Interest-bearing Central Bank Digital Currency and Banking Redux

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Abstract

I develop a general equilibrium model to study the optimal design and implementation of central bank digital currencies (CBDCs) in an economy where CBDCs and private bank deposits coexist as competing payment instruments. The findings suggest that the welfare consequences of CBDCs and the policies required for first-best implementation depend on specific parameters. In sufficiently patient economies, a passive monetary policy with non-interest bearing CBDC can achieve the first-best allocation without crowding out bank deposits. Conversely, in more impatient economies, active policies with positive inflation and nominal interest rates are necessary, and interest-bearing CBDC could potentially crowd out bank deposits if the interest rate on CBDC is too high relative to deposit rates. The results highlight the importance of carefully designing CBDCs with incentive-feasible policies intended to maximize welfare and minimize risks to financial intermediation and stability. In a calibrated model representing the US economy, I find that increasing the CBDC interest rate from 0% to 5% leads to a decline of about 5% in bank deposits, while improving overall welfare by 1.4%.

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1 Introduction

Central bank digital currencies (CBDCs) have emerged as a topic of significant interest among policy makers and economists in recent years. The potential benefits and risks of CBDCs have been widely discussed, with many central banks actively exploring the possibility of issuing their own digital currencies.[1] Major economies worldwide are actively researching CBDCs, which would represent a third form of currency accessible to the public, akin to cash, and also accessible to many financial institutions, similar to central bank reserves.[2] Several central banks have conducted or are in the process of planning pilot programs, and operational CBDCs already exist in Caribbean-island countries, such as DCash in the Eastern Caribbean Currency Union (ECCU), the Sand Dollar in the Bahamas, and Jam-Dex in Jamaica. China stands out as a key player among populous nations, aiming to extend financial services across extensive sectors of its economy. Additionally, India and Indonesia are currently conducting trials for digital versions of their respective currencies, the rupee and the rupiah.[3]

A wide range of technological designs for CBDCs have been proposed, but a fundamental characteristic of a CBDC is that it must be universally accessible, meaning it can be held by anyone for any purpose. A second feature relates to whether CBDCs are interest-bearing. The interest rate can serve as an additional policy tool, expanding upon the existing monetary toolkit to stabilize inflation and output. One frequent policy concern surrounding CBDCs is their potential impact on the banking system and financial intermediation. Specifically, many economists and policymakers have expressed concerns about whether CBDCs could lead to a crowding out of bank deposits, as households substitute traditional bank accounts for holding CBDCs. This disintermediation effect could have significant implications for bank lending, investment, and overall financial stability.[4]

In this paper, I develop a general equilibrium model to study the conditions under which the introduction of a CBDC could crowd out bank deposits and lead to a reduction in bank-financed investment. I build on the framework of Andolfatto et al. (2016) and the subsequent New Monetarist literature, by introducing a CBDC that competes with bank deposits as a means of payment. In the model, households can voluntarily choose their portfolio allocation between CBDC and bank deposits based on their relative returns and liquidity properties. Banks, in turn, make investment decisions based on the level of deposits they attract. I distinguish between

[1]According to Boar and Wehrli (2021), a survey conducted by the Bank for International Settlements involved 65 central banks. The survey revealed that 86% of these banks are actively engaged in initiatives related to CBDCs, with 60% having initiated experiments or proofs-of-concept for CBDCs. Additionally, 14% of the banks have progressed to the stage of developing and piloting CBDC arrangements.

[2]See Auer et al. (2020) for an overview of the policy discussion surrounding CBDCs and their different designs across a diverse mix of countries.

[3]Updated lists of countries investigating or issuing CBDCs are reported by `https://www.atlanticcouncil.org/cbdctracker/`.

[4]See Meaning et al. (2018) for a discussion on the potential impact of CBDC on monetary policy transmission and the risks CBDC poses to the banking sector.

the liquidity properties of bank deposits by introducing checkable deposits (more liquid) and time deposits (less liquid). A key feature of the model is that I explicitly incorporate the design choices around CBDC, such as the interest rate it pays and any fees associated with its use. This allows us to study how these policy parameters interact with household portfolio choices and banks' investment decisions. Importantly, the government is assumed to lack a lump-sum tax instrument to implement its monetary policy rule. This implies that households' currency choices must adhere to sequential rationality constraints, as there is no coercion or forced participation. Monetary policy, in this sense, must be incentive-feasible.

I focus my attention on implementing first-best allocations. The main results characterize the government policies required to implement the first-best allocation in an economy where CBDC and bank deposits coexist as payment instruments. In sufficiently patient economies, a passive policy with constant money supply is enough to achieve the first-best outcome. No inflation or taxes are needed, and CBDC does not crowd out bank deposits if CBDC is non-interest bearing. Banks are willing to issue deposits and pay interest rates to households at a sufficiently high level, as households are willing to sacrifice enough of their consumption for labor in the production of the output. Moreover, asset prices are priced at their fundamental level. There is no liquidity shortage as the consumer debt-constraints are slack.

In impatient economies, active policies with positive inflation and nominal interest rates are required to implement a first-best allocation with competing payment instruments. Binding debt-constraints lead to a shortage of liquidity. Banks issue fewer deposits to households and offer excessively high interest rates to motivate households to work hard and produce enough output. Liquidity premium arises due to asset scarcity as a result of the binding debt-constraints. In the absence of lump-sum taxes, the government must use some combination of seigniorage revenue, labor income taxes, and CBDC fees to incentivize efficient production by households and deposit issuance by banks. In this case, interest-bearing CBDC can potentially crowd out bank deposits if the interest rate on CBDC is too high relative to deposit rates. Distortionary taxes and CBDC fees are necessary to relax the debt-constraints of households.

To quantify these theoretical predictions, I calibrate the model to match key features of the US economy using data from 1987 to 2008. Consistent with the theoretical prediction that interest-bearing CBDC can crowd out bank deposits, the quantitative analysis shows that raising the nominal CBDC interest rate from 0% to 5% leads to an approximately 5% decline in bank deposits. Despite this disintermediation effect, PM market production improves slighlty, and overall welfare increases by 1.4% in consumption-equivalent terms. These findings validate the theoretical prediction that while CBDC introduction can lead to some bank disintermediation, the efficiency gains through improved payment mechanisms can outweigh these costs when CBDC is properly designed.

1.1 Related literature

There has been a burgeoning number of CBDC papers recently that is impractical to review here, but my paper complements the CBDC papers in the New Monetarist literature. Keister and Sanches (2023) study the potential effects of introducing a CBDC on the banking system and monetary policy in perfectly competitive markets. In their model, banks face a pledgeability constraint. They find that a CBDC can lead to a disintermediation effect, where households substitute private bank liabilities for CBDC holdings, which can lead to a reduction in productive investment and social welfare. In contrast, my paper focuses on the conditions under which a CBDC could crowd out bank deposits and the policies required to implement the first-best allocation in an economy where CBDC and bank deposits coexist.

Chiu and Davoodalhosseini (2023) investigate the macroeconomic benefits of a cash-like CBDC design. They find that a CBDC can improve welfare by reducing the cost of holding and using money, as well as by promoting financial inclusion. However, they also note that a CBDC may lead to a reduction in bank deposits and a decline in bank lending. In my paper, I explicitly incorporate the design choices around CBDC, such as the interest rate it pays and any associated fees, and studying how these policy parameters interact with household portfolio choices and banks' investment decisions. Household portfolio choices are also voluntary, meaning that there is no coercion or forced participation, so that sequential rationality is respected.

There are some influential papers that study the effect of CBDC issuance in economies with imperfect competition among banks. Andolfatto (2021) examines the impact of a CBDC on the banking system and monetary policy transmission when there is monopoly power in the banking system. He argues that a CBDC could discipline the banks by compelling them to increase their deposit rate, leading to an increase in bank deposits and financial inclusion. In Chiu et al. (2023), banks also have market power in the deposit market. They find that CBDC issuance could expand bank intermediation if the interest rate on CBDC lies within an intermediate range and causes disintermediation only if the interest rate is too high.

Williamson (2022) focuses on efficiency where the central bank competes with the private sector for safe assets. Welfare is increased through households substituting CBDC for private bank liabilities and a CBDC may disintermediate banks when there is an overaccumulation of capital. This implies that disintermediation comes at the expense of improving economic efficiency. In my paper, the financial frictions in the banking sector themselves lead to an overproduction of goods but investment is too low to satisfy the demand. Lower investment in my model then reduces economic efficiency and bank deposits are priced at a premium.

Many studies have also investigated various aspects of CBDCs, such as their optimal design, their impact on monetary policy transmission, and their potential risks to financial stability. Barrdear and Kumhof (2022) examine the macroeconomic effects of CBDC issuance in a DSGE model, while Davoodalhosseini (2022) examines the coexistence of cash and CBDC with balance-contingent transfers. Fernández-Villaverde et al. (2021) explore the effects of a

CBDC on financial stability and bank runs within the framework established by Diamond and Dybvig (1983). Similar papers that also study CBDC and financial stability include Keister and Monnet (2022), Rahman (2024), and Williamson (2022). Brunnermeier and Niepelt (2019) and Niepelt (2023) show that a CBDC might not impact macroeconomic outcomes, including bank intermediation. Jiang and Zhu (2021) delve into the interactions between CBDC and reserves as tools for monetary policy. Various papers, including those by Agur et al. (2022), Davoodalhosseini and Rivadeneyra (2020), Wang (2023), and Kumhof and Noone (2018) contribute to understanding CBDC motivations and designs. However, none of these papers assume the absence of a government lump-sum transfer and consider individual rationality behind the interaction between CBDC and bank deposits. This is how my paper differs from this literature, and I also study the coexistence issues and which conditions are necessary for first-best allocations.

2 The Physical Environment

The physical environment is based on Andolfatto et al. (2016) and Chiu and Davoodalhosseini (2023). Time is discrete and the horizon is infinite. As is typical in models like those in Lagos and Wright (2005), each period is divided into two subperiods. In this context, I refer to these subperiods as AM and PM, respectively. Search friction is abstracted away and agents meet in centralized locations in both subperiods. Two distinct perishable goods (or outputs) are produced and consumed in each subperiod called the AM good and the PM good.[5]

There is a continuum of infinitely-lived households, distributed uniformly on the unit interval $[0, 1]$. In each AM subperiod, a unit measure of new competitive bankers enters the economy, and they exit in the subsequent AM. Households are identical *ex ante*, but may differ *ex post*. Let $\{c_t(i), y_t(i)\} \in \mathbb{R}^2_+$ denote consumption and production of the PM good, respectively, at date t by agent i. Households discount utility payoffs across periods with the discount factor $0 < \beta < 1$; so that the preferences for household i are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(x_t(i)) - Ah_t(i) + \pi \left[u(c_t(i)) - g(y_t(i)) \right] \right\}. \tag{1}$$

At the beginning of the PM, each member of a household experiences an idiosyncratic shock that determines their types. Let the types be classified as *consumers*, *producers*, and *idlers*.[6] A member of a household can become a consumer or a producer with equal probability π, so

[5]They can also be thought of *day good* and *night good*, respectively, as described in Andolfatto (2010). Provided that the two goods are unique and pertain to separate subperiods, the specific terminology used to label them is not critical to the analysis.

[6]The idlers are inactive agents or nonparticipants who are intended to mimic the unmatched agents in Lagos and Wright (2005).

that the probability of becoming an idler is $1 - 2\pi$, where $0 < \pi \leq 1/2$. A consumer derives flow utility $u(c_t(i)) \in \mathbb{R}_+$ from consuming the PM good, where $u'' < 0 < u'$, and $u(0) = 0$, $u'(0) = \infty$. A producer derives flow utility $-g(y_t(i)) \in \mathbb{R}_+$ from producing the PM good, where $g(0) = g'(0) = 0$, $g' > 0$ for $y > 0$, and $g'' \geq 0$. The PM flow utility for idlers is normalized to zero. Since there is an equal measure of consumers and producers, feasibility and efficiency imply $c = y$.

In the AM subperiod, all households share identical preferences and opportunities. Their utility flow in the AM is given by $U(x_t(i)) - Ah_t(i)$, where $x_t(i) \in \mathbb{R}$ denotes the consumption of the AM good by individual i at date t, and $h_t(i)$ denotes their labor at date t. Assume that $U'' < 0 < U'$ with $U(0) = -\infty$ and $U'(0) = \infty$. The parameter A represents the relative emphasis households place on consumption versus labor in their utility preferences, a key factor that significantly influences the outcomes analyzed in this paper.

Bankers live for two periods, participate only in the AM, and consume only in old age. They are endowed with an investment technology. Households can consume the AM good, but they can also transfer these goods to young bankers. Young bankers can transform k units of the AM good into $f(k)$ units of the AM good in the next AM. The banker then consumes k in date $t+1$ when he becomes old. The aggregate resource constraint in the AM is given by

$$X_t + k_{t+1} \leq H_t + f(k_t), \tag{2}$$

where $X \equiv \int x_t(i)di$ and $H \equiv \int h_t(i)di$.

As the PM good is perishable, another aggregate resource constraint requires

$$\int c_t(i)di \leq \int y_t(i)di \tag{3}$$

for all $t \geq 0$.

Consider a planner who weights all the agents equally and maximizes the *ex ante* utility of the agents with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(X_t) - AH_t + \pi \left[u(c_t(i)) - g(y_t(i)) \right] \right\} \tag{4}$$

subject to the aggregate resource constraints (2) and (3). The steady-state first-best allocation constitutes a set of numbers (X^*, k^*, y^*) satisfying:

$$U'(X^*) = A, \tag{5}$$

$$\beta f'(k^*) = 1, \tag{6}$$

$$u'(y^*) = g'(y^*). \tag{7}$$

Lemma 1 is directly derived from the results presented in equations (5) through (7).

Lemma 1 *X^* is strictly decreasing in A. k^* and y^* are determined independently of A.*

3 Agent Decision-making

I will impose restrictions on the environment that will render trade by credit to become infeasible, so that a medium of exchange is essential. A medium of exchange is essential in the sense that it will allow society to achieve desirable outcomes that could not be achieved in its absence. Firstly, I assume limited commitment among household members, contrasting with banks' ability to commit and enforce debt repayment. Limited commitment implies that all trade is voluntary, respecting sequential rationality. This leads to the absence of a lump-sum tax instrument, a point I will discuss later. Secondly, I assume household anonymity, which, combined with the first assumption, rule out private debt between households and makes a medium of exchange essential. Thirdly, I assume that households engage in a sequence of competitive spot market trades, exchanging bank deposits and interest-bearing CBDC for goods in both subperiods. In the following discussion, I will explore how bank deposits and CBDC, as voluntary payment instruments chosen by individual household members, possess distinct properties crucial in determining the welfare consequences of monetary policy.

3.1 Banker decision-making

I first consider the decision-making of bankers who derive utility from consuming the AM good in old age. All markets are assumed to be competitive. Each of the bankers possesses an investment technology that allows them to invest in k units of the AM good at date t. The banker then produces $f(k)$ units of the AM good at date $t+1$. Assume $f'' < 0 < f'$, $f'(0) = \infty$, and $f'(\infty) = 0$. The banker finances the investment by issuing deposits d and pays a gross real interest rate R^D to each member of a household who is using bank deposits for payment. Figure 1 presents the timeline for all agents.

Each banker takes the deposit price ϕ_1 in the AM as given and maximizes their profit

$$\max_{k} \quad \left\{\phi_1 f(k) - R^D k\right\}. \tag{8}$$

The first-order condition is then given by

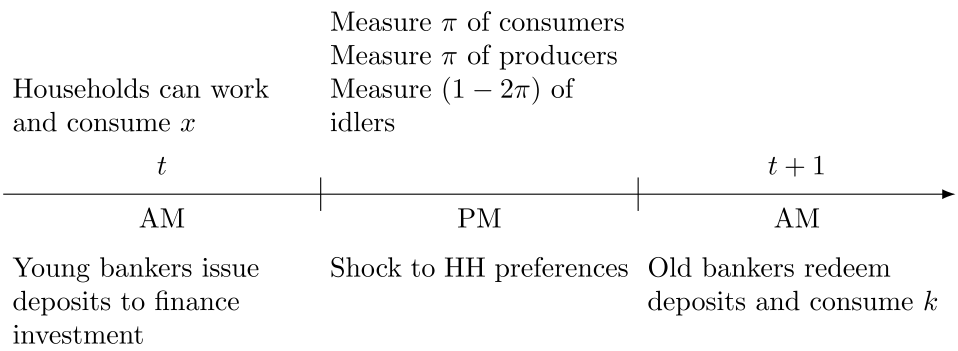

Figure 1: Timeline

$$f'(k) = \frac{R^D}{\phi_1}. \tag{9}$$

Given that f is an increasing and strictly concave function, condition (9) implies that the demand for investment k is decreasing in R^D.

Lemma 2 *The bankers' demand for investment spending $k(R^D)$ is decreasing in the deposit rate R^D.*

3.2 Household-member decision-making

Members of a household use CBDC and bank deposits as payment instruments. Denote by $\{(v_1, v_2), (\phi_1, \phi_2)\}$ the price of CBDC and bank deposits in the AM and PM markets, respectively.

Government policy pertinent to household decision-making will be described in detail below. Here, I outline key policy elements that impact the choices of individual household members. The government's policy rule operates before the start of the AM-market trading. A household member enters the AM with money balances in the form of CBDC and bank deposits, denoted by z and a respectively. The individual then has the option to approach either a government or a bank counter to transform these balances into $R^M z - \tau$ or $R^D a + f(a)$ units of money, respectively. Household members also have to pay a labor income tax $\tau_h \in [0, 1)$ on their labor income wh, where h is an individual's labor supply and and w is the market price for leisure. If $R^M > 1$ and $\tau > 0$, then CBDC here is akin to an interest-bearing bond subject to a fixed fee, as in Andolfatto (2010).

If an individual member of a household decides not to use CBDC, he simply uses bank deposits. Subsequently he enters the AM-market with $R^D a + \phi_1 f(a)$ units of money in the form of bank deposits. In contrast to CBDC, bank deposits are partially illiquid financial instruments not issued by the government. The $f(a)$ component captures the illiquid aspect of bank deposits.

I model this illiquidity aspect to capture the real-world diversity of bank deposits, differentiating between more liquid forms like checkable deposits and less liquid forms such as time deposits.[7]

After activities in the AM market, household members carry their CBDC and bank deposit balances into the PM market, where their roles as producers, consumers, or idlers are realized. Subsequently, following the PM market transactions, individuals retain their remaining CBDC and deposit balances, moving into the next AM. There, they are again faced with the choice between using CBDC or bank deposits, each offering different interest rates.

3.2.1 The AM market

Denote by $(z, a) \geq 0$ a household member's CBDC and bank deposit balance, respectively, in the AM at date t; and denote by $(m, d) \geq 0$ the CBDC and bank deposit balance, respectively, that the individual household member carries forward into the PM market. Let $\sigma \in [0, 1]$ denote the probability of an individual household member exercising the CBDC interest vehicle option. This σ also represents the probability of paying the fixed CBDC fee. Depending on the decision to pay the fee or not, the individual household member can then purchase or sell output x at either the market price v_1 if selecting CBDC as currency, or the market price ϕ_1 if selecting bank deposits as the preferred currency of use for transactions. The AM-market budget constraint is then given by

$$x = (1 - \tau_h)\, wh + \sigma \left(R^M v_1 z - \tau \right) + (1 - \sigma)\phi_1 \left(R^D a + f(a) \right) - v_1 m - \phi_1 d. \tag{10}$$

A recursive representation of a household member's optimal choice problem is as follows. Let $W(z, a)$ denote the value function in the AM with CBDC and bank deposit balances, $(z, a) \geq 0$, respectively; and let $V(m, d)$ denote the value function in the PM before the household member realizes his type. The value functions $W(z, a)$ and $V(m, d)$ must satisfy the following recursive relationship:

$$\begin{aligned} W(z, a) \equiv \max_{x,h,\sigma,m,d} \quad & \{U(x) - Ah + V(m, d)\} \\ \text{s.t.} \quad & x = (1 - \tau_h)\, wh + \sigma \left(R^M v_1 z - \tau \right) \\ & + (1 - \sigma)\phi_1 \left(R^D a + f(a) \right) - v_1 m - \phi_1 d. \end{aligned} \tag{11}$$

Assuming that $V(m, d)$ is strictly concave, substituting out for h yields the following first-order conditions

[7] For a more detailed exploration of the dynamics of various forms of exchange media, consider the insights offered in the studies by Chiu et al. (2023) and Wright (2010), which delve into the complexities and implications of different exchange mechanisms.

$$U'(x) = \frac{A}{(1-\tau_h)\, w}, \tag{12}$$

$$v_1 = \frac{(1-\tau_h)\, wV_1(m,d)}{A}, \tag{13}$$

$$\phi_1 = \frac{(1-\tau_h)\, wV_2(m,d)}{A}. \tag{14}$$

The demand for both CBDC and bank deposits, respectively, is independent of a household member's initial CBDC and deposit holdings, z and a, respectively. This implies that all household members enter the PM market with identical CBDC and deposit holdings. By comparing conditions (5) and (12), it becomes evident that labor taxes introduce distortions that lead to overconsumption in the equilibrium compared to the optimum, as indicated by $x > x^*$. The optimal CBDC interest vehicle choice must satisfy

$$\sigma = \begin{cases} 1 \\ [0,1] \\ 0 \end{cases} \quad \text{if } R^M v_1 z - \tau \begin{cases} > \\ = \phi_1 \left(R^D a + f(a)\right), \\ < \end{cases} \tag{15}$$

so that the act of CBDC fee payment is sequentially rational if, and only if,

$$R^M v_1 z - \tau \geq \phi_1 \Big(R^D a + f(a)\Big). \tag{16}$$

For a given CBDC interest vehicle choice σ, by the envelope theorem:

$$W_1(z,a) = \frac{A\sigma R^M v_1}{(1-\tau_h)\, w}, \tag{17}$$

$$W_2(z,a) = \frac{A\,(1-\sigma)\,\phi_1 \left(R^D + f'(a)\right)\phi_1}{(1-\tau_h)\, w}. \tag{18}$$

Given the assumptions that $R^M > 1$ and $\tau > 0$, the function $W(z,a)$ is characterized as piece-wise linear and convex in z and a. Furthermore, $W(z,a)$ is non-differentiable at the point $R^M v_1 z - \tau = \phi_1 \left(R^D a + f(a)\right)$.

3.2.2 The PM market

After AM-market activity, the household member carries CBDC and deposit balances with him into the PM market. Just before entering the PM market, the individual experiences a stochastic shock, where he realizes he is a consumer, a producer, or an idler. Following

PM-market activity, the individual carries any remaining CBDC and deposit balances forward to the next AM, where he once again decides whether to exercise the CBDC interest vehicle option. Let $V^C(m,d)$, $V^P(m,d)$, and $V^I(m,d)$ denote the utility value associated with being a consumer, a producer, and an idler, respectively. The ex ante value function associated with entering the PM market is given by

$$V(m,d) = \pi V^C(m,d) + \pi V^P(m,d) + (1-2\pi)\, V^I(m,d). \tag{19}$$

A consumer who enters the PM with a wealth portfolio (m,d) faces the budget constraint $c = v_2(m - z^+) + \phi_2(d - a^+)$. Substituting out for c, the choice problem can be stated as

$$V^C(m,d) \equiv \max_{z^+\geq 0, a^+\geq 0} \left\{u\left(v_2\left(m - z^+\right) + \phi_2\left(d - a^+\right)\right) + \beta W\left(z^+, a^+\right)\right\}. \tag{20}$$

In what follows, the consumer's debt-constraint $\{(z^+, a^+) \geq (0,0)\}$ will play an important role in the results below. It is also important to note that if $z^+ = 0$, then $a^+ = 0$, and conversely. The implication of this assumption is that a consumer returning to the AM-market will likely find it optimal to refrain from exercising the CBDC interest vehicle option, meaning they will not pay the CBDC fee. By making use of (17) and (18), the PM consumption is characterized by

$$\begin{aligned} v_2 u'(c) &= \tfrac{\beta A R^M v_1^+}{(1-\tau_h)w} \\ & && \text{if} \qquad v_2 m + \phi_2 d \geq c \\ \phi_2 u'(c) &= \tfrac{\beta A(R^D + f'(a^+))\phi_1^+}{(1-\tau_h)w} \\ \\ c &= v_2 m + \phi_2 d && \text{otherwise.} \end{aligned} \tag{21}$$

By the envelope theorem:

$$V_1^C(m,d) = v_2 u'(c), \tag{22}$$

$$V_2^C(m,d) = \phi_2 u'(c). \tag{23}$$

A producer who enters the PM with a wealth portfolio (m,d) faces the budget constraint $y = v_2(z^+ - m) + \phi_2(a^+ - d)$. Substituting out for y, the choice problem can be stated as

$$V^P(m,d) \equiv \max_{z^+\geq 0, a^+\geq 0} \left\{-g\left(v_2\left(z^+ - m\right) + \phi_2\left(a^+ - d\right)\right) + \beta W\left(z^+, a^+\right)\right\}. \tag{24}$$

Since a producer has no desire to consume, his debt-constraint is necessarily slack. Therefore, a producer must strictly prefer to exercise his CBDC interest vehicle option, meaning he will pay the CBDC fee the next AM. Utilizing (17) and (18), the PM production is characterized by

$$v_2 g'(y) = \frac{\beta A R^M v_1^+}{(1-\tau_h)w}, \tag{25}$$

$$\phi_2 g'(y) = \frac{\beta A(R^D + f'(a^+))\phi_1^+}{(1-\tau_h)w}. \tag{26}$$

Idle household members entering the PM market with a wealth portfolio (m, d) simply carry their CBDC and bank deposit balances forward to the next AM. Consequently, we have $V^I(m, d) \equiv \beta W(m, d)$. The envelope theorem yields the following equations, applicable to both idlers and producers:

$$V_1^P(m, d) = V_1^I(m, d) = v_2 g'(y), \tag{27}$$

$$V_2^C(m, d) = V_1^I(m, d) = \phi_2 g'(y). \tag{28}$$

As the choice of a preferred payment instrument also comes into question, I want to restrict attention to equilibria where both bank deposits and CBDC coexist. For this to occur, the following rate-of-return equality condition must be satisfied:

$$\frac{R^M v_1^+}{v_2} = \frac{\left(R^D + f'(a^+)\right)\phi_1^+}{\phi_2}. \tag{29}$$

That is, for both assets to be accepted as payment, the expected rate of return on assets from the PM to the next AM must be the same. Consequently, at the individual level, portfolio composition becomes indeterminate in equilibrium.

4 Government Policy

I will now outline the government's policy. As a reminder, the government's operational approach involves intervening before AM-market trading begins. The policy entails offering a nominal interest rate of R^M on CBDC balances to household members willing to pay the fixed CBDC fee τ. Additionally, the government has the authority to impose labor income taxes, denoted as τ_h, on the labor earnings (wh) of individual household members, irrespective of their choice of currency.[8]

[8] In this context, the approach to implementing a labor tax differs significantly from that presented in Rahman and Wang (2023). In their paper, τ_h can also be viewed as a sales tax.

Let M^{-} denote the supply of outside money in the form of CBDC at the beginning of the AM-market (prior to any injection or withdrawal). Assume that this digital money supply grows at the constant (gross) rate $M = \mu M^{-}$, where M denotes the supply of digital money in the "next" period. Based on the assumptions, the initial CBDC supply M^{-} is entirely held by producers and idlers at the beginning of the AM. This is because both producers and idlers find it optimal to pay the CBDC fee τ. Consequently, the government bears an aggregate interest obligation of $(R^M - 1)M^{-}$, along with revenue from labor income tax $\tau_h wH$, and revenue from CBDC fee payments, $(1 - \pi)\tau$.

The government can also earn seigniorage revenue by printing new digital money $M - M^{-}$. Thus, a feasible government policy will have to satisfy the government budget constraint:

$$\underbrace{\left(R^M - 1\right) M^{-}}_{\text{Government spending}} = \underbrace{\tau_h wH}_{\text{Labor income tax revenue}} + \underbrace{(1 - \pi)\,\tau}_{\text{CBDC fee revenue}} + \underbrace{M - M^{-}}_{\text{Seigniorage revenue}} . \tag{30}$$

By defining $\delta \equiv {}^{R^M}\!/\!_{\mu}$ and rearranging the equation above, the government budget constraint may alternatively be expressed as:

$$\tau = \frac{(\delta - 1)M - \tau_h wH}{1 - \pi}. \tag{31}$$

Invoking the results derived from the aforementioned assumptions, which are established to be valid within this class of quasilinear models, the joint equilibrium distribution of CBDC and bank deposit holdings (z, a) will be massed over points: $\{(0, 0), (M, D), (2M, 2D)\}$. This means that the mass π of PM consumers enter the AM with zero units of CBDC and deposit holdings, the mass $(1-2\pi)$ of PM idlers enter with (M, D) units of wealth, and the mass π of PM producers enter with $(2M, 2D)$ units of wealth. Hence, an incentive-feasible government policy is one designed to ensure that both the fraction $(1 - 2\pi)$ of idlers and the fraction π of producers voluntarily pay the CBDC fee τ, while also satisfying (31) with the policy parameters (δ, τ, τ_h). It is worth noting that unlike the CBDC fee τ, the labor income tax τ_h will be voluntarily paid by all household members.

I define a *passive policy* as a government policy with the property $(\delta, \tau, \tau_h) = (1, 0, 0)$. In a passive policy, CBDC is non-interest bearing. Any policy that does not meet this criterion is referred to below as an *active policy*, where CBDC bears interest.

5 Stationary Monetary Equilibrium

In this section, I will examine the characteristics of stationary monetary equilibria under different currency regimes. These regimes include economies where only bank deposits serve as exchange media, economies where only CBDC is used, and economies where both bank deposits and CBDC coexist and compete as exchange media. My primary focus is on analyzing the properties of a stationary equilibrium where both CBDC and bank deposits coexist, given an incentive-feasible government policy. Briefly outlined, such equilibria must meet the following requirements: (i) Household and banker decisions are optimal; (ii) decisions are symmetric across all producers and consumers; (iii) markets clear at every date; and (iv) all real quantities remain constant over time.

5.1 A CBDC economy

Suppose that outside money, specifically CBDC, can only be used for payment in the PM. In particular, the rate of return on CBDC is higher than that on bank deposits. That is

$$\frac{R^M v_1^+}{v_2} > \frac{\left(R^D + f'(a^+)\right)\phi_1^+}{\phi_2}.$$

The market-clearing conditions for the money market are given by $m = M$ and $c = y$, as well as $v_2 = y/M$.

Gathering restrictions implied by individual behavior, I combine (13), (22), (27) to form

$$\frac{Av_1}{(1-\tau_h)w} = v_2\left[\pi u'(c) + (1-\pi)g'(y)\right]. \tag{32}$$

Updating the latter expression by one period and combining with (25) yields

$$g'(y) = \beta R^M \left(\frac{v_2^+}{v_2}\right)\left[\pi u'(y^+) + (1-\pi)g'(y^+)\right]. \tag{33}$$

Restricting our attention to steady-state ($y = y^+ > 0$) it follows that $v_2^+/v_2 = v_1^+/v_1 = 1/\mu$. This then together with the market-clearing conditions yields

$$\beta\delta L(y) = 1, \tag{34}$$

where

$$L(y) = \frac{\pi u'(y) + (1-\pi)g'(y)}{g'(y)}. \tag{35}$$

Market clearing implies

$$v_2 M \geq y^* \text{ or } v_2 M = y < y^*. \tag{36}$$

Conditions (16), (34) and (36) characterize the monetary equilibrium as a function of parameters, contingent upon an incentive-feasible policy $\delta \equiv {}^{R^M}/_{\mu}$ in an economy with CBDC as the only medium of exchange. Note that $L'(y)$ can either increase or decrease with respect to y, and $L(y^*) = 1$. Furthermore, it should be emphasized that the "standard" Friedman rule, where $(R^M, \mu) = (1, \beta)$ or $\delta = {}^1/_{\beta}$, is not incentive-feasible, as the CBDC fee is not voluntarily paid by every individual.

5.2 A bank credit economy

Now, let us consider the case where inside money, specifically bank deposits, can only be used for payment in the PM. Conversely, the condition below holds:

$$\frac{R^M v_1^+}{v_2} < \frac{\left(R^D + f'(a^+)\right)\phi_1^+}{\phi_2}.$$

This implies that the return on bank deposits is higher than that on CBDC. The market-clearing conditions for the deposit market are given by $d = D$ and $c = y$.

Gathering restrictions implied by individual behavior, I combine (14), (23), (28) to form

$$\frac{A v_1}{(1-\tau_h)w} = \phi_2 \left[\pi u'(c) + (1-\pi)g'(y)\right]. \tag{37}$$

Updating the latter expression by one period and combining with (26) yields

$$g'(y) = \beta\left(R^D + f'(a^+)\right)\left(\frac{\phi_2^+}{\phi_2}\right)\left[\pi u'(y^+) + (1-\pi)g'(y^+)\right]. \tag{38}$$

Restricting our attention to steady-state ($y = y^+ > 0$, $a = a^+ = 0$) it follows that $\phi_2^+ = \phi_2 > 0$ and $\phi_1^+ = \phi_1 > 0$. This then together with the market-clearing conditions yields

$$\beta\left(R^D + f'(a)\right)L(y) = 1. \tag{39}$$

To solve for the AM price of bank deposits assume that $L(y^*) = 1$, so that there is zero-liquidity premium at the first-best allocation and that assets are efficiently priced at their "fundamental level". By combining the banker's first-order condition (9) with (39), we obtain:

$$\phi_1^* = \frac{\beta R^D}{1 - \beta R^D} > 0, \tag{40}$$

which bears resemblance to the standard asset-pricing formula derived for risk-neutral agents. Note that ϕ_1 is increasing in the bank interest rate R^D. Furthermore, we need $1 < R^D < 1/\beta$ to satisfy $0 < \phi_1 < \infty$ for a bank credit equilibrium to exist.

To solve for the PM price of bank deposits, we can combine (26) and (40) to find:

$$\phi_2^* = \frac{\beta A R^D}{(1 - \beta R^D)(1 - \tau_h) w g'(y^*)} > 0. \tag{41}$$

An immediate observation from the above equation is the influence of labor income tax τ_h on deposit prices. Once again, market clearing implies

$$\phi_2^* D \geq y^* \text{ or } \phi_2 D = y < y^*. \tag{42}$$

Conditions (9), (39), (40), (41), and (42) constitute the key restrictions that characterize the general equilibrium allocation and price system in this competitive economy, where only bank deposits are used as the medium of exchange. If the debt-constraints are binding, bank deposits may become overvalued ($\phi_1 > \phi_1^* \implies \phi_2 > \phi_2^*$) due to a shortage in their supply. Consequently, this creates a liquidity premium on the price of bank deposits, leading to a lower expected rate of return. The following condition must hold for deposit prices to be overvalued with binding debt-constraints:

$$\frac{1}{\beta} > \frac{R^D(1 + \phi_1)}{\phi_1} > 1.$$

5.3 A mixed CBDC and bank credit economy

I now consider an economy where CBDC and bank deposits coexist as payment instruments, with condition (29) being satisfied. Market clearing now implies

$$v_2 M + \phi_2^* D \geq y^* \text{ or } v_2 M + \phi_2 D = y < y^*. \tag{43}$$

To confirm whether the conjecture made regarding (43) holds in equilibrium, we can use (21) to derive:

$$A \geq \frac{(1-\tau_h)wg'(y^*)y^*}{\beta\left[\delta v_1 M + (R^D + f'(a))\phi_1^* D\right]}. \tag{44}$$

Since we can use (40) to derive an equilibrium value for $\phi_1 > 0$, then all we require is any value $v_1 < \infty$ satisfying (44) to obtain an equilibrium in which both CBDC and bank deposits coexist. In economies with a sufficient level of A, multiple assets can coexist and hold value.

To derive the consumption allocation across household types in each AM, we can combine the joint steady-state distribution of wealth above with the household budget constraint (10). For those who were consumers in the previous PM, $\sigma = 0$ and $(z^+, a^+) = (z, a) = (0, 0)$, so:

$$x = (1-\tau_h)\,wh - v_1 M - \phi_1 D. \tag{45}$$

For those who were idlers in the previous PM, $\sigma = 1$ and $(z^+, a^+) = (z, a) = (M, D)$, so:

$$x = (1-\tau_h)\,wh + (R^M - 1)v_1 M - \tau - \phi_1 D. \tag{46}$$

For those who were producers in the previous PM, $\sigma = 1$ and $(z^+, a^+) = (z, a) = (2M, 2D)$, so:

$$x = (1-\tau_h)\,wh + (2R^M - 1)v_1 M - \tau - \phi_1 D. \tag{47}$$

We can easily verify that the population-weighted sum of (45), (46), and (47) is nonzero.[9]

Recall that we want to restrict our attention to incentive schemes that satisfy (16), ensuring that producers strictly prefer to pay the CBDC fee, while idle household members weakly prefer to do so. Consider that idlers will find it optimal to pay the CBDC fee the next AM. In equilibrium, idlers enter the AM market with wealth $(z, a) = (M, D)$. By appealing to (31), the CBDC fee constraint $R^M v_1 z - \tau \geq \phi_1\left(R^D a + f(a)\right)$ can be written as follows:

$$(1-\pi)\left[R^M v_1 M - \phi_1(R^D D + f(D))\right] + \tau_h w H \geq (\delta - 1)M. \tag{48}$$

In this class of models, it is crucial to consider the sequential participation of a consumer who enters the AM market with zero CBDC and deposit balances. To accumulate wealth, this agent must exert significant effort, sacrificing transferable utility. Let $W(0, 0)$ denote the payoff for rebalancing asset holdings in the AM on the equilibrium path. If the cost or sacrifice of

[9]Although the process may appear similar, the result differs significantly from that of Andolfatto (2010).

rebalancing is excessively high, an individual household member would forego that opportunity and enter the next PM with zero CBDC and deposit holdings. However, the individual can still consume in the AM and opt to work in the PM market. If he chooses to work in the PM, then he takes his CBDC and deposit holdings and spend them in the AM. Along this alternative path, the individual never consumes in the PM. Let $\widehat{W}(0,0)$ represent the payoff of this alternate strategy. Then, sequential rationality must satisfy:

$$W(0,0) \geq \widehat{W}(0,0). \tag{49}$$

Another constraint that must be met is that producers in the PM market must be willing to produce good y for (z, a) units of money. It is straightforward to verify that this constraint is satisfied in equilibrium.

Condition (43) and (49), along with conditions (9), (16), (34), (39), (40), and (41) derived earlier, constitute the key restrictions characterizing the general equilibrium allocation and price system in an economy where both CBDC and bank deposits hold value.

In the next section, I will examine the various policies required for first-best implementation in a mixed CBDC and credit economy.

6 Optimal Policies

I now study the implementation of first-best allocation in a mixed CBDC and bank credit economy. Depending on parameters, the level of output may or may not be efficient. The goal is to identify the policies that are required for a first-best implementation.

6.1 Passive policy

I first determine the conditions under which a passive policy $(\delta, \tau, \tau_h) = (1, 0, 0)$ is optimal. With a passive policy where CBDC is non-interest bearing, $\sigma = 1$ holds trivially for all household members. Hence, the CBDC fee constraint (48) can be ignored. The government budget constraint (31) will also be satisfied, so that the passive policy is trivially an incentive-feasible policy. The question is whether the consumer's debt-constraint (21) will bind or not. From (44), if the consumer debt-constraint is slack then the conjecture that needs to be satisfied is the following:

$$A \geq \frac{wg'(y^*)y^*}{\beta\left[v_1 M + (R^D + f'(a))\phi_1^* D\right]}. \tag{50}$$

Proposition 1 *Under a passive government policy* $(\delta, \tau, \tau_h) = (1, 0, 0)$ *with non-interest bearing CBDC, there exists a unique* $0 < A_0 < \infty$ *that satisfies* $v_2 M + \phi_2 D = y^*$.

For economies with $A \geq A_0$*, the competitive monetary equilibrium is efficient. That is,* (40) *and* (41) *hold, as well as*

$$v_2 M + \phi_2 D \geq y^*. \tag{51}$$

For economies with $0 < A < A_0$*, the competitive monetary equilibrium is inefficient. That is,*

$$v_2 M + \phi_2 D < y^*, \tag{52}$$

$$\phi_1 > \phi_1^*, \tag{53}$$

$$\phi_2 > \phi_2^*. \tag{54}$$

According to Proposition 1, if $A \geq A_0$, the household's preference for the AM good is sufficiently high for the AM good to be produced at its efficient level, as there are no distortionary effects from labor income tax. As, the real rate of return on money with a non-interest bearing CBDC is high enough, the debt-constraint for consumers remain slack. The prices of CBDC and bank deposits are at their fundamental level. Bankers will issue deposits and pay an interest rate R^D that is sufficiently high, as household members are willing to sacrifice enough of their consumption for labor to produce the AM good. Furthermore, producers will be willing to produce the first-best amount of goods in the PM.

On the other hand, if $A < A_0$, the household's preference for the AM good is too low for the AM good to be produced efficiently, even without distortionary effects from labor income tax. The assets are priced inefficiently due to a shortage of liquidity. Bankers issue fewer deposits and offer an excessively high interest rate R^D, which results in a liquidity premium of deposit prices. Consequently, binding consumer debt-constraints will lead to the producers delivering an inefficient level of PM goods, and consumption in the PM is too small.

Proposition 1 is related to the nonmonetary and monetary equilibria discussed in Lagos and Rocheteau (2008) and Andolfatto et al. (2016). Lagos and Rocheteau (2008) find that if the capital stock is small, there is an overaccumulation of capital in the equilibrium, leading to lower consumption in the PM. Conversely, Andolfatto et al. (2016) assume an efficient capital stock, resulting in an overvaluation of money (backed by the capital stock) in equilibrium and lower consumption in the PM. Here, based on Lemma 1, since the AM good X is strictly decreasing in A, if $A < A_0$, this results in an overproduction of the AM good, meaning $X > X^*$. This occurs because with a higher interest rate R^D, there is a lower demand for the AM good from the bankers via the investment channel, as outlined in Lemma 2. Consequently, there will be lower consumption of the PM good with $y < y^*$. Finally, the liquidity premium of deposits when the debt-constraints are binding, that is, when $A < A_0$, is also similar to that of Keister and Sanches (2023), except for the fact that their result depends on the parameter β instead of A here.

When $A < A_0$, policies need to be necessarily inflationary, that is, $\delta > 1$ ($R^M > 1, \mu > 1$), to restore efficiency. Moreover, the private banks will have to lower R^D to not reduce their investment demand.

6.2 Active policies

If first-best implementation under a passive policy is infeasible for economies with low A, then our only recourse is a range of active policies necessitating strictly positive inflation and nominal interest rates. This implies an interest-bearing CBDC is necessary to improve welfare. I now focus solely on economies where $A < A_0$, specifically in regions of the parameter space where a passive policy fails to implement the first-best allocation. This is because the real rate of return is too low to motivate bankers to issue deposits and producers to supply the first-best level of output. The government has three instruments to finance its CBDC interest obligation: seigniorage, labor income tax revenue and CBDC fee revenue. Below, I restrict attention to policies that satisfy $1 < \delta < 1/\beta$; since otherwise, a monetary equilibrium will fail to exist, except in the limiting case of $\delta \nearrow \beta^{-1}$.

Case 1: $1 < \delta < 1/\beta$, $\tau = 0$, *and* $\tau_h > 0$. In what follows, I ask whether seigniorage and labor income tax revenue help to implement the first-best allocation in the absence of CBDC fees. Note that from (44) it is easy to verify that A is decreasing in both δ and τ_h. As $A < A_0$, we will have

$$A_0 \equiv \frac{(1-\tau_h) w g'(y_0) y_0}{\beta\left[\delta v_1 M + (R^D + f'(a))\phi_1 D\right]}. \tag{55}$$

We simply need to reduce the right-hand side of condition (55) to relax the debt-constraint for consumers, ensuring that (44) holds. This can be accomplished through a strictly inflationary policy and a strictly positive labor income tax. Since A is decreasing in both δ and τ_h, there exists a unique critical value $0 < A_1 < A_0$ so that the consumer debt-constraint is slack. Note that since the labor income tax is distortionary (see condition (12)), AM consumption in the equilibrium is lower than in the optima. Unlike Andolfatto et al. (2016) and Andolfatto (2010), money injected in this manner is not superneutral when it is introduced in the form of interest. This is because seigniorage and labor income tax expand the set of economies that can attain the first-best allocation.

Case 2: $1 < \delta < 1/\beta$, $\tau > 0$, *and* $\tau_h = 0$. In the absence of labor income tax with $\tau_h = 0$, I want to now show how CBDC expenditures can be financed by seigniorage and CBDC fees to improve the allocation. For this instance, we have

$$A_0 \equiv \frac{wg'(y_0)y_0}{\beta\left[\delta v_1 M + (R^D + f'(a))\phi_1 D\right]}. \tag{56}$$

The following lemma characterizes the optimal CBDC fee necessary to implement the first-best allocation.

Lemma 3 *For economies with $A_1 \leq A < A_0$, the optimal CBDC fee τ that can be attained to implement the first-best allocation is*

$$\tau^*(\delta, R^D, A) = (1-\pi)^{-1}\left\{(\delta - 1)M + \frac{\beta AH\left[\delta v_1 M(1-\beta R^D) + R^D D\right]}{(1-\beta R^D)g'(y^*)y^*} - wH\right\}. \tag{57}$$

$\tau^*(\delta, R^D, A)$ is increasing in A. $\tau^*(\delta, R^D, A)$ is increasing in δ and the effect of R^D on $\tau^*(\delta, R^D, A)$ is ambiguous.

Proof. To derive $\tau^*(\delta, R^D, A)$, set $A = wg'(y^*)y^*\left\{\beta\left[\delta v_1 M + (R^D + f'(a))\phi_1 D\right]\right\}^{-1}$ in (50) to obtain

$$\tau_h = 1 - \frac{A\beta[\delta v_1 M + (R^D + f'(a))\phi_1^* D]}{wg'(y^*)y^*}.$$

Use the government budget constraint (31) to substitute out τ_h to obtain

$$\tau = (1-\pi)^{-1}\left\{(\delta - 1)M + \frac{\beta AH[\delta v_1 M + (R^D + f'(a)\phi_1^*)D]}{g'(y^*)y^*} - wH\right\}.$$

Use (40) to substitute ϕ_1^* and simplify to obtain (57). Differentiating (57) with respect to A leads to

$$\frac{\partial \tau^*(\delta, R^D, A)}{\partial A} = \frac{\beta H(1-\pi)^{-1}[\delta v_1 M(1-\beta R^D) + R^D D]}{(1-\beta R^D)g'(y^*)y^*} > 0.$$

Now, differentiating (57) with respect to δ leads to

$$\frac{\partial \tau^*(\delta, R^D, A)}{\partial \delta} = (1-\pi)^{-1}M + \frac{\beta AH(1-\beta R^D)(1-\pi)^{-1}v_1 M}{(1-\beta R^D)g'(y^*)y^*} > 0.$$

Finally, differentiating (57) with respect to R^D gives us

$$\frac{\partial \tau^*(\delta, R^D, A)}{\partial R^D} = \frac{(1-\beta R^D)g'(y^*)y^*[\beta AH(1-\pi)^{-1}D - \beta^2 A(1-\pi)^{-1}\delta v_1 M]}{\left[(1-\beta R^D)g'(y^*)y^*\right]^2} + \frac{\beta^2 AH(1-\pi)^{-1}\left[\delta v_1 M(1-\beta R^D) + R^D D\right]g'(y^*)y^*}{\left[(1-\beta R^D)g'(y^*)y^*\right]^2} \lesseqgtr 0.$$

■

Unlike in Andolfatto et al. (2016), the CBDC fee $\tau(\delta, R^D, A)$ here is increasing in A. Although the interpretation of A here and in their paper is similar, A enters slightly differently in my model. The CBDC fee is also not independent of inflation μ and interest rates R^M and R^D. As δ (the ratio of nominal interest on CBDC to money growth rate) increases, the CBDC fee also increases.

The positive effect of R^M on τ^* suggests that when the government pays a higher interest rate on CBDC, it can afford to charge a higher CBDC fee without significantly reducing the demand for CBDC. This is because the higher interest rate on CBDC compensates households for higher CBDC fees, maintaining the overall attractiveness of CBDC as a payment instrument. The effect of R^D on τ^* can potentially induce both income and substitution effects. If R^D increases, bank deposits become relatively more attractive compared to CBDC, prompting the government to potentially reduce CBDC fees to incentivize households to use CBDC instead of bank deposits (substitution effect). Conversely, as households earn more interest income from their bank deposits, they may have additional disposable income to spend on transaction fees. Consequently, the government could potentially charge a higher CBDC fee without significantly discouraging CBDC use (income effect). Ultimately, the sign of ${}^{\partial \tau^*}/_{\partial R^D}$ will depend on which effect dominates. However, for $A \geq A_0$, no CBDC fee income and inflation are necessary to implement the first-best allocation.

Since coercion is ruled out and all trade must be voluntary, it is necessary to determine the conditions under which the CBDC fee τ^* can be collected through voluntary contributions. Specifically, the CBDC fee τ^* must satisfy the CBDC fee constraint given by equation (16). This constraint ensures that the fee is set at a level that incentivizes agents to participate in the CBDC system voluntarily, without the need for coercion or forced participation.

It is also crucial to consider how CBDC could potentially displace bank deposits in an environment with strictly positive inflation and positive nominal interest rates. If $R^M > 1$, maintaining the rate-of-return equality condition (29) requires an increase in R^D. However, as Lemma 2 suggests, an increase in R^D would lead to a decline in investment demand. This is a channel through which bank deposits could be crowded out by CBDC, even though sequentialy rationality is respected.

Case 3: $1 < \delta < 1/\beta$, $\tau > 0$, *and* $\tau_h > 0$. Suppose now that $A < A_1$, indicating that the government cannot implement the first-best allocation exclusively with either labor income tax (and seigniorage) as in *Case 1*, or with a fixed fee (and seigniorage) as in *Case 2*. With further restrictions in the economy, the government will have to utilize all the available policy instruments at its disposal. This comes at the expense of giving up more degrees of freedom than desired.

Note that with the both $\tau_h > 0$ and $\tau > 0$, the CBDC fee constraint (48) needs to be satisfied to induce voluntary participation. Rearranging (48) further we can obtain the expression

$$\tau_h \geq \frac{(1-\pi)(R^D + f(D)) - [(1-\pi)R^M v_1 + 1 - \delta]M}{wH}. \tag{58}$$

Condition (58) is an expression for the minimum labor income when the government uses all the policy tools available in a constrained equilibrium. A higher CBDC fee and labor income tax relaxes the CBDC fee constraint, which is a channel through which households can relax their debt-constraint. Since $A < A_1$ and A decreases with both δ and τ_h, there exists a unique critical value $0 < A_2 < A_1 < A_0$ that will relax the consumer debt-constraint. The following proposition identifies the regions of the parameter space in which active government policies alone, within a constrained CBDC and credit equilibrium, are sufficient for achieving the first-best allocation.

Proposition 2 *If* $A_0 > A \geq A_1$*, the first-best allocation can be implemented with* $1 < \delta < 1/\beta$*,* $\tau = 0$*, and* $\tau_h > 0$*, or with* $1 < \delta < 1/\beta$*,* $\tau > 0$*, and* $\tau_h = 0$ *where CBDC is interest-bearing. If* $A_1 > A \geq A_2$*, implementing the first-best allocation requires* $1 < \delta < 1/\beta$*,* $\tau > 0$*, and* $\tau_h > 0$*. Price stability is achieved with* $\phi_1 = \phi_1^*$ *and* $\phi_2 = \phi_2^*$ *satisfying* (40)*,* (41) *and* (50)*.*

Propositions 1 and 2 identify three regions of the parameter space with different rates of returns on CBDCs. For $A \geq A_0$, a passive policy $((\delta, \tau, \tau_h) = (1, 0, 0))$ is sufficient to deliver a first-best allocation where CBDC bears zero interest. There is adequate liquidity provisioning in the economy, so having a constant supply of CBDC is enough to induce household members to work hard and produce the PM good at the first-best level. Bankers will issue deposits and pay interest rate R^D high enough so that household members sacrifice enough of their consumption to produce the AM good. Moreover, all trade is voluntary among agents as sequential rationality is respected. CBDC in this case is non-interest bearing with $R^M = 1$ and can be viewed as digital cash with which agents cannot hide their money balances. The unbacked nature of this form of government-issued fiat money over private money (bank deposits) is an advantage. There is no disintermediation in the banking system in this case as the rate-of-return equality condition (29) holds.

For $A_0 > A \geq A_1$, the constrained-efficient policy entails positive inflation and non-zero nominal interest rates, as a constant supply of CBDC is insufficient to restore efficiency. In this case, the government will require a combination of seigniorage with labor income tax ($1 < \delta < 1/\beta$, $\tau = 0$, and $\tau_h > 0$) or a combination of seigniorage with fee income ($1 < \delta < 1/\beta$, $\tau > 0$, and $\tau_h = 0$) to induce producers to produce an efficient level of PM goods and for bankers to issue enough deposits. CBDC in this case must be interest-bearing to deliver efficiency. However, CBDC could crowd out bank deposits if R^M is too high relative to R^D and decrease investment. For $A_1 > A \geq A_2$, the constrained-efficient policy requires the government to use both labor income tax and fee income (and seigniorage) to induce producers to produce y^* and for bankers to issue deposits k^*. Any optimal policy in this region must have a CBDC that bears interest. In the equilibrium, however, there will be an overproduction of the AM good X^* due to the distortionary effects of labor income tax. The optimal CBDC fee income and labor income tax through voluntary contributions are incentive-feasible. Strictly positive inflation and nominal interest rates encourage households to use CBDC and pay its associated fees, as it relaxes their CBDC fee constraint so that their debt-constraint does not bind. Disintermediation of banks could still occur with this policy as it is strictly inflationary with strictly positive interest rates.

Finally, it is worth pointing out that achieving the first-best allocation is impossible under deflation within any parameter space, in contrast to Andolfatto et al. (2016). The government policy here mirrors that of Andolfatto (2010), where an incentive-feasible policy precludes deflation. Unlike in Andolfatto et al. (2016), there is no dividend fee income from holding assets to encourage voluntary contributions for a small CBDC fee. Additionally, in Andolfatto et al. (2016), there are no competing payment instruments; however, in this model, both CBDC and bank deposits compete as exchange media, and their choice must adhere to individual rationality. Thus, positive inflation and non-zero nominal interest rates are always necessary in more impatient economies when CBDC and bank deposits compete as exchange media, as deflation is infeasible.

7 Quantitative Analysis

In this section, I calibrate the theoretical model to the US economy in order to assess the impact of introducing different versions of a CBDC. As noted earlier, the CBDC in my paper can either be non-interest-bearing or interest-bearing.

7.1 Calibration

For the quantitative exercise, I consider an annual model with utility functions $U(y) = B\log(y+\epsilon)$ in the AM and $u(c) = ((c+\epsilon)^{1-\chi} - 1)/(1-\chi)$ in the PM, PM disutility $g(y) = y^{1+\gamma}/(1+\gamma)$, and production technology $f(k) = 2\sqrt{k}$, where ϵ is a utility normalization parameter set to 0.001.

The calibration procedure consists of two parts, involving the calibration of ten parameters: $(A, B, \beta, \chi, \gamma, \mu, \pi, R^M, \tau, \tau_h)$. In the first step, some parameters are set directly. These include the discount factor, money growth rate, nominal interest rate, labor income tax, CBDC fee, and preference shock parameter. Following the literature, I set $\beta = 0.97$ and choose a money growth rate of $\mu = 1.02$, which aligns with the historical long-term inflation rate in the United States. The nominal interest rate on CBDC, $R^M = 1.021$, is set to reflect the FRED data on the Interest Rate on Excess Reserves (IOER) in 2019, ensuring that $\delta > 1$. The labor income tax, τ_h, is set to 0.24, taken directly from Christiano et al. (2014). Second, I jointly calibrate four parameters (A, B, χ, γ) to match the empirical money demand curve using data from 1987 to 2008. I use the new M1 series from Lucas and Nicolini (2015), excluding the post-crisis period when M1 demand rose sharply due to non-transactional motives. The calibration employs a grid search followed by numerical optimization to minimize a weighted objective function that combines two criteria: the root mean squared error between model-predicted and actual M1/GDP ratios, and the difference between their averages. Table 1 summarizes all the parameter values along with their calibration targets. Figure 2 shows the predicted money demand curve from the model compared to the actual data from 1987 to 2008.

Parameters	Notation	Value	Calibration Targets
Calibrated externally			
Discount factor	β	0.97	Standard in literature
Money growth rate	μ	1.02	2% inflation
Nominal interest rate	R^M	1.021	2.1% IOER rate
Labor income tax	τ_h	0.24	Christiano et al. (2014)
CBDC fee	τ	0.01	Set directly
Preference shock	π	0.55	Set directly
Calibrated internally			
PM utility curvature	χ	0.30	Money demand curve
AM utility parameter	B	5.20	Money demand curve
Production curvature	γ	10.00	Money demand curve
AM preference	A	1.00	Money demand curve

Table 1: Calibration Results

The model captures the downward-sloping relationship between the M1 to GDP ratio and

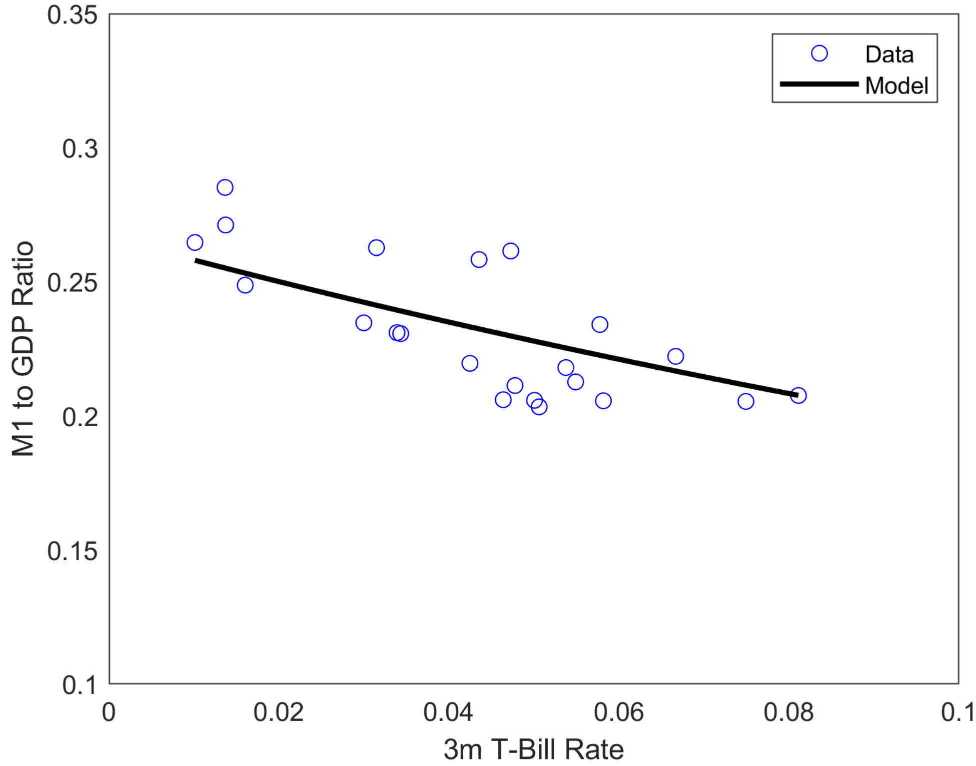

Figure 2: Money Demand Curve: Model vs Data

the 3-month T-bill rate. The model reasonably approximates the data, with the average money demand broadly consistent with empirical observations. The efficiency measures indicate that the calibrated economy exhibits slight overconsumption and overproduction in both AM and PM markets, respectively, while showing some underinvestment relative to first-best. Using these calibrated parameters, I analyze the quantitative effects of introducing different versions of CBDC policy.

7.2 Effects of an interest-bearing and non-interest-bearing CBDC

Taking the economy without interest-bearing CBDC ($\delta = 1$) as my benchmark, I find several key effects when introducing CBDC policy. Figure 3 shows the results for varying the CBDC policy parameter δ, while Figure 4 displays the effects of changing the CBDC interest rate R^M.

AM consumption remains essentially unchanged across both policy variations. In Figure 3, AM consumption stays constant at approximately 6.85 as δ increases from 1.0 to 1.03. Similarly, in Figure 4, AM consumption remains flat at about 6.85 as R^M increases from 1.0 to 1.05 (representing a 5 percentage point increase in the CBDC interest rate). This stability is consistent with the theoretical prediction that AM consumption is determined by labor-leisure tradeoffs and is largely insulated from monetary policy.

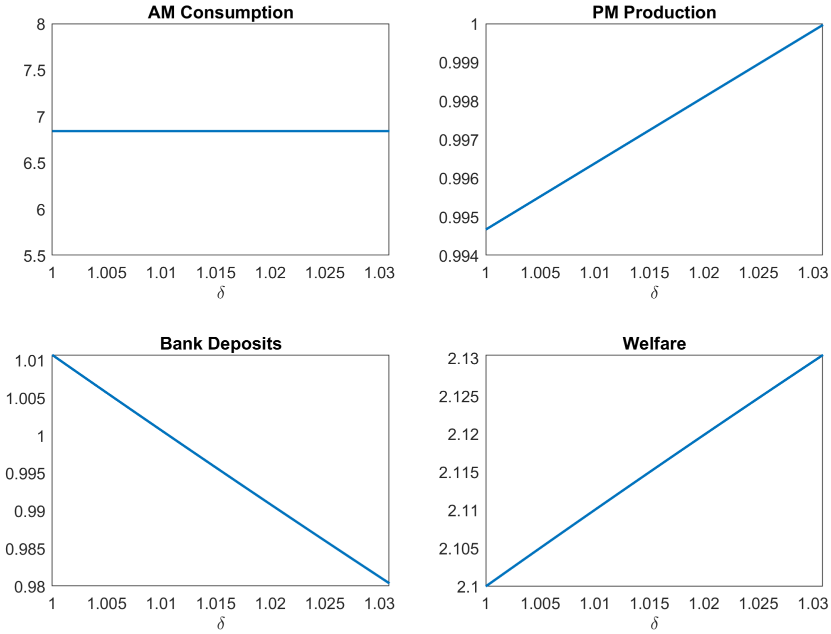

Figure 3: Effects of Interest-bearing CBDC through δ

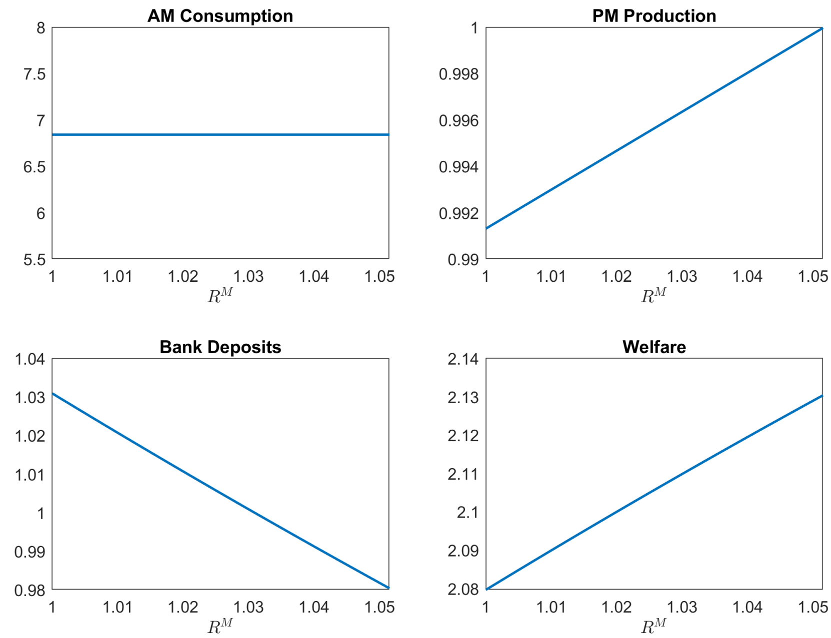

Figure 4: Effects of Interest-bearing CBDC through R^M

PM production shows meaningful positive responses to CBDC policy in both scenarios. In Figure 3, PM production increases from approximately 0.995 to 1.0 as δ rises from 1.0 to 1.03, representing about a 0.5% increase. Figure 4 shows similar results, with PM production rising from about 0.991 to 1.0 as the CBDC interest rate increases by 5 percentage points (from $R^M = 1.0$ to $R^M = 1.05$). This aligns with the theoretical model where monetary policy affects PM market efficiency through the interest rate channel.

My model generates substantial bank disintermediation effects. In Figure 3, bank deposits decline from approximately 1.01 to 0.98 as δ increases to 1.03, representing about a 3% decrease. Figure 4 shows an even more pronounced effect, with deposits falling from about 1.031 to 0.981 as R^M rises by 5 percentage points, representing approximately a 5% decline. This supports the theoretical prediction that as CBDC becomes more attractive, agents substitute away from traditional bank deposits.

Despite this disintermediation, I find that welfare effects are positive. In Figure 3, welfare increases from approximately 2.1 to 2.13 as δ rises to 1.03. Figure 4 shows a similar pattern, with welfare rising from about 2.08 to 2.13 as the CBDC interest rate increases by 5 percentage points. This represents roughly a 1.4% welfare improvement, suggesting that in sufficiently impatient economies, active CBDC policies with positive interest rates can enhance welfare even though they reduce bank intermediation.

These findings quantify an important policy tradeoff in my theoretical model: while interest-bearing CBDC can improve welfare through increased payment efficiency in the PM market, it comes at the cost of reduced bank deposits. The calibrated results suggest that the efficiency gains outweigh the costs of disintermediation within the policy ranges considered, whether we examine the effects through δ or R^M.

7.3 Welfare costs of active and passive policies

Following Wang (2016), I measure the welfare cost of CBDC issuance by increasing the policy parameter δ from 1 to α percent by compensating consumption. This allows me to quantify the economic significance of moving from passive to active policy choices in consumption-equivalent terms.

For any policy parameter α, total welfare in the economy, derived from (4), is given by:

$$\text{Wel}(\alpha) = (1-\beta)^{-1}[U(X(\alpha)) - AH(\alpha) + \pi[u(y(\alpha)) - g(y(\alpha))]], \tag{59}$$

where $X(\alpha)$, $H(\alpha)$, and $y(\alpha)$ represent the equilibrium allocations of AM consumption, AM labor, and PM production for any α, respectively. This welfare function captures the total utility from AM market activity through $U(X) - AH$ and PM market activity through $\pi[u(y) - g(y)]$.

I measure the welfare cost of implementing active policies ($\delta > 1$) with positive inflation and nominal interest rates relative to the passive policy benchmark ($\delta = 1$) with constant money supply as the value $1 - \Delta_1$ that solves $\text{Wel}(\delta) = \text{Wel}_{\Delta_1}(1)$, where the scaled welfare is:

$$\text{Wel}_{\Delta_1}(1) = (1-\beta)^{-1}[U(\Delta_1 X(1)) - AH(1) + \pi[u(y(1)) - g(y(1))]]. \tag{60}$$

In equation (60), only AM consumption is scaled by factor Δ_1, while AM labor and PM production remain at their passive policy levels. This follows the standard approach in calculating the welfare cost, where the welfare cost $1 - \Delta_1$ represents the percentage of consumption that agents would need to give up to move from the passive policy to the active policy regime.

Figure 5 illustrates the welfare cost results. Active CBDC policies ($\delta > 1$) generate welfare *gains* relative to the passive policy benchmark. At $\delta = 1.02$, the welfare gain is 0.391% of consumption, rising to 0.931% at $\delta = 1.05$. Note that $\delta = 1$ is not the Friedman rule, which would require $\mu = \beta$ and $R^M = 1$, yielding $\delta = 1/\beta \approx 1.03$ in my calibration.

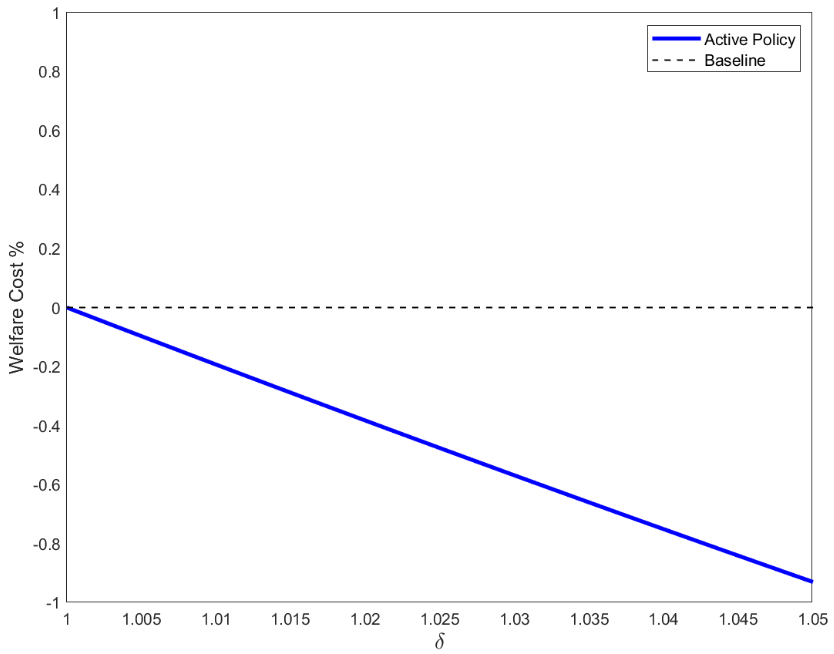

Figure 5: The Welfare Cost of CBDC

The welfare gains from active CBDC policies arise through several channels. Table 2 summarizes the key quantitative results. AM consumption remains perfectly stable across all policy parameter values. PM production increases monotonically from 0.995 under the passive policy to 1.00 under the most active policy ($\delta = 1.05$), representing a 0.854% improvement in production efficiency. This finding is consistent with the theoretical result that active policies with positive inflation and nominal interest rates help achieve efficient PM allocation y^* by inducing the producers to produce more and relaxing their debt constraints. However, investment falls from 1.01 to 0.963 as δ increases to 1.05, representing a 4.75% decline in bank intermediation. This crowding-out effect confirms the theoretical prediction from Lemma 2 that higher CBDC interest rates can reduce bank deposit demand. The mechanism works through the rate-of-return equality condition (29): as R^M increases, maintaining equilibrium requires either higher R^D or acceptance of some disintermediation.

Despite significant banking disintermediation, the overall welfare effect is positive and increasing in δ. The welfare gain reaches 0.931% at $\delta = 1.05$, indicating that PM market efficiency improvements more than compensate for reduced financial intermediation. Overall, these results highlight the importance of incentive-feasibility constraints in CBDC design. The negative welfare costs shown in Figure 5 demonstrate that passive policy ($\delta = 1$) with constant money supply is indeed not optimal when agents must voluntarily choose to pay CBDC fees. The calibrated economy falls into the region where $A < A_0$, requiring active monetary policy with positive inflation and nominal interest rates to achieve efficiency.

Policy Type	δ	Welfare Cost (%)	AM Consumption	PM Production	Investment
Passive	1.00	0.00	6.84	0.995	1.01
Active	1.02	-0.391	6.84	0.998	0.991
Active	1.05	-0.931	6.84	1.00	0.963

Table 2: Welfare Cost Percentage Changes and Key Outcomes

Based on this exercise, we can conclude that even though the welfare gains are somewhat small, we can properly design active CBDC policies with positive inflation and nominal interest rates to improve upon passive policies in more impatient economies. While active CBDC policies improve welfare through increased payment efficiency in the PM market, they can potentially lead to disintermediation in the banking sector. This suggests that the efficiency gains outweigh the costs of disintermediation within the policy ranges considered, confirming the central insight that incentive-feasible policies requiring voluntary participation can dominate theoretically optimal but non-implementable passive policies with non-interest bearing CBDC.

8 Conclusion

Many policymakers are debating whether CBDCs should be interest-bearing and universally accessible. In this study, I extend a fairly standard monetary model in the New Monetarist tradition to explore the optimal design and implementation of central bank digital currencies in an economy where both CBDCs and private bank deposits coexist as competing payment instruments. The main findings suggest that the welfare consequences of CBDCs and the policies required for first-best implementation depend crucially on specific parameters. Another issue I address is whether private intermediaries should be left to provide exchange media to facilitate trade when commitment and record-keeping are limited. There is no obvious answer to this, as it depends on the state of the economy.

In sufficiently patient economies, a passive monetary policy with constant money supply and non-interest bearing CBDC can achieve the first-best allocation. In these economies, CBDC does not disintermediate or crowd out bank deposits, as banks are willing to issue deposits and pay sufficiently high interest rates. Asset prices reflect their fundamental values and there are no liquidity shortages.

Conversely, in more impatient economies, active policies with positive inflation and nominal interest rates are necessary to implement first-best allocations when CBDC competes with bank deposits. In these economies, binding debt constraints lead to liquidity shortages, causing banks to issue fewer deposits at excessively high interest rates. Consequently, asset prices become overvalued, reflecting liquidity premia. To restore efficiency, the government must use a combination of seigniorage revenue, labor income taxes, and CBDC fees to incentivize households to increase production. Importantly, interest-bearing CBDC in these environments could potentially crowd out bank deposits if the interest rate on CBDC is too high relative to deposit rates. The disintermediation of banks then comes at the expense of improving efficiency. This tradeoff is one that central banks should consider when designing policies regarding CBDCs that are incentive-feasible

The findings of this paper contribute to the growing literature on CBDCs and provide valuable insights for policymakers considering the introduction of CBDCs. The results highlight the importance of carefully designing CBDC policies based on the specific characteristics and preferences of the economy to maximize welfare and minimize potential risks to financial intermediation and stability. Future research could extend this framework to incorporate additional features such as heterogeneous agents, financial frictions in the banking sector such as bank market power and pledgeability constraints, and monetary policy transmission channels to further deepen our understanding of the macroeconomic implications of CBDCs.

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