Econo-info-metrics[1]

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Expected payoffs

- In the context of choice theory, consider the problem of a decision-maker who can choose among a finite number of actions $a_1, \ldots, a_K$, knowing the payoff $u[a_k]$ for each.
- The term "utility" carries with it both the sense of "payoff" and the sense of "welfare". Since welfare is not the issue in this context, I will use the term "payoff" to describe the variable that influences choice behavior.
- In general we can represent the behavior of the decision-maker as a mixed strategy or behavior frequency distribution assigning some non-negative frequency $f_k \geq 0$ to each of the actions, and resulting in an expected payoff $\sum_k f_k u[a_k]$.

Maximum entropy subject to an expected payoff constraint

- The quantal response Gibbs distribution can be derived by maximizing the entropy of the behavior frequency distribution subject to a constraint on expected payoff, $\bar{u}$.

$$\underset{\{f_1,\ldots,f_K\}\geq 0}{\text{Max}} - \sum f_k \text{Log}\,[f_k] \tag{1}$$

$$\text{subjectto} \sum f_k = 1 \tag{2}$$

$$\sum f_k u\,[a_k] \geq \bar{u} \tag{3}$$

- This has the Lagrangian:

$$\mathcal{L}[f, \mu, \beta] = -\sum f_k \text{Log}\,[f_k] - \mu\,(\textstyle\sum_k f_k - 1) + \beta\,(\sum f_k u\,[a_k] - \bar{u})$$

- Effectively the same Lagrangian corresponds to maximizing expected payoff subject to a lower bound on the entropy of the behavior frequency distribution.

First-order conditions

- The first-order conditions can be solved to yield:

$$f\left[a_k\right] = \frac{e^{\beta u[a_k]}}{\sum_k e^{\beta u[a_k]}} \tag{4}$$

- This is the *Gibbs distribution*, which leads the decision-maker to choose each available action with a positive frequency, with the logarithm of frequency proportional to the product of the payoff and the Lagrange multiplier β.
- The Lagrange multiplier corresponding to expected payoff can be regarded as a *responsiveness*.
- The higher the responsiveness, the more concentrated the decision-maker's behavior is on the payoff-maximizing action.

Visualizing choice behavior

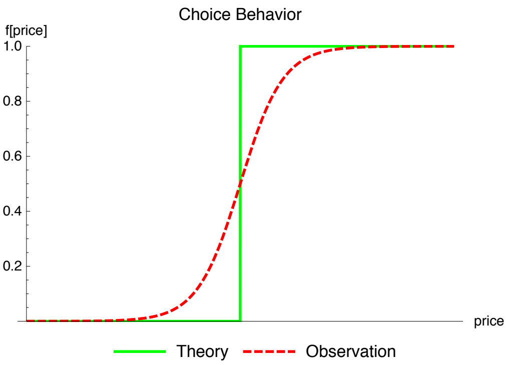

Figure 1: Conventional choice theory predicts a sharp step-function response in the frequency with which subjects sell a good depending on the price offered, according to the green curve. Observations invariably show a logistic quantal response to price, according to the red dashed curve, governed by a parameter that expresses responsiveness.

Quantal response and "rational choice"

- Logistic quantal response behavior can be regarded as a generalization of rational choice theory, in so far as the decision-maker, as in rational choice theory, has well-defined payoffs over actions, and maximizes expected utility in choosing a mixed strategy, which results in more frequent choices of higher-payoff actions.
- The new element in entropy-constrained behavior is the behavior temperature, which limits the degree to which the decision-maker can concentrate frequency on the highest-payoff action.

Approximation

- It is tempting to think that because entropy-constrained logistic quantal response behavior approximates full payoff-maximizing behavior the converse is true, so that models assuming full payoff-maximizing behavior are reliable guides to entropy-constrained behavior in the real world.
- But this logic does not hold, because the limiting case where responsiveness goes to infinity has important qualitative differences from entropy-constrained behavior at any finite responsiveness.
- For example, at any finite responsiveness the entropy-constrained model predicts that we will observe every available action with some positive (though possible very low) frequency.
- But unconstrained payoff-maximization predicts that we will observe only payoff-maximizing actions.

Spontaneous trading in conventional theory

- A striking example of the implications of quantal response behavior is the question of whether identical agents who have the same valuation of a tradable asset will ever transact with each other.
- In the case where agents are payoff-maximizers without an entropy constraint, identical agents have no incentive to transact because there are no potential gains from trade between them.

Spontaneous trading with entropy-constrained behavior

- The case of entropy-constrained agents is, however, qualitatively different.
- Suppose the typical agent values the good at μ, has a payoff of buying (selling) a unit of the good at price p equal to $u[p] = \mu - p$, which is her consumer surplus when her valuation of the good is μ and operates at a responsiveness $\beta < \infty$.
- Then the frequencies of buying, $f[p]$, and selling, $1 - f[p]$, at price p are:

$$f[p] = \frac{1}{1 + \exp[-\beta(\mu - p)]} \tag{5}$$

$$1 - f[p] = \frac{1}{1 + \exp[\beta(\mu - p)]]} \tag{6}$$

Conditions for transactions

- The frequency of a transaction at price p is $f[p](1-f[p])$, the frequency with which one agent buys at that price and a counterpart agent sells.
- The frequency of transactions at price p, $\tau[p]$ is, assuming $\mu = 0$, so that p represents the difference between the transaction price and the typical agent's value of the good:

$$\tau[p] = \frac{1}{1+\exp[\beta p]} \frac{1}{1+\exp[-\beta p]} = \frac{1}{2+2\cosh[\beta p]}$$

Visualizing noise trading

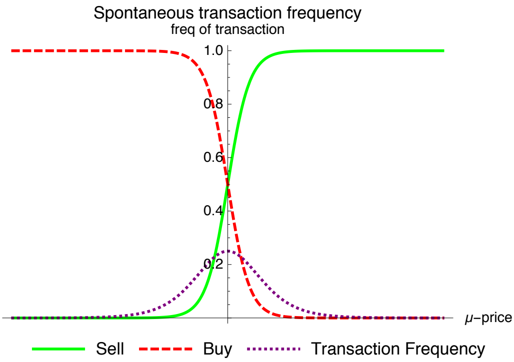

Figure 2: Identical transactors with the same valuation of a good or asset and the same positive behavior temperature will generate non-zero spontaneous transactions due to quantal response effects. The frequency of transactions is equal to $\frac{1}{\beta}$, the area under the frequency curve.

Intensity of noise trading and responsiveness

- Integrating the frequency over all prices shows that the frequency of spontaneous transactions among entropy-constrained transactors who have the same valuation of a good or asset and operate is equal to the inverse of their responsiveness, $\frac{1}{\beta}$.
- Entropy-constrained behavior provides an explanation for the widely-observed phenomenon of "noise-trading" in asset and other markets.

Social interaction

- Another important consequence of responsiveness is the multiplicity and stability of equilibria in a social interaction scenario.
- When the frequency with which other identical agents choose some action influences the frequency with which a typical agent chooses that action, responsiveness determines the number and stability of equilibria (which occur on the 45° line where all agents act identically).

Visualizing social interaction

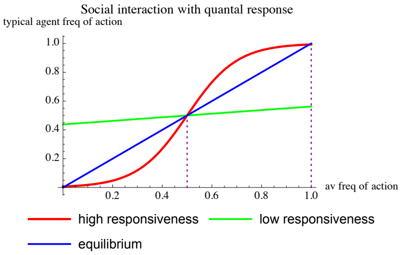

Figure 3: The horizontal axis measures the frequency with which other agents choose the action and vertical axis the frequency with which the typical agent's best response is to choose the action, represented by two quantal response curves. The green quantal response represents a low responsiveness: the typical agent does not respond very much to the behavior of other agents, and there is a single stable interior equilibrium. The red quantal response represents a high responsiveness: the typical agent responds sensitively to the behavior of other agents and the interior equilibrium becomes unstable and bifurcates into two stable extreme equilibria. Changes in responsiveness can transform an interaction from a Prisoners' Dilemma-like single equilibrium interaction into an Assurance Game-like multiple equilibrium interaction with path dependency.

Social coordination and responsiveness

- In this scenario the agent responds not to an offered price, but to the average behavior of other identical agents, measured by the frequency of their choosing some action.
- Because the agents are identical, equilibria occur where the quantal response frequency of the typical agent is equal to the frequency of the other agents, represented by the 45° line in Figure 3.
- When responsiveness of the typical agent is low, she does not respond much to other agents' actions, and there is a single stable interior equilibrium

Prisoners' dilemma outcomes

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- Given the strategic complementarity implied by the upward-sloping quantal best response function, this equilibrium will in general be a Prisoners' Dilemma–like outcome with agents choosing the action too rarely.

Assurance game outcomes

- When the responsiveness of the typical agent is high enough, her quantal best response cuts the equilibrium locus from below at the interior equilibrium, which becomes unstable, bifurcating into two stable extreme equilibria. In typical situations, one or the other of the stable equilibria is preferred by the typical agent, but which one prevails depends on the initial starting point so that the system is a path-dependent Assurance Game-like interaction.
- A change in responsiveness transforms the Prisoners' Dilemma-like interaction into an Assurance Game-like interaction.

A Markov process social interaction model

- The quantal response of the typical agent induces a Markov chain on the state space of profiles of agent behavior in the social interaction model.
- Given any starting profile of agent actions determining the average frequency of the action, the quantal best response of the typical agent determines the frequency with which any agent chooses the action in the next period, andthe frequency of any particular profile of agent actions in the next period.
- If there are n agents each with responsiveness β and a payoff $\mu - P$, where P is the average frequency of agents choosing the action, the state of the system in any round of interaction can be described as the number of agents choosing the action, $k = 0, \ldots, n$, the average frequency of taking the action will be $P = \frac{k}{n}$, the frequency with which each agent will choose the action is $f[P] = \frac{1}{1+exp[\beta(\mu-P)]}$ and the transition probabilities from state k to state k', $t_{k,k'}$ are:

$$t_{k,k'} = \left(\begin{array}{c} n \\ k' \end{array} \right) f[\frac{k}{n}]^{k'}(1 - f[\frac{k}{n}])^{n-k'} \tag{7}$$

- The resulting Markov chain has an ergodic distribution, which describes the long run evolution of the system, as Figure 4 illustrates.

Visualizing the ergodic distribution of a social interaction

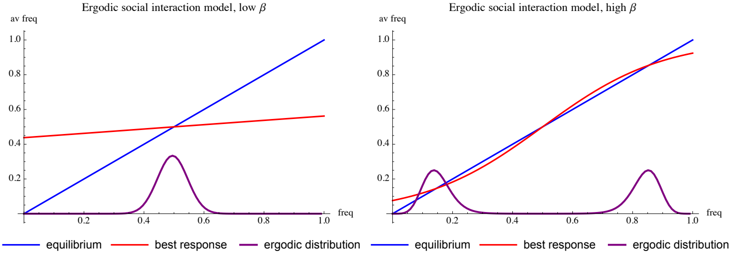

Figure 4: Ergodic distributions of the social interaction model at low and high responsiveness. The red line is the best response, which is flat for low responsiveness, and steep for high responsiveness. The blue line is the equilibrium locus. The purple distribution dots represent the ergodic distribution induced by the social interaction in a population of $n = 100$. For low responsiveness the ergodic distribution is centered on the interior stable equilibrium. For high responsiveness the ergodic distribution is bi-modal concentrated on extreme outcomes in which most of the agents choose either to take or not to take the action. In the high responsiveness case there is a positive frequency of transition between the extreme quasi-equilibrium configurations.

Dynamics and equilibrium through Markov ergodic distributions

- The ergodic distribution for low responsiveness social interactions is centered on the interior stable equilibrium, but there is a finite frequency for any outcome due to the fact that the quantal response has a non-zero entropy.
- The ergodic distribution for high responiveness social interactions is bi-modal, with concentrations at quasi-equilibria corresponding to the extreme stable equilibria.
- Again, due to the fact that the quantal response has non-zero entropy, there is a finite frequency of transition between the extreme quasi–equilibria.