Exercise Sheet 3

Due: 28.11.2018, 09:00

Download the files **f31.csv, f32train.csv, f32test.csv, f33.csv** from ISIS. The last column of each file contains output values, all other columns are input features.

Exercise 3.1

The goal of this exercise is to investigate the generalization error in dependence of the number of training examples.

Consider the following function

$$f: [-1,1]^2 \to \mathbb{R}, \quad \boldsymbol{x} = (x_1, x_2) \mapsto x_1 \sin(\pi x_2)$$

We define the following single experiment $E(m)$ for number of training examples m:

- Generate a training set with m examples (*1)
- Use multiple linear regression to fit a linear model to the training data
- Compute the training error of the fitted model
- Estimate the generalization error of the fitted model using the test data f31.csv

(*1) Generate training data by drawing independent samples $\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_m}$ from the uniform distribution on $[-1,1]^2$ and computing the corresponding output values $y_i = f(\boldsymbol{x_i})$.

For each $m = 2, 3, 4, \dots, 80$ conduct the single experiment $E(m)$ 100 times and plot the average training and test errors in dependence of m. Discuss the results.

Exercise 3.2

In this exercise we compare model selection with train-test split and with cross validation on the f32-data.

1. For each $k = 1,2,3, \dots, 10$ perform polynomial regression of order k on the f32train-data. Compute the test-MSE for each of the 10 models on the f32test-data.
2. For each $k = 1,2,3, \dots, 10$ conduct 10-fold cross validation on the f32train-data using polynomial regression of order k.

Plot the test-MSE and the cross validation error in dependence of the order k. Discuss your results with respect to model selection.

Exercise 3.3

Apply polynomial regression of order $k = 10$ with L_2 -regularization to the f33-data. Specify a suitable regularization parameter and estimate the generalization error using nested cross validation. Use 5 folds for both, inner and outer cross validation. Present and discuss your results.