,dS = \iint_R f(x, y, g(x, y))\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\,dA \qquad (8)$$

**15.6.3 THEOREM** *Let $\sigma$ be a smooth surface of the form $z = g(x, y)$, $y = g(z, x)$, or $x = g(y, z)$, and suppose that the component functions of the vector field $\mathbf{F}$ are continuous on $\sigma$. Suppose also that the equation for $\sigma$ is rewritten as $G(x, y, z) = 0$ by taking $g$ to the left side of the equation, and let $R$ be the projection of $\sigma$ on the coordinate plane determined by the independent variables of $g$. If $\sigma$ has positive orientation, then*

$$\Phi = \iint_\sigma \mathbf{F}\cdot\mathbf{n}\,dS = \iint_R \mathbf{F}\cdot\nabla G\,dA \qquad (11)$$

**15.7.1 THEOREM (The Divergence Theorem)** *Let $G$ be a solid whose surface $\sigma$ is oriented outward. If*

$$\mathbf{F}(x, y, z) = f(x, y, z)\mathbf{i} + g(x, y, z)\mathbf{j} + h(x, y, z)\mathbf{k}$$

*where $f$, $g$, and $h$ have continuous first partial derivatives on some open set containing $G$, and if $\mathbf{n}$ is the outward unit normal on $\sigma$, then*

$$\iint_\sigma \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_G \operatorname{div}\mathbf{F}\,dV \qquad (1)$$

**15.8.1 THEOREM (Stokes' Theorem)** *Let $\sigma$ be a piecewise smooth oriented surface that is bounded by a simple, closed, piecewise smooth curve $C$ with positive orientation. If the components of the vector field*

$$\mathbf{F}(x, y, z) = f(x, y, z)\mathbf{i} + g(x, y, z)\mathbf{j} + h(x, y, z)\mathbf{k}$$

*are continuous and have continuous first partial derivatives on some open set containing $\sigma$, and if $\mathbf{T}$ is the unit tangent vector to $C$, then*

$$\oint_C \mathbf{F}\cdot\mathbf{T}\,ds = \iint_\sigma (\operatorname{curl}\mathbf{F})\cdot\mathbf{n}\,dS \qquad (2)$$