Punctuated non-equilibrium and niche conservatism explain biodiversity fluctuations through the Phanerozoic

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Abstract

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Fluctuations in biodiversity, both large and small, are pervasive through the fossil record, yet we do not understand the processes generating them. Here we use a novel extension of theory from non-equilibrium statistical physics to show that three universal properties of macroevolution, punctuated adaptive radiation, niche conservatism and resultant heterogeneity of diversification rates between taxa, are sufficient to explain previously unaccounted for biodiversity patterns throughout the Phanerozoic. Using this theory, known as super-statistics, we identify taxonomic orders as largely autonomous evolutionary units, each likely experiencing its own unique and conserved adaptive landscape. This indicates that while neutral processes could adequately explain within-order diversification, between-order diversification is likely driven by major evolutionary innovations. Compared to other approaches that have used simple birth-death processes, equilibrial dynamics or non-linear theories from complexity science, super-statistics is superior in its ability to account for both small and extreme fluctuations in fossil diversity. Its success opens up new research directions to better understand the universal nature of non-equilibrium dynamics across disperate systems of interest—from societal to physical to biological—and, specifically in the biological case, to understand the evolutionary processes leading adaptive landscapes to be conserved within orders and undergo punctuated innovations between orders.

Biodiversity has not remained constant nor followed a simple trajectory through geologic time^{1–6}. Instead, it has been marked by fluctuations in the number of extant taxa, both positive in the case of net origination or negative in the case of net extinction. Major events, such as adaptive radiations and mass extinctions have received special attention^{7,8}, but fluctuations of all sizes are ubiquitous^{2,5,9}. Predicting the magnitude of these fluctuations continues to illude paleobiologists and biodiversity theoreticians.

Several approaches have been taken to study the complex trajectory of paleo-biodiversity ranging from the hypothesis that biological systems self-organize to the brink of critical phase-transitions^{10,11} to invocations of non-linear environmental perturbations¹² and escalatory co-evolutionary interactions¹³. New data and analyses have not supported any of these hypotheses at the scale of the entire Phanerozoic marine invertebrate fauna^{5,14,15}. Other studies have modeled the mean trend in diversity as tracking a potentially evolving equilibrium^{2,5,6,16} and yet ignore the potential role of stochasticity and non-equilibrium dynamics in producing observed patterns^{4,9,17}. As such, we still lack a synthetic theory of evolving biodiversity through the fossil record.

Despite the heterogeneity of explanations of Phanerozoic biodiversity, consensus has emerged on three properties of macroevolution: i) gross ecological and life history attributes of clades are often maintained, a phenomenon known as niche conservatism^{18,19}; ii) long periods of niche conservatism are interrupted by adaptive diversification and exploration of new ecological niche space^{19–21}; and iii) as a consequence of the interaction between their life history characteristics and the environments they inhabit²² (conserved through niche conservatism) different clades experience different rates of morphological evolution, speciation and extinction^{2,3,23,24}.

Here we show that these simple and well-supported mechanisms are all that are needed to describe pervasive fluctuations in diversity throughout the marine Phanerozoic. These biological mechanisms have a precises correspondence to non-equilibrial theory, known as "superstatistics" derived in statistical mechanics²⁵ and applied across the physical and social sciences^{26,27}. We leverage this correspondence to derive a robust prediction of the distribution of fluctuations in the standing diversity of marine invertebrates preserved in the Phanerozoic fossil record.

Superstatistics²⁵ proposes that non-equilibrial systems can be decomposed into locally equilibrial sub-systems. The distribution of equilibria across sub-systems determines the dynamics of the complete system²⁵. When these sub-systems are superimposed the resulting system can no longer be described by a single equilibrial model. In the context of macroevolution we propose that a clade with conserved life history characteristics corresponds to a locally equilibrial sub-system. If a certain region of niche space can only contain a finite diversity of taxa^{16,23,28,29} then diversity within clades should fluctuate stochastically about this equilibrium due to random origination and extinction. The magnitude of these macroevolutionary rates should be a function of the life history and ecological characteristics that define that region of niche space. Larval type³⁰, body plan¹⁷, body size³¹, range size^{31,32} and substrate preference¹⁹ have all been shown to influence such rates. Thus different regions of niche space, and the clades occupying them, will experience different magnitudes of stochastic fluctuation in diversity. As clades occasionally split to fill new regions of niche space their punctuated diversification determines the non-equilibrium nature of the entire biota.

To uncover the superstatistical nature of the marine invertebrate Phanerozoic fauna we

analyze the distribution of diversity fluctuations using two canonical databased of fossil biodiversity, the Paleobiology Database⁵ and Sepkoski's compendium³³ of fossil marine invertebrates (results from Sepkoski's compendium are presented in Appendix 2.3). We define potentially equilibrial sub-systems based on taxonomic hierarchies, as a full phylogenetic hy-68 pothesis for all marine invertebrates is lacking. Taxa ideally represent groups of organisms that descend from a common ancestor and share similar ecologically and evolutionary rele-70 vant morphological traits^{34,35}. For Phanerozoic marine invertebrates, the taxonomic level of 71 orders is a likely candidate for equilibrial sub-system delineation²⁴. However, to evaluate the 72 optimal taxonomic level for sub-system designation, we test our superstatistical theory using 73 taxonomic levels from order to phylum. Additionally, we compare our results to randomized taxonomies and confirm that the observed fit of superstatistical theory is not an artifact of ar-75 bitrary classification but instead represent real, biologically relevant diversification processes within and between clades.

Results

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In statistical mechanics, local sub-systems can be defined by a simple statistical parameter β often corresponding to inverse temperature. In the context of macroevolution we define the β_k of clade k as the inverse variance of fluctuations x_k in the number of genera within that clade. The β_k thus represent the inverse variances of homogeneous origination-extinction processes, which will be approximately Gaussian if clades' diversification dynamics are independent and in local equilibrium (see Appendix 1). Independence could result from neutral-like processes³⁶, where the dynamics of one taxon are unaffected by those of another, or from dampening mechanisms that stabilize complex networks of interacting taxa³⁷. This is in direct contrast to the instability hypothesis underlying the self-organized criticality theory of paleo-biodiversity^{10,11}. Three exemplar dynamics taken from a bias-corrected (see methods section) aggregation of the Paleobiology Database (PBDB)⁵ are shown in Figure 1, and all diversity fluctuations within orders are well characterized by a Gaussian distribution 1.

To predict the super-statistical behavior of the entire marine invertebrate Phanerozoic fauna we must integrate over all possible local equilibria that each clade could experience. The stationary distribution of β_k values describes these possible equilibria, specifying the probability that a given clade, chosen at random, will occupy a region of niche space characterized β_k . The form of this stationary distribution could shed interesting light on the biological processes that lead different clades to explore different regions of adaptive land-scapes, and thus different equilibria, as discussed below.

We estimate the distribution of β_k 's simply as the maximum likelihood distribution describing the set of inverse variances for all orders. In both the PBDB and Sepkoski's compendium, Phanerozoic marine invertebrate orders clearly follow a Gamma distribution in their β_k values (Fig. 1).

Using the observation of within order statistical equilibrium and Gamma-distributed β_k parameters we can calculate, without further adjusting free parameters, the distributions of order-level fluctuations for the entire marine Phanerozoic, P(x), as

$$P(x) = \int_0^\infty p_k(x \mid \beta) f(\beta) d\beta \tag{1}$$

where $p_k(x \mid \beta)$ is the distribution of fluctuations within an order and $f(\beta)$ is the stationary distribution of inverse variance in the magnitude of order-level fluctuations in diversity. This leads to a non-Gaussian, fat-tailed prediction for P(x) which matches both the PBDB and Sepkoski data closely (Fig. 2 and Appendix 2.3).

To quantitatively evaluate how well the super-statistical prediction matches the data we constructed a 95% confidence envelope from bootstrapped maximum likelihood estimates for P(x). Observed fluctuations fall within this 95% confidence envelope (Fig. 2), indicating that the data do not reject the super-statistical prediction. For further comparison, we fit a Gaussian distribution to the observed fluctuations, which corresponds to the equilibrium hypothesis that all orders conform to the same statistic. Using Akaike Information Criterion (AIC) we find that observed fluctuations are considerably better explained by the super-statistical prediction than by the Gaussian hypothesis (Δ AIC = 11285.18). Thus, as expected under the superstatitical hypothesis, the fat tailed distribution of fluctuations arise from the superposition of independent Gaussian statistics of fluctuations within orders.

Computing the distribution of fluctuations using classes instead of orders leads to a substantially poorer fit to the observed data (Fig. 4). We quantify this shift with the Kolmogorov-Smirnov statistic, which changes from 0.041 for orders to 0.062 for classes (Fig. 3). However, if super-statistical theory explains the data, this worsening fit with increasing taxonomic scale is expected as the different classes are not well defined subsystems in their fluctuation dynamics. Instead, classes aggregate increasingly disparate groups of organisms, and thus effectively mix their associated Gaussian fluctuations, meaning that one statistic should no longer be sufficient to describe class-level dynamics. Our analysis indicates that orders are evolutionarily coherent and independent entities, with all subsumed taxa sharing key ecological and evolutionary attributes allowing them to diversify concertedly and independently from other orders. Both the good fit at the order level and worsening fit at higher taxonomic levels is confirmed in Sepkoski's compendium, which also allows analysis of phylum-level patterns (Fig. 5).

To further test the evolutionary coherence of orders we conducted a permutation experiment in which genera were randomly reassigned to orders while maintaining the number of genera in each order. For each permutation, we calculated the super-statistical prediction and the Kolmogorov-Smirnov statistic. The permutation simulates a null model in which common evolutionary history is stripped away (genera are placed in random orders) but the total number of observed genera per order is held constant. Repeating this permutation 500 times yields a null distribution of Kolmogorov-Smirnov statistics that is far separated from the observed value (Fig. 3) suggesting the good fit at the order level is not merely a statistical artifact of classification but carries important biological information.

141 Discussion

Our analysis makes no assumption that orders should correspond to super-statistical subsystems, but identifies them as the appropriate level for marine invertebrates. Holman²⁴
has also shown that orders are "evolutionarily coherent" in that subtaxa within orders share
common diversification dynamics. As we show, orders differ only in the variances of their
diversity fluctuations (Fig. 1).

Our study is the first to demonstrate that complex patterns in the sequence of origination and extinction events in the fossil record are the result of a simple underlying process analogous to the statistical mechanisms by which complexity emerges in large, non-equilibrium physical²⁶ and social systems²⁷. We do so by identifying the biological scale at which clades conform to equilibrial dynamics. This scale is likely determined by the process of niche conservatism^{18,19} within orders, and equilibrium the likely outcome of neutrality or processes that dampen—rather than exacerbate—fluctuations in complex ecological networks³⁸. We then show that punctuated shifts to different equilibria between order, a consequence of punctuated exploration of niche space by newly evolving clades^{19–21}, leads to a characteristically non-equilibrial distribution of diversity fluctuations when the marine Phanerozoic fauna is viewed as a whole macro-system.

The stationary distribution describing this process of punctuated non-equilibrium is clearly Gamma. A Gamma distribution, while consistent with multiple processes³⁹, could result from evolution of diversification rates across an adaptive landscape that promotes niche conservatism and punctuated exploration of niche space. Specifically, if β_k values are associated with a clade's physiological and life history traits, and those traits evolve via Ornstein-Uhlenbeck-like exploration of an adaptive landscape, the resulting stationary distribution of β_k will be Gamma^{39,40}.

Our work highlights the importance of both niche conservatism and punctuated adaptive radiation in producing the statistical behavior of the Phanerozoic; our theory thus provides new motivation for identifying the eco-evolutionary causes of innovations between lineages and how those innovations are eventually conserved within lineages. Armed with an understanding of the statistical behavior of diversification we can go on to examine mechanisms underlying additional patterns in the mean trend of biodiversity through the Phanerozoic. In particular, clades have been shown to wax and wane systematically through time^{4,9}, a pattern that we cannot explain with super-statistics alone.

Superstatistics could also be applied to other areas of evolution and macroecology. For example new phylogenetic models already consider heterogeneous rates of diversification⁴¹. The superstatistics of clades in adaptive landscapes could provide a means to build efficient models that jointly predict morphological change and diversification. This framework could also provide a new paradigm in modeling the distributions of diversity, abundance and resource use in non-neutral communities. Non-neutral models in ecology are criticized for their over-parameterization rosindell2011TREE, yet a persistent counter argument to neutral theory³⁶ is the unrealistic assumption of ecological equivalency⁴² and poor prediction of real dynamics⁴³. If ecosystems are viewed as the super-position of many individualistically evolving clades, each exploiting the environment differently and thus obeying a different set of non-equivalent statistics, then diversity dynamics could be parsimoniously predicted with superstatistics while incorporating real biological information on ecological differences between taxa.

Methods

Paleobiology Database data download and filtering

Data were downloaded from the Paleobiology Database (PBDB; www.pbdb.org) on 28 May 2013. Collections were filtered using the same approach as Alroy⁵ to insure that only well preserved marine invertebrate occurrences were used in subsequent analysis resulting in 221202 genus occurrences. These were further filtered to exclude those occurrences without order-level taxonomy and those collections with age estimate resolutions outside the 10my default bins of the PBDB resulting in 189516 occurrences left for analysis. To avoid basic sampling concerns we excluded the earliest Cambrian and the Cenozoic.

To focus attention on the variance of fluctuations we center each clade's fluctuation distribution. Because "equilibrium" in the statistical mechanical sense means a system undergoes coherent, concerted responses to perturbation the mean trend line is of less interest than deviations from it. We also note that most clades are already close to centered and so centering has little influence on clade-specific fluctuation distributions.

Three-timer and publication bias correction

We use a new and flexible method, described below, to correct for known sampling biases in publication-based specimen databases^{5,6}. We were motivated to use this method because rarefaction has been shown to under-perform compared to the more data-intensive shareholder quorum subsampling (SQS) method⁶. However, subsampling cannot be applied to small orders (i.e. the majority) because SQS becomes increasingly unreliable as sample size decreases⁶. We therefore develop a simple method based on first correcting for detection bias using the "three timer" correction⁵ in which the rate of failure to observe a genus is estimated by the number of times a gap occurs in the occurrence history of each genus. To eliminate further bias due to preferential publication of novel taxa we divide observed number of genera per order per time period by the expected number of genera given publications in that time period. The expected number is calculated by regressing the log-transformed number of genera on log-transformed number of publications. There is a weak trend toward higher diversity with more publications (Fig. 1) meaning that the most important correction comes from the three timer correction.

Our new method effectively re-scales each genus occurrence from 0/1 (absent/present), to a weighted number continuously ranging between 0 and 1. This method achieves similar results to more computationally intensive sub-sampling procedures^{5,6}. We directly compare our predicted time series of global genus diversity with results derived from SQS⁶ and the raw data (Fig. 2). Our method shows minor differences with the SQS prediction, However, these discrepancies do not have impact the distribution of fluctuations (Fig. 2) and superstatistical analysis on uncorrected PBDB data (see section 2.1) produces a similar result to the analysis on corrected PBDB data presented in the main text.

Numerical methods

To fit our super-statistical prediction we use the method of least squares instead and maxi-224 mum likelihood. When building the prediction for P(x) by calculating order-level Gaussian distributions and integrating over them, we use least squares to fit the variance term to each 226 order. We do so because orders potentially show asymmetries in their distribution of fluctuations. Least squares is more flexible in fitting such distributions compared to maximum 228 likelihood which will always estimate the empirical variance as the best-fitting parameters. 229 We also estimate P(x) directly from the raw data using maximum likelihood to compare 230 the fit of our super-statistical prediction and that of a simple Gaussian distribution using 231 AIC. To calculate a likelihood-based confidence interval on our prediction we bootstrapped

the data, subsampling fluctuations with replacement from all orders combined.

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332 Figures

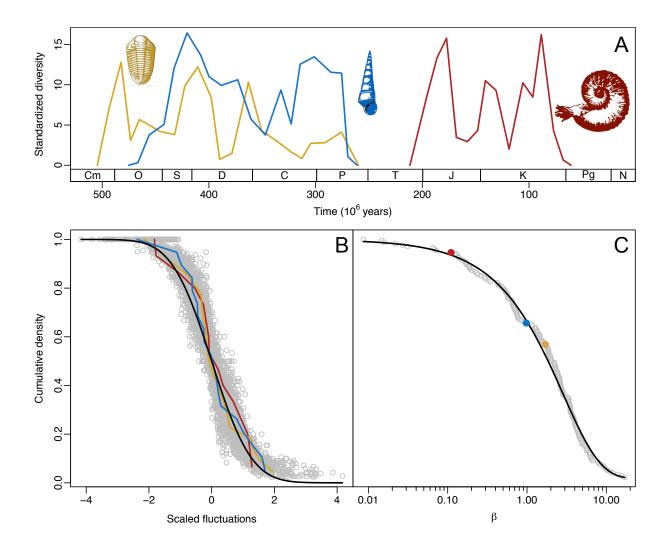


Figure 1: The distributions of within-order fluctuations in genus diversity shown for the trajectories of three exemplar orders (A) and shown as an empirical cumulative density aggregated across all orders (B). To display all orders simultaneously we simply collapse their fluctuation distributions by dividing by their standard deviations. If orders conform to the Gaussian hypothesis their scaled fluctuations should fall along the cumulative density line of a normal N(0, 1) distribution, as shown in (B). In (C) the distribution of inverse variances β_k across all orders matches very closely to a Gamma distribution (black line); exemplar orders are again highlighted.

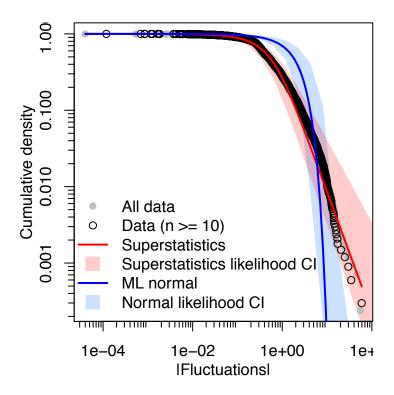


Figure 2: Distribution of fluctuations in genus diversity within orders of marine invertebrates in the Paleobiology Database⁵ after bias correction. The distribution is fat-tailed as compared to the maximum likelihood estimate of the normal distribution (blue line). At the order level the empirical distribution of fluctuations are well described by our super-statistical approach, both when computed from integrating over the distribution of observed variances (red line) and when fit via maximum likelihood (95% confidence interval; red shading).

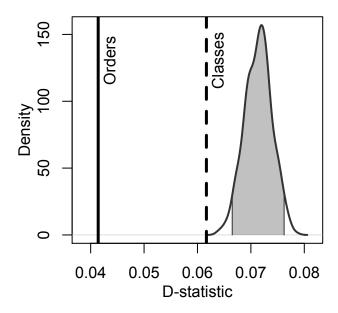


Figure 3: Distribution of Kolmogorov-Smirnov (KS) statistics from randomly permuting genera within orders (gray shading represents 95% confidence interval). Solid black line is observed KS statistic at the order level, while the dashed black line shows the observed KS statistic at the class level.