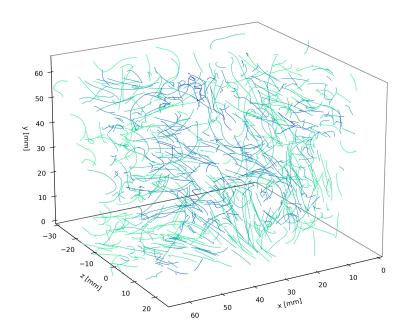


User Manual



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Github repository: https://github.com/ronshnapp/MyPTV

Get help & interact with our community: https://github.com/ronshnapp/MyPTV/discussions

Contact the developers: ronshnapp@gmail.com

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1 Introduction

1.1 3D-PTV principles

The 3D-PTV method is used to measure the trajectories of particles in 3D space. It utilizes the principles of stereoscopic vision to reconstruct 3D positions of particles from images taken from several angles. A scheme of a typical 3D-PTV experiment using a four-camera system is shown in Fig. 1a. The "workhorse" behind the 3D-PTV method is the co-linearity condition, the 3D model. In principle, if we know what the position is and what is the orientation of the camera in 3D space $(O' \text{ and } \theta \text{ in Fig. 1b})$, we can use the pin-hole camera model to relate the image space coordinates of a particle $(\eta, \zeta \text{ in Fig. 1b})$ to the ray of light connecting the imaging center and the particle. Then, if we have more than one camera, the particle will be located at the intersection of the two rays. Detailed information is given in [1, 2].

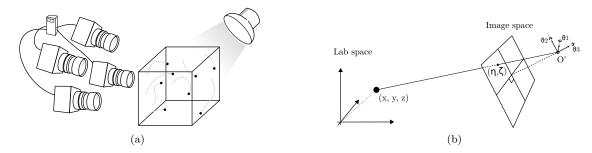


Figure 1: (a) A schematics of a 3D-PTV experiment. (b) A schematic description of the 3d model, the pin-hole camera model.

Once the experiment, namely data acquisition, is done, there are six intrinsic steps to follow in order to complete the analysis. The six steps are outlined in Fig. 2. In the camera calibration step, we use images of known calibration targets to estimate the position, orientation, and internal parameters of the cameras. In particle segmentation, we use image analysis to obtain the particles' image space coordinates (η, ζ) . In the Particle matching step, we use the ray crossing principle to decide which particle image in each camera corresponds to the same physical particle, and triangulate their positions through stereo matching. In particle tracking, we connect the positions of particles in 3D space to form trajectories. In data conditioning, we might use smoothing and re-tracking algorithms to enhance our data quality according to some physical heuristics. Lastly, we can analyze the data to obtain information on the physics of the particles we are studying. The MyPTV package is meant to handle the first five of these steps.

The sections that follow outline the code used to handle the 3D-PTV method in MyPTV.

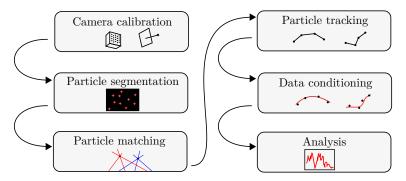


Figure 2: Basic steps in the analysis of PTV raw data into particle trajectories and scientific output. The first five steps are handled by MyPYV.

1.2 3D model

We use a pinhole camera model to transform the 2D particles seen in the camera images. In particular, the following expression relates the coordinates in the 2D image space coordinates and a direction vector pointing out from the camera imaging center:

$$\vec{r} - \vec{O} = \begin{pmatrix} \begin{bmatrix} \eta + x_h \\ \zeta + y_h \\ f \end{bmatrix} + \vec{e}(\eta, \zeta) \end{pmatrix} \cdot \begin{bmatrix} R \end{bmatrix}$$
 (1)

where the description of the notations is given in Table 1. The matrix $[R] = [R_1] \cdot [R_2] \cdot [R_3]$ is the rotation matrix calculated with the components of the orientation vector, $\vec{\theta} = [\theta 1, \theta 2, \theta 3]$. In addition, the correction term \vec{e} is assumed to be a quadratic polynomial of the image space coordinates:

$$\vec{e}(\eta,\zeta) = [E] \cdot P(\eta,\zeta) = \begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} & E_{15} \\ E_{21} & E_{22} & E_{23} & E_{24} & E_{25} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \eta \\ \zeta \\ \eta^2 \\ \zeta^2 \\ \eta \zeta \end{bmatrix}$$
(2)

where [E] is a 3×5 matrix that holds the correction coefficients; the last row is filled with zeros because we do not attempt to correct f.

Symbol	Description
\vec{r}	Particle position in the lab space coordinates
$ec{O}$	Position of a camera's imaging center
η, ζ	image space coordinates (pixels) of a particle
x_h, y_h	Correction to the camera's imaging center (in pixels)
f	The camera's principle distance divided by the pixel size
$\vec{e}(\eta,\zeta)$	A nonlinear correction term to compensate for image distortion and multimedia problems.
[R]	The rotation matrix which corresponds to the camera orientation vector.

Table 1: Description of mathematical notation.

1.3 MyPTV usage and structure

MyPTV is used to conduct the post-processing steps in which the experimental raw data (images) are transformed into particles' trajectories. The five steps include camera calibration, image segmentation, particle matching, particle tracking, and, data conditioning which is divided into smoothing and stitching of broken trajectories. This section outlines how these steps are to be followed and how MyPTV is structured to facilitate them. Section 2 gives instructions on how to efficiently use MyPTV.

At the current stage, MyPTV does not include a graphical user interface. Instead, the package includes a script that can be used to efficiently and conveniently run the various post-processing steps. Knowledge of Python is not required to operate MyPTV.

The internal structure of MyPTV is made of separate "modules", each of which deals with a particular step out of the list described above. Each module consists of a Python script file that includes one or several classes. Detailed information on each of these classes is given in Sections 3–9, and the code itself includes documentation to a level appropriate for development. In addition to the source code, MyPTV includes a workflow script that is used to operate the software in an organized fashion. Thus, MyPTV can be used either by using the workflow script or by directly evoking classes from the source code.

2 The Workflow: guidelines for a 3D-PTV experiment

MyPTV can be operated using the workflow.py script. The script is found under the *example* folder under the home directory of the MyPTV package. The workflow is used through command line given instructions and a file that outlines the parameters to be used in each step.

2.1 Preparation of an experiment folder

To begin post processing of the 3D-PTV results, prepare the following directory structure:

- 1. Make a new directory; in it, make a new directory called Calibration and place inside it the calibration images, and make another set of directories, one for each camera, and place inside it the particle images of each camera.
- 2. Locate the example under the root folder of the MyPTV package. This can be used as a template to the experiment directory made in the previous step.
- Copy the files workflow.py and params_file.yml from the example folder into your experiment's folder.

2.1.1 The workflow.py script

workflow.py is a Python script used by the user to run the various steps of the post-processing. The script is used through the terminal or command line. To use MyPTV's various functions, go to the experiment's directory with:

```
cd \path\to\experiment\folder
```

and then, use the following syntax:

```
python workflow.py params_file.yml "command"
```

where "command" should be replaced with one of the following options, depending on the particular step of the post processing:

```
help
initial_calibration
final_calibration
calibration_with_particles
segmentation
matching
tracking
smoothing
stitching
2D_tracking
manual_matching
```

Each of these commands initiates one of MyPTVs functionalities, and a detailed explanation on them is found in Sections 2.2–2.8. Use help to print the available list of commands.

2.1.2 The params_file.yml file

The params_file.yml outlines all the parameters needed in order to run MyPTV using the work-flow script. In every step conducted, go through the lists of parameters and make sure that their values are correct. The definitions per every step are tabulated in the sections below.

Table 2: The params_file.yml parameters for the calibration step. All paths to files are relative to the workflow.py script.

Parameter	Description
camera_name	the name of the camera to be calibrated; for example, cam1, etc.
target_file	path to the file that lists the calibration target's lab points
calibration_image	path of the calibration image
resolution	camera resolution; for example: 1280, 1024

2.2 Calibration guide

A good calibration is key to having success in your PTV experiment! Thus, follow this guide to calibrate your camera using MyPTV.

In the process called camera calibration, our goal is to determine what are the external $(\vec{O}, \vec{\theta})$ and internal $(f, x_h, y_h, [E])$ parameters of each of our cameras. This process is done by taking an image, a *calibration image*, of an object that has points with known coordinates marked on it *calibration target*. Therefore, its important to make sure that for each experiment and each camera we have a calibration image ready, as seen, for example, in Fig. 7a.

Importantly, since our goal in calibration is to find the cameras' positions, once calibration images were taken in an experiment it is crucial that the cameras are not moved. Even minor vibrations can ruin an experiment. Also, the calibration target needs to be in the same medium in which the experiment will be performed to ensure that any refraction is corrected by the error terms (for example, if you are using a tank of water, place the calibration target inside the tank of water).

The calibration in MyPTV is performed in two steps: *initial calibration* and *final calibration*. The initial calibration is used to obtain a rough estimation of some of the camera's parameters using a small number of calibration points, as well as to extract the information for all the calibration points on the calibration target. In the final calibration we fine tune the camera's parameters in order to minimize the *calibration error*. The process is explained in detail in the following sub sections.

2.2.1 Preparing a target_file

The target file is a text file (tab separated values), in which each row shows the lab space coordinates of a calibration point. An example for the format is shown in Fig. 6b. If there is no target file in the experimental folder, this is the time to generate one. Prepare such a file and save it in the experiment's calibration folder.

2.2.2 Preparing the parameters in the params_file

As in any step of MyPTV, we make sure that the params_file.yaml in our experiment folder has the correct parameters, namely file names under the calibration tab; see Tab. 2. Each camera is calibrated separately, and so, for the calibration of each camera we need to change the file names in params_file.yaml appropriately (alternatively, one could have separate parameter files for each camera).

2.2.3 Initial calibration

To start the initial calibration, we start the *initial calibration GUI*. Using the workflow file in the terminal, use the following command:

python workflow.py params_file.yml initial_calibration

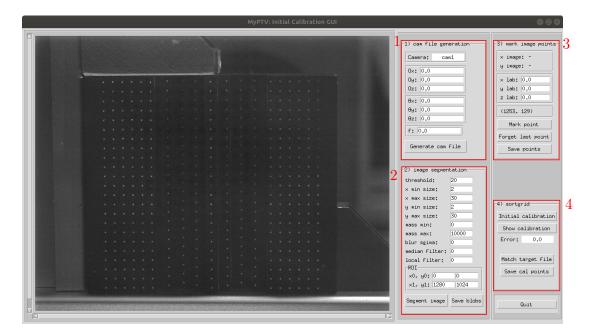


Figure 3: The initial calibration GUI.

which should open the a window like the one shown in Fig. 3. The left most panel shows the calibration image; we can zoom in and out by pressing the "+" and "-" keys.

The initial calibration is composed of four steps, and each one has a dedicated panel in the GUI. The steps should, in general, be followed along the order of the steps: $1 \to 4$.

Here are the details regarding each of the initial calibration steps:

- 1. cam file generation: Here we produce a text file that stores the camera parameters. We first have to give a first guess for the six different external parameters, \vec{O} and $\vec{\theta}$, and the focal length, f. It's important to give a reasonably good initial guess, because the calibration problem often has local minima that make the convergence to solution difficult. To understand the how to choose the values, consult Section 2.2.4. When done, click the Generate cam file button.
- 2. image segmentation: In this step we use MyPTV to detect the points on the calibration image and to store their pixel coordinates in a file. Choose and tweak the segmentation parameters until all the calibration points are detected (or at least most of them if the images are not ideal). Detected points are indicated by green squares on the image. Also, a reference for the segmentation parameters can be found in Section 2.3. Once done, save the file using the save button.
- 3. mark image points: Here, we start by choosing six points (at least) of the calibration target. The points should not be coplanar and it is best to choose them such that there is some asymmetry to reduce the chance of false convergence. For each point chosen, we mark it by clicking on it on the image and we write its' lab space coordinates in the x lab, y lab, z lab fields, and then click the Mark point button. The marked points will be shown as blue crosses on the image. After marking all the chosen points, we save their coordinates using the Save points button.
- 4. sortgrid: In this last step, MyPTV tries to calibrate the camera using only the points we marked in step 3, and then to match the calibration points we segmented in step 2 to the points we gave in the target file. Thus, click the initial calibration button, make sure that the calibration error is low (aim for about 1.0), and click show calibration. If the results are good, click the Match target file button, and you should see the segmented

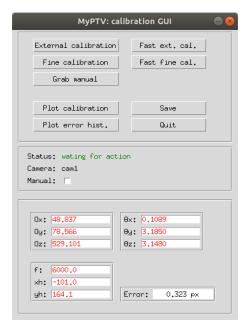


Figure 4: An image of the calibration GUI. The calibration GUI has several functionalities to run the minimization functions, plot the calibration error and the calibration particle's images, as well as change manually the camera parameters. See Tab. 3 for a description of the operation of each button.

points and the associated points from the target file paired by a red line on the image. Make sure that there are no errors in the pairing, and then save the results. If you detect errors or the initial calibration did not reach low enough error, it might be needed to generate an improved initial guess.

This concludes the initial calibration step.

2.2.4 Understanding camera external parameters

The external parameters of a camera in MyPTV include its position, its orientation, and its focal length, \vec{O} , $\vec{\theta}$, and f, respectively. In the calibration process, it is important to understand these parameters to be able to give a good initial guess and judge the results of the calibration procedure. Thus, let us explain how these parameters are defined.

Each camera is associated with an image space reference frame, shown in Fig. 5 as the x', y', and z' axes. The z' axis is normal to the imaging plane (the camera's sensor), while x', and y' are tangent to it; the system is right-handed.

The position parameter, \vec{O} , is defined as a vector pointing from the origin of the lab space coordinate system to the origin of the camera's reference frame. It is measured in lab-space units (e.g. millimeters). Therefore, one could estimate \vec{O} directly by using a ruler or a measuring tape to measure where the camera is in the lab-space coordinates.

The orientation parameter, $\vec{\theta}$, is a vector of three angles that describe the rotation of the camera's reference frame with respect to the lab space frame, namely the camera's Euler angles. The first angle describes the rotation around the x' axis, the second describes rotation around y', and the third angle describes rotation around z'. Rules of thumb in trying to guess $\vec{\theta}$ are: 1) ask yourself "how should one rotate the lab's frame of reference so that it aligns with the camera's frame of reference?"; 2) we first rotate around the x' axis, then around y' and then around z'. Note: in many cases, the origin of the images is on the top left corner, so half a turn rotation around the z' axis is often needed. See Fig. 5 for an example.

Table 3: The various commands available in the calibration GUI (Fig. 4).

Parameter	Description
External calibration	Will minimize the calibration error by optimizing the external parameters of the camera using all the calibration points. This might require several runs to reach good results.
Fast ext. cal.	Will optimize the external camera parameters using a faster stochastic optimizer. This option is good for rapidly obtaining a reasonable calibration when there are a lot of calibration points.
Fine calibration	Will minimize the calibration error by optimizing the nonlinear error term of the camera parameters. This might require several runs to reach good results.
Fast fine cal.	Will optimize the nonlinear error terms of the camera parameters using a faster stochastic optimizer. This option is good for rapidly obtaining a reasonable calibration when there are a lot of calibration points.
Grab manual	This allows to manually change camera parameters. to do this, first tick the Manual checkbox. Then, manually change the values of the camera parameters. Finally, click the button and watch the effect on the calibration error.
Plot calibration	Will generate a plot of the calibration points projected on the camera space, and the known calibration points.
Plot error hist.	Will generate a histogram of the calibration errors for the various calibration points.
Save	Will save the results of the current session in the camera file.
Quit	Will quit the GUI.

The focal length parameter, f, is essentially an extension factor. In MyPTV f is measured in units of pixels. The focal length is normally more or less equal to the camera's lens focal length. For example, if one uses a camera with a 60mm lens in their experiment, and the size of one camera pixel is $10\mu\text{m}{=}0.01\text{mm}$, then f would roughly be equal to 6000 (pixels). Potentially, one could also estimate f in an experiment: 1) place the camera in a known position facing a calibration target (as e.g. in Fig.4a) and take a picture 2) prepare a calibration points file for a few of the points on the calibration target; 3) preparing a camera file using the known position and orientation 4) insert some initial value for f; 5) using the workflow.py file to compare the distribution of the points using this value of f; 6) repeat steps 4 and 5 until a fair agreement is found.

2.2.5 Final calibration

In the final calibration we use all the calibration points to find the parameters of the camera. We do this by minimizing the calibration error, defined as the root mean square of the distances between the image-space calibration points positions and the projection of the lab-space points on the image plane. The minimization is solved numerically using the *final calibration gui*.

To start the final calibration gui, use the workflow script with the following command:

python workflow.py params_file.yml final_calibration

which should start the interface shown in Fig. 4. The description of each of the possible functionalities are given in Table 3. In principal, the lower the calibration error, the results of the measurements will be more reliable, and it will become possible to track a higher number of particles in each frame. A good calibration will have an error lower than about 0.5, or even 0.3 pixels. Once a good error value is achieved, the calibration process is complete.

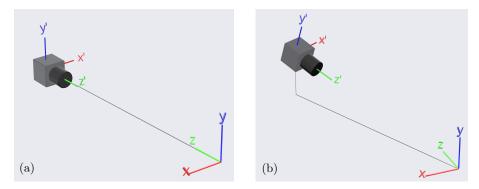


Figure 5: Two examples for camera setups. The (x, y, z) system represents the lab space, and the (x', y', z') represents the camera's system of reference. In (a) the camera's position is roughly $\vec{O} = (0, 0, 1)$, and an appropriate rotation vector could be $\vec{\theta} = (0, \pi, 0)$. In (b) the camera's position might be $\vec{O} = (1, 1, 1)$, and the orientation vector might approximately be $\vec{\theta} = (-\frac{1}{4}\pi, -\frac{3}{4}\pi, 0)$.

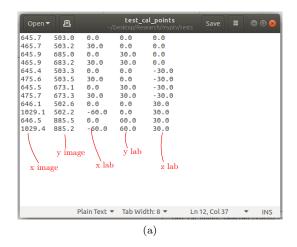
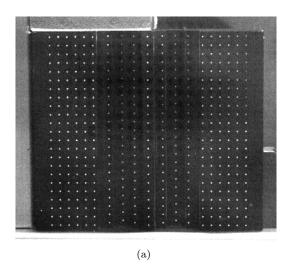




Figure 6: (a) Format of the target file, which lists the lab-space coordinates of the calibration target as tab-separated values. The order at which points are given is not relevant. (b) An example of a text file holding the calibration point data. (c) The results of matching a target file points (blue circles) and the segmented blobs (red crosses). Here, we know that the match is correct because each cross overlays a circle. In location where circles are not overlaid by a cross, the segmentation did not recognize a calibration target; having such missing points may slightly reduce the quality of the calibration but it is not critical for the target file matching.

2.2.6 Important notes

- 1. A "good" calibration will have a low error value less than 0.5 pixels.
- 2. Optional validation After all the cameras have been calibrated, it is also good practice to verify the calibration solution by stereo matching the calibration points. To do this, we use the match_blob_files (Section 6) to stereo match the files of the segmented calibration target points. We can then calculate the so-called *static calibration error* by computing the RMS of the distance between the triangulated calibration points and the real data from the target file. See Fig. 7b for an example. This validation step is not yet implemented in the workflow script.



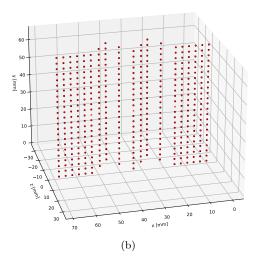


Figure 7: (a) An example of a calibration image. The points on the target have known lab space coordinates. Not also that the points are distributed over several plains (3 different z values in this case). (b) An example plot of a calibration error estimation. The red crosses represent the known positions of the calibration points given in the target file, and the black circles mark the positions of the stereo-matched segmented calibration points. In this case, the root mean square of the static calibration error was $84\mu m$.

2.3 Segmentation

2.3.1 How segmentation works

In the segmentation step, the program extracts the location of particles in the image. Segmentation in MyPTV is performed in one of two methods: *labeling*, or *dilation*, and includes several optional image processing steps.

- 1. In the labeling method, we first choose all pixels in an image whose grey value is higher than a given threshold. Then, such pixels that are touching each other are considered to be "blobs", so we group them together. For each blob, we calculate a *center of mass*, being the grey value weighted average of the blob, a *bounding box* size, and a *mass*, being the sum of grey values of the blobs' pixels.
- 2. In the dilation method, we are looking for pixels that their grey value is higher than that of all their neghbours within a box of a given size (particle_size), and thet their grey value is higher than a given threshold. After that, all the neighbouring pixels inside the particle size box are considered to be a blob. We then calculate the center of mass of the blob to achieve a better estimation of its position, thus redifining the blob. This process is iterated

a maxinum of 3 times untill convergence is obtained. Finally, we calculate the same blob's center of mass, bounding box size, mass.

To improve the results of segmenting over non-ideal images, MyPTV can be set to apply image processing before the blob extraction. The filters that can be used are a Gaussian blur filter, a median noise removal filter, local mean subtraction filter, and a mask. The first three filters may increase the computational time, however they may be necessary depending on the experimental conditions, such as uneven illumination or noisy images. Examples of the results of using several combinations of the filters on a calibration image is shown in Fig. 8.

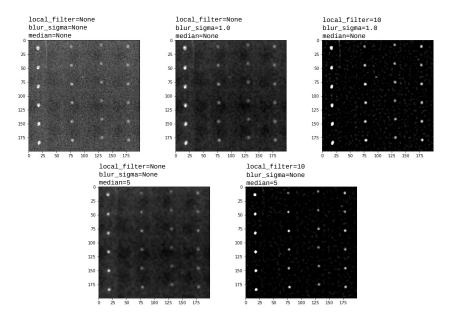


Figure 8: The effect oft various image filters on the segmentation.

In this step we extract the particles' image-space coordinates from the images and save the coordinates in dedicated files. Use the workflow.py script with the segmentation command to start the process:

- 1. We perform the segmentation first on a single image by specifying Number_of_images: 1, and giving an image name with single_image_name. We then tune the segmentation parameters and the image processing filters used (e.g. ROI, threshold, blur_sigma, method etc; the full list is given in Table 4) to obtain optimal results; optionally repeat over several more images to ensure the results are satisfactory. To view the segmented particles over the image, use plot_result: True, and make sure the results are not saved by using save_name: None.
- 2. Once the parameters are determined, we set Number_of_images: None, and run the segmentation workflow again. This will then loop over all the images in the given folder (under images_folder), and save the results in a text file by setting save_name: /file/name/to/use

2.4 Matching

In the matching step, particles images in the various images are used to stereo-locate (triangulate) the 3D positions of particles in the lab-space coordinates. As before, run first only on several images to find optimal parameter values, and only then iterate over all frames by setting N_frames: None and save the results.

Note: to perform easy stereo matching of points in images, use the manual matching GUI. See Section 2.10 and Fig. 10.

Table 4: The params_file.yml parameters for the **segmentation** step. All paths to files are relative to the workflow.py script.

Parameter	Description
image_start	The number of the image from which the loof begins. If this is None (default) the first image is set to 0.
Number_of_images	Number of images over which to do the segmentation. If it is 1, segmentation is done only on the image with the name given in single_image_name. If this is an integer, segmentation is performed on the first N images found in the directory images_folder. If this is None, segmentation is performed over all the images in the directory images_folder
images_folder	path to the folder containing the images
single_image_name	If Number_of_images is 1, the segmentation will be done on the image whose name is given here. If Number_of_images is any other value, this parameter is not used.
image_extension	extension of the images; for example, .tif
plot_result	if False will not plot the results; if True the results will be plotted but only if number of images is 1
mask	if this is 1.0, no mask is used; if this is set to a path to a file, the file will be used as a mask for the segmentation
ROI	region of interest; specify by indicating the xmin, xmax, ymin, ymax coordinates
threshold	the brightness value for which a point is considered a particle after a local mean subtraction is performed
median	the intger window size of a median noise removal filter; set to None in order to not apply the filter
blur_sigma	the float standard deviation of a Gaussian blur filter; set to None in order to not apply the filter
local_filter	the integer widow size of a local mean subtraction filter; set to None in order to not apply the filter
min_xsize	minimum particle size (pixels) in x direction
min_ysize	minimum particle size (pixels) in y direction
min_mass	minimum particle mass (mass is the sum of pixel grey values)
max_xsize	maximum particle size (pixels) in x direction
max_ysize	maximum particle size (pixels) in y direction
max_mass	maximum particle area (mass is the sum of pixel grey values)
plot_result	if True the segmented particles will be plotted over the processed image (after applying blur, local mean subtraction and masking); if False will not plot the results
method	ihe name of the method used for segmenting the blobs; can be either 'labeling' or 'dilation'
particle_size	in the 'dilation' method, this gives the diameter of blobs in the image (an integer number of pixels); this parameter is not used when method is set to 'labeling'
save_name	if None the results will not be saved in a file; if path/to/file will save the results in the given file name

Table 5: The params_file.yml parameters for the matching step. All paths to files are relative to the workflow.py script.

Danamatan	Description
Parameter	<u> </u>
	names of files that hold the segmented particles for each camera,
blob_files	separated by commas; for example: blobs_cam1, blobs_cam2,
Parameter blob_files frame_start N_frames camera_names cam_resolution ROI voxel_size max_blob_distance max_err save_name	blobs_cam3
frame start	if None will start matching from the first available frame. If an
Trame_boaro	integer it will start the matching from this given number.
N framos	if None will match particles in all available frames; if an integer
N_II dillep	will match only particles in the first N frames.
comoro nomos	names of cameras used separated by commas; for example, cam1,
Camera_names	cam2, cam3
cam_resolution	camera resolution; for example, 1280, 1024
	region of interest for the matching in lab space coordinates, and
ROI	according to the format of xmin, xmax, ymin, ymax, zmin,
	zmax
	the side length of voxels used in the Ray Traversal algorithm; note
voxel_size	- too high values lead to long computation times and too low value
	result in matching errors and long computation times.
	the distance particle usually undergo during each frame in image
max_blob_distance	space coordinates (pixels)
	maximum value of the RMS triangulation error in lab space coor-
max_err	dinates
	path name used for saving the results; if None the results are not
save_name	saved

2.5 Tracking

The tracking step is used to link particle in the lab space coordinates in time, thus forming the 3D trajectories.

2.6 Calibration with particles

After some trajectories have been obtained, we can refine the calibration of cameras by using this obtained data and re-run the calibration procedure with the measured particles. For more detailes see Sec. 4.2. To start calibration with particles enter the command calibration_with_particles in the command line following the workflow command. Once this is done, a calibration sequence is initiated that uses the trajectory data.

2.7 Smoothing

The smoothing step is used to smooth trajectories in time and to calculate the velocity and acceleration of particles. This is done by fitting polynomials over sliding windows of the trajectory where velocities and acceleration are calculated through a direct differentiation of the polynomial (see [3]).

2.8 Stitching

This step is used to re-track trajectories once velocities are known and to "stitch" broken trajectories following the algorithm of Ref. [4].

Table 6: The params_file.yml parameters for the tracking step. All paths to files are relative to the workflow.py script.

Parameter	Description
particles_file_name	path name of the file which holds the 3D coordinates of particles, namely the results of the matching step
frame_start	if None will start tracking from the first available frame. If an integer it will start the tracking from this given number.
N_frames	if $None$ will iterate over particles in all frames; if an integer will only track particles in the first N frames of the particles file
d_max	the maximum translation in lab space coordinates
dv_max	the maximum allowable change in velocity in lab space coordinates per frame (e.g. $\rm mm/frame)$
mean_flow	Adds a constant mean flow parameter that helps the tracking algorithm to track particles in flows with a constant drift; the format is [vx, vy, vz] in e.g. mm/frame.
plot_candidate_graph	If this is set to True, once tracking is done, it will show a graph of all the linking candidates. Red lines are links that were made and blue lines were candidates to be linked but they were rejected.
save_name	name of the file in which the results shall be saved; if None the results are not saved on the drive

Table 7: The params_file.yml parameters for the calibration with particles step.

Parameter	Description
traj_filename	the name of the file containing the trajectories from which the calibration points are taken
camera	an instance of the camera we wish to try and re-calibrate
cam_number	int, $>=1$; the number (index) of the camera to be calibrated. For example, to calibrate camera2, this should be set to 2
blobs_fname	The name of the file that contains the segmented particles' data
min_traj_len	Only trajectories longer then this number will be used in the calibration
max_point_number	the maximum number of points that shall be taken to re-calibrate the camera. Note that too many points (say above 1000) might lead to long calculation times

2.9 2D tracking guide

In addition to the 3D particle tracking capabilities, MyPTV can also be used to track particles in 2D, namely using only one camera view of the particles. The main advantages of 2D experiments are that they are easier and since only one camera is needed, and that the tracking is easier in 2D because there are no particle occlusions. Note that 2D particle tracking in MyPTV doesn't simply track the particle's pixel locations, since this might give off errors due to camera aberrations or miss-alignment of the camera with the imaging plane. In practice this is done by making use of the optical model used in the 3D version, so this methodology is advantageous over simpler, pixel location, tracking approaches.

To track particles in 2D in MyPTV, we assume that all the particles shown in the image are located on a two-dimensional plane. In practice, this is done by setting a single z coordinate (given by

Table 8: The params_file.yml parameters for the smoothing step. All paths to files are relative to the workflow.py script.

Parameter	Description
trajectory_file	file name of the trajectories file, namely the results of the tracking step
window_size	window size of the sliding polynomial (must be an odd number); if a trajectory is shorter than window_size but longer than min_traj_length, then we use a window the size of the trajectory length
polynom_order	degree of the polynomial used in the smoothing
min_traj_length	the length of the shortest trajectories to be smoothed; has to be larger than min_traj_length
repetitions	the smoothing operation will be repeated this many times over each trajectory
save_name	file name to save the results; if $None$ the results will not be save on the disk

Table 9: The params_file.yml parameters for the smoothing step. All paths to files are relative to the workflow.py script.

Parameter	Description
trajectory_file	name of the smoothed trajectory file used in the stitching
max_time_separation	maximum time separation over which to stitch broken trajectories
max_distance	maximum distance in the position-velocity space
save_name	file name to save the results; if None the results will not be save on the disk

the user) for all particles and then tracking their motion over the x and y coordinates only (see Fig. 9).

The 2D particle tracking procedure requires following these steps:

- 1. Prepare a calibration folder as described in Sec. 2.1
- 2. Go through camera calibration as shown in Sec. 2.2
- 3. Perform particle segmentation of the particle images, as described in Sec. 2.3
- 4. Track the particles' coordinates using the workflow script using the 2D_tracking command (see Sec. 2.1.1). The various parameters of the params_file associated with 2D tracking are given in Tab. 10
- 5. If needed, follow smoothing (Sec. 2.7) and stitching (Sec. 2.8) of the results

2.10 Manual Matching GUI

In certain applications a user might wish to probe the 3D positions of elements in the images manually. Examples might be, to test the quality of the calibration on regions outside of the calibration target, or to measure certain geometrical features of objects in the images. For that purpose MyPTV features a graphical user interface dedicated to doing this easily. To run the GUI, use the manual_matching command of the workflow.py script.

To match a point we need to identify it from the different camera views. In the GUI we do this by clicking on the point we are after in the various images. See Fig. 10 for instructions.

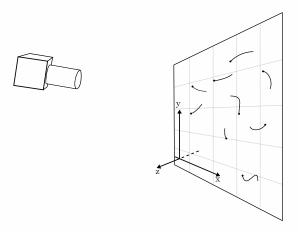


Figure 9: An example for experimental setup for 2D tracking. Note that the camera doesn't have to be perpendicular to the particles' plane and that the particles don't have to be at z=0.

Table 10: The params_file.yml parameters for the 2D tracking step. All paths to files are relative to the workflow.py script.

Parameter	Description
blob_file	The name of the file that contains the segmentation results
frame_start	The number of the first frame from which we want to perform 2D tracking
N_frames	The number of frames to be analyzed; If None, then all the frames will be analyzed
camera_name	the name of the camera from which the images were taken (must be a calibrated camera)
camera_resolution	A tuple representing the camera resolution, for example, (1280, 1024)
$z_{\mathtt{particles}}$	The value of the z coordinate of the plane on which the particles are found, e.g. 0.0
d_max	The maximum allowable particle translation between frame in lab- coordinates
dv_max	The maximum allowable change of velocity for the particles in lab-space coordinates per frame (e.g. mm/frame)
save_name	The name of the file used to save the results

Table 11: The params_file.yml parameters for the manual matching operation. All paths to files are relative to the workflow.py script.

Parameter	Description
	a list that holds the names of the cameras used; for example [cam1,
cameras	cam2, cam3]. Note that the camera files need to be in the same
	folder where we are running the workflow script
images	the paths of the images from which matching should be performed.

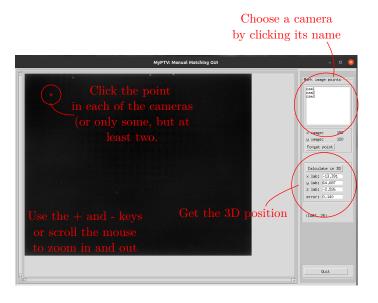


Figure 10: Instructions of how to use the manual matching GUI

3 Imaging module - imaging_mod.py

The imaging module is used to handle the translation from 2D image space coordinates to lab space coordinates and vice versa through the 3D model.

3.1 The camera object

An object that stores the camera external and internal parameters and handles the projections to and from image space and lab space. Inputs are:

- name string, name for the camera. This is the name used when saving and loading the camera parameters.
- 2. resolution tuple (2), two integers for the camera number of pixels
- 3. cal_points_fname string (optional), path to a file with calibration coordinates for the camera. The format fro the calibration point file is given in Section 3.3 (see Fig. 6a).

The important functionality options are:

- 1. get_r(eta, zeta) Will solve eq. 1 for the orientation vector $\vec{b} = \vec{r} \vec{O}$, given an input of pixel coordinates (η, ζ) .
- 2. projection(x) Will reverse solve equation (1) to find the image space coordinates (η, ζ) , of an input 3D point, $(x=\vec{r})$.
- 3. save(dir_path) Will save the camera parameters in a file called after the camera name in the given directory path, see Fig. 11.
- 4. load(dir_path) Will load the camera parameters in a file called after the camera name in the given directory path, see Fig. 11.

After calibration we can save the camera parameters on the hard disc. The camera files have the structure shown in Fig. 11.

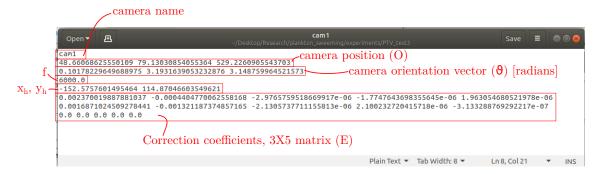


Figure 11: The structure of a camera file. The files are simple text files where each row corresponds to a specific parameter and the values in each row are separated by a white space.

3.2 The imsys object

An object that holds several camera instances and can be used to perform stereo-matching. The important functionalities are:

 stereo_match(coords, d_max) - Takes as an input a dictionary with coordinates in image space from the several cameras and calculates the triangulation position.
 The coordinate dictionary has keys that are the camera number and the values which are the coordinates in each camera. d max is maximum allowable distance for the triangulation.

3.3 The Cal_image_coord object

This is a class used for reading information given in the optional argument cal_points_fname of the camera class (Sec. 3.1). It is used internally and generally users will not have to deal with this. This class will read and interpret text files with tab separated valued, where the columns' meanings are: [x image space, y image space, x lab space, y lab space, z lab space], and each row is a single point of some known calibration target.

The input for this class is:

1. fname - String, the path to your calibration point file. The file is holds tab separated values with the meaning of: [x image, y image, x lab, y lab, z lab], see Fig. 6a.

4 Camera calibration - calibrate_mod.py

The calibrate_mod.py module, with the calibrate object, is used to find the camera calibration parameters. We calibrate each camera by taking an image of a calibration target - a body with markings of known coordinates in lab space - and search for the camera parameters that minimize the distance between the projection of the known points in image space and the image taken with the camera. In addition to that, after a calibration is made and trajectories have been obtained, the calibration might be refined by using the calibrate_with_particles object; this is particularly helpfull in cases where some of the cameras slightly moved in between the taking the calibration images and the recording of the tracer particles.

4.1 The calibrate object

Used to solve for the camera parameters given an input list of image space and lab space coordinates. The inputs are:

- 1. camera An instance of a camera object which we would like to calibrate.
- 2. lab_coords a list of lab space coordinates of some known calibration target.
- 3. img_coords a list of image space coordinates that is ordered in accordance with the lab space coordinates.

The important functionalities are:

- 1. searchCalibration(maxiter=5000, fix_f=True) When this is run, we use a nonlinear least squares search to find the camera parameters that minimize the cost function (item 3 below). This function is used to find the $\vec{O}, \vec{\theta}, f$, and x_h, y_h parameters (in case fix_f=False, it will not solve for f. maxiter is the maximum number of iterations allowed for the least squares search.
- 2. fineCalibration(maxiter=500) This function will solve for the coefficients of the quadratic polynomial used for the nonlinear correction term ([E]).
- 3. mean_squared_err This is our cost function, being the sum of distances between the image space coordinates and the projection of the given lab space coordinates.

To find an optimal calibration solution, we might need to run each function several times, and run the coarse and fine calibrations one after the other until a satisfactory solution is obtained. Once it is obtained, we should keep in mind to save the results using the save functionality of the camera object.

4.2 The calibrate_with_particles object

A class used to refine the calibration using particles data. In short, after the primary clibration is done, matching and tracking can be used to obtain trajectories from the experimental data. Assuming that this resulted in trajectories that were successfully measured, we can leverage the trajectories obtained to minimize further the calibration error. The refinement is done by assuming that the positions of particles along the resolved trajectories are "true" positions in lab-space, and thus, the blobs corresponding to these particles are used in a calibration sequence to minimize the calibration error. The assumption is that longer trajectories with low triangulation error are considered more reliable as compared to shorter trajectories, so we use only "long enough" trajectories in this process.

The inputs are:

- 1. traj_filename the name of the file containing the trajectories from which the calibration points are taken.
- 2. camera an instance of the camera we wish to try and re-calibrate
- 3. cam_number int, >= 1; the number (index) of the camera to be calibrated. For example, to calibrate camera2, this should be set to 2.
- 4. blobs_fname The name of the file that contains the segmented particles' data.
- 5. min_traj_len Only trajectories longer then this number will be used in the calibration
- 6. max_point_number the maximum number of points that shall be taken to re-calibrate the camera. Note that too many points (say above 1000) might lead to long calculation times.

The important functionalities are:

1. get_calibrate_instance - This function returns an instance of the calibrate object with the calibration points taken from the trajectory file. We then use this object to refine the calibration using the regular procedure (Sec. 4.1).

4.3 The gui_final_cal.py file

The gui_final_cal.py script is used to launch a tkinter based graphical user interface that is utilized to run various functionalities of the calibration process. The GUI is shown in Fig. 4, and the various buttons are explained in Tab. 3.

4.4 The gui_initial_cal.py file

The gui_initial_cal.py script is used to launch a tkinter based graphical user interface that is utilized to run various functionalities of the calibration process. The GUI is shown in Fig. 3 and it is explained in section 2.2.3.

5 Particle segmentation - segmentation_mod.py

This module handles the image analysis part of MyPTV, taking in raw camera images containing particles and output their image space coordinates. For the segmentation we first blur the image to remove salt and pepper noise, then we highlight particles using a local mean subtraction around each pixel, and then use a global threshold to mark foreground and pixels. Finally, the connected foreground pixels are considered to be particles, and we estimate the blob's center using a brightness weighted average of blob pixels.

5.1 The particle_segmentation object

Used to segment particles in a given image. This class is used internally to iterate over frames in a single folder by the loop_segmentation class. However, it is recommended to check the segmentation parameters manually using this particle_segmentation over several images in order to tune the particle searching parameters. The inputs are:

- 1. image the image for segmentation
- 2. threshold=10 the global filter's threshold brightness value, so pixels with brightness higher than this number are considered foreground
- 3. mask=1.0 A mask matrix can be used to specify rigions of interest within the image
- 4. sigma=None If float, this is taken as the standard deviation of a Gaussian blurring filter; if it is None, no bluring is performed
- 5. median=None If integer this is taken as the window size for a median noise removal filter; if None, no median filter is performed
- 6. local_filter=None Parameters for a local mean subtraction. If it's an interger it is taken as the window size; is it None, no mean subtraction is performed.
- 7. a bunch of threshold pixel sizes and mass in all directions and in area.
- 8. method String. The name of the method used for segmentation (either 'labeling' or 'dilation', see Sec. 2.3). Default is 'labeling'.
- 9. particle_size=3 The particle size used in the dilation method.

The important functionalities are:

- 1. get_blobs Will return a list of blob centers, their box size and their area.
- 2. plot_blobs() Uses matplotlib to plot the results of the segmentation. A very useful functionality in the testing of segmentation parameters!
- 3. save_results(fname) Will save the segmented particles in a text file. The file is arranged in six columns with the following attributes: (x center position, y center position, x size, y size, area, image number), see Fig. 12.

5.2 The loop_segmentation object

An object used for looping over images in a given directory to segment particles and save the results in a file. The inputs are nearly identical to those of particle_segmentation.

important functionalities are:

- 1. segment_folder_images() Will loop over the images in the given directory and segment particles according to the given parameters
- 2. save_results(fname) Will save the segmented particles in a text file. The file is arranged in six columns with the following attributes: (x center position, y center position, x size, y size, area, image number), see Fig. 12.

6 Particle matching - particle_matching_mod.py

The module used to link particles in the different images through stereo matching and estimating their 3D positions. One of the main issue in this process is that stereo matching all possible candidates is an NP hard problem, so to track numerous particles in each frame we have to choose which particles are likely to produce a 3D particle coordinate. Thus, particle matching in MyPTV

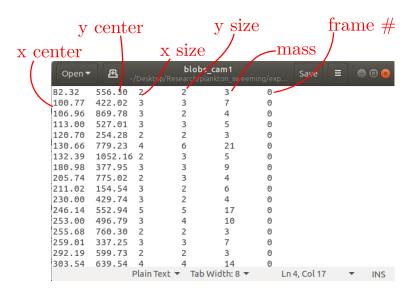


Figure 12: An example of a text file holding the segmentation resuls and the description of the different columns.

uses two algorithms in conjunction. First is a novel algorithm that uses 2D time tracking of blobs to deduce which candidates are more likely to produce traceable particles in 3D. Second is the Ray Traversal algorithm proposed in Ref [5], in which the lab space volume is divided to voxels and stereo matching is attempted for rays within each voxel. Using the two algorithms in conjunction was found to yield a 50% reduction in computational time and more traceable trajectories (45% more trajectories were found in a test).

6.1 The match_blob_files object

This is the object that we use to get triangulated particles results from the segmented blob files (a file as the one in Fig. 12 for each camera). For each frame it first runs the first algorithm using time information, and only then uses the Ray traversal algorithm on the blobs that were not successfully connected. The inputs are:

- 1. blob_fnames a list of the (srting) file names containing the segmented blob data. The list has to be sorted according the order of cameras in the img_system.
- 2. img_system an instance of the img_system class with the calibrated cameras.
- 3. RIO A nested list of 3X2 elements. The first holds the minimum and maximum values of x coordinates, the second is same for y, and the third for z coordinates.
- 4. voxel_size the side length of voxel cubes used in the ray traversal algorithm. Given in lab space coordinates (e.g. mm). Note a too large voxel size will result in high computational times due a high number of candidates, while a too small voxel size might lead to erroneous intersection of rays, leading to matching errors. Thus, this parameter should be optimized.
- 5. max_blob_dist the largest distance for which blobs are considered neighbours in the image space coordinates (namely, the largest permissible blob displacement in pixels).
- max_err=None Maximum acceptable uncertainty in particle position. If None, (defult), than no bound is used.
- 7. reverse_eta_zeta=False Should be false if the eta and zeta coordinates need to be in reverse order so as to match the calibration. This may be needed if the calibration data points were given where the x and y coordinates are transposed (as happens, e.g., if using matplotlib.pyplot.imshow).

The important functionalities are:

- 1. get_particles() Use this to match blobs into particles in 3D.
- 2. save_results(fname) Save the results in a text file. The format has 4 + number of cameras columns separated by tabs: (x, y, z, [N columns corresponding to the blob number in each camera], frame number, see Fig. 13).

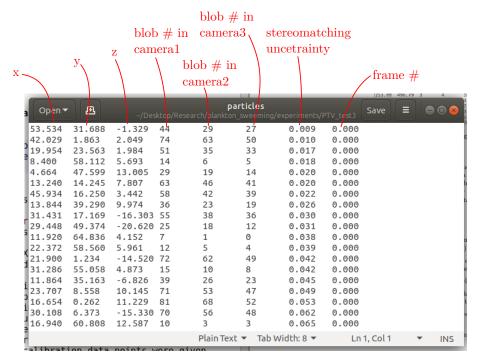


Figure 13: An example of a text file holding the triangulated particles' resuls and the description of the different columns. In this example there were three cameras. The blob number columns give the index of the blobs corresponding to any particle at the this specific frame number; a value of -1 in one of the rows means that no blob was used to stereo-match the particle in this row for this particular camera.

6.2 The matching object

This object is the "engine" used to match particles using the Ray Traversal algorithm. In practice we run the relevant functions: $get_voxel_dictionary() \rightarrow list_candidates() \rightarrow get_particles()$, and after that the results are held in the attribute matched_particles.

6.3 The matching_using_time object

This object performs the matching of blobs using the 2D tracking heuristic. In principle, it is given a list of blobs that were successfully used to form 3D particles in the previous frame. Then, for each of the given blobs it searches for nearest neighbours in the current frame, and stereo-matches those blobs that were found (using the .triangulate_candidates() method).

6.4 The initiate_time_matching object

This object is used to initiate the time searching algorithm on the first frame. It goes over the blobs at the first frame and searches for blobs that have nearest neighbours at the second frame.

Those that have neighbours are used in a first run of the Ray Traversal algorithm, thus they are given priority in the search.

7 Tracking in 3D - tracking_mod.py

This is the module that is used to track particles in 3D. There are currently three tracking methods implemented, nearest neighbour, two-frame, and four-frame, see Ref. [6]. Users are welcome to choose their preferred method and use it.

7.1 The tracker_four_frames object

An object used to perform tracking through the 4-frame best estimate method [6]. Input:

- 1. fname a string name of a particle file (e.g. Fig. 13
- 2. mean_flow=0.0 either zero (default) of a numpy array of the mean flow vector, in units of the calibrations spatial units per frame (e.g. mm per frame). The mean flow is assumed not to change in space and time.
- 3. d_max_=1e10 maximum allowable translation between two frames for the nearest neighbour search, after subtracting the mean flow.
- 4. $dv_{max}=1e10$ maximum allowable change in velocity for the two-frame velocity projection search. The radius around the projection is therefore dv_{max}/dt (where dt=1 frame⁻¹)
- 5. store_candidates = False Boolean indicator. If it is true, then the tracker stores all the possible links that correspond to the tracking thresholds. In this case we can then plot a particle linking graph to analyze the tracking quality. If it is False, the tracker does not keep this information.

The important functionality is:

- 1. track_all_frames() Will track particles through all the frames.
- 2. return_connected_particles() Will retur the list of trajectories that were established.
- 3. save_results(fname) Will save the results on the hard drive. The results are saved in a text file, where each row is a sample of a trajectory. The columns are specified as follows: [trajectory number, x, y, z, frame number], see Fig 14.
- 4. plot_candidate_graph() If the candidate links were stored, this method will plot a graph that shows the links that were made and the candidate links that could have been made but were eventually not made due to duplicate links for the same particle.

7.2 The tracker_two_frames object

An object used for tracking through the 2-frame method. The description is the same as in Section 7.1

7.3 The tracker_nearest_neighbour object

An object used for tracking through the nearest neighbour method. The description is the same as in Section 7.1

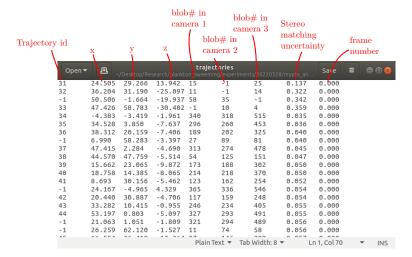


Figure 14: Example of a trajectory file and the column definitions. For Trajectory id being a non-negative integer, rows with the same Trajectory id correspond to the same trajectory; rows with Trajectory id being -1 are samples that could not be linked with the given tracking parameters.

8 Trajectory smoothing - traj_smoothing_mod.py

This module is used to smooth trajectories and to calculate the velocity and acceleration of the particles. For the smoothing we are using the polynomial fitting method proposed and used in Refs. [3, 7]. In short, each component of the particle's position is fitted with a series of polynomials with a sliding window of fixed and the derivatives are calculated by analytically differentiating the polynomial. The end result is a new file with smoothed trajectories. However note that we smooth and calculate velocities and accelerations only for trajectories longer than the window size for the smoothing (a user decided parameter).

8.1 The smooth_trajectories object

A class used to smooth trajectories in a list of trajectories. Due to the smoothing we also calculate the velocity and acceleration of the trajectories. The input trajectory list structure is the same as the files produced by the classes in tracking_mod.py.

Note - only trajectories whose length is larger than the window size will be smoothed and saved. Shorter trajectories are saved with zero velocity and accelerations.

The inputs are:

- 1. traj_list a list of samples organized as trajectories. This should have the same data structure used in the saving function of the tracking algorithms (see Section 7.1).
- 2. window The window size used in the sliding polynomial fitting.
- 3. polyorder The order of the polynomial used in the fitting.

The important functionality is:

- 1. smooth() performs the actual smoothing
- 2. save_results(fname) Saves the results on the hard drive using the given (string) file name. The resulting file is a text file such that each row is a sample of a trajectory, and with 11 columns. The columns have the following meaning: [traj number, x, y, z, v_x , v_y , v_z , a_x , a_y , a_z , frame number], where v_i and a_i denote components of the velocity and acceleration vectors respectively, see Fig. 15.

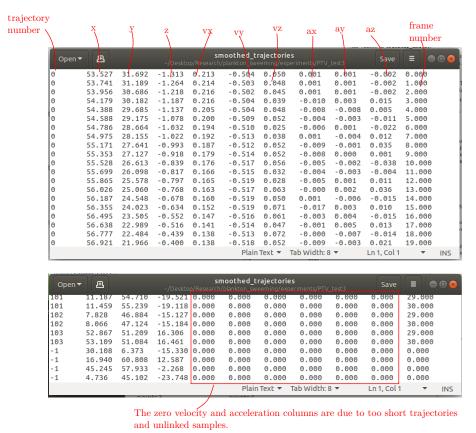


Figure 15: Example file holding the results of smoothed trajectories, and the description for each column. Note also the unsmoothed samples at the bottom of the file.

9 Trajectory stitching - traj_stitching_mod.py

This module applies the algorithm by Ref. [4] to connect trajectories that were broken along the process by tracking the trajectories again in the position–velocity space. We also extend this by interpolating the missing samples using a 3rd order polynomial, that is fitted to the existing 4 data points at the tips of the broken trajectories. This module is applied after the trajectory smoothing step.

9.1 The traj_stitching object

This object performs the stitching process. Inputs:

- traj_list the list of smoothed trajectory, given as a Numpy array of shape (N,11), where N is the number of samples. The format is the same as the format generated after the smoothing process.
- 2. Ts The maximum number of broken samples allowed in the connection.
- 3. dm The maximum distance between the trajectory for which connections are made.

The important functionalities are:

1. stitch_trajectories() - Will search for candidates and stitch the best fitting candidates. Run this function to perform the stitching. After running the new trajectory list is held as the attribute new_traj_list.

 save_results(fname) - Will save the stitched trajectories in a text file with a given file name. The format for the saved file is the same as the one used in the smoothing trajectories (Fig. 15).

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