GENERAL ASSEMBLY

INTRO TO DATA SCIENCE
NETWORKS & GRAPHS

I. NETWORKS
II. NETWORK STATICS
III. NETWORK DYNAMICS

IV. EXERCISE

I. NETWORKS

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Given a large set of social network data, what types of questions do you think would be interesting to ask?

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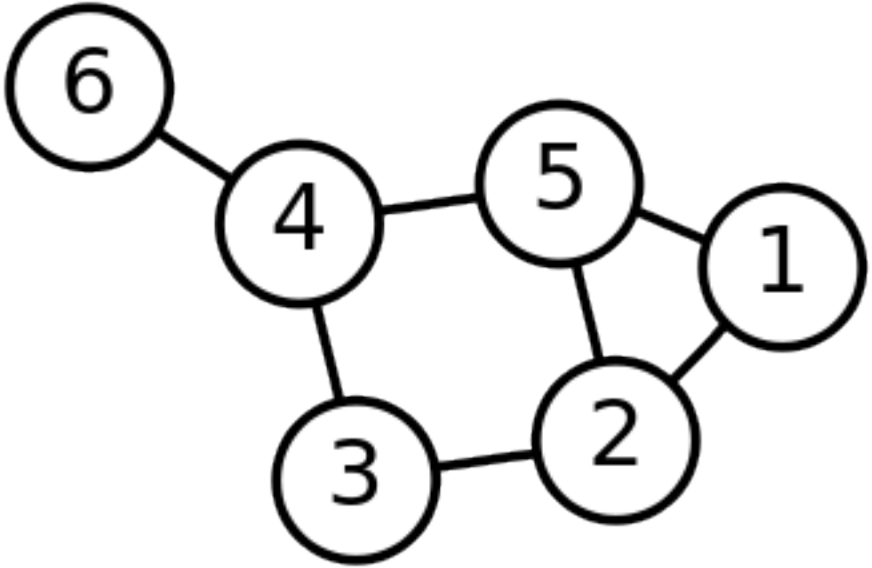
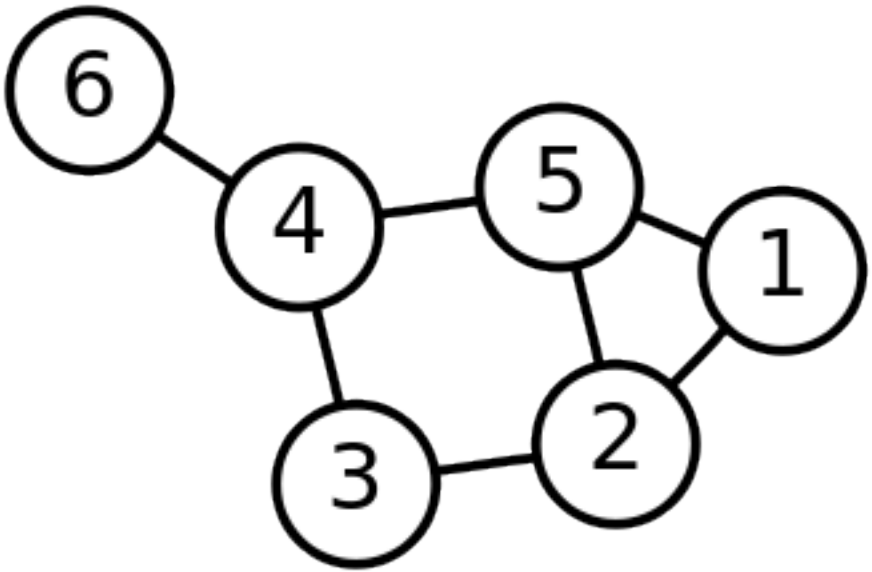
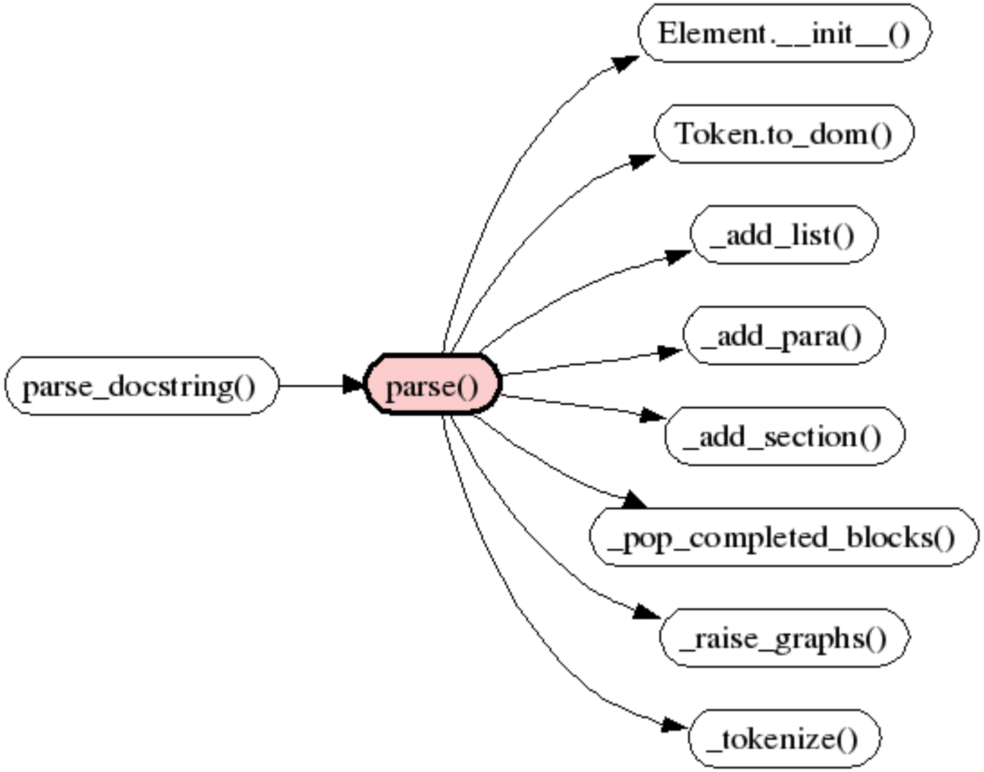
These are questions about network *behavior*.

The mathematical representation of a network is an object called a **graph**, which is a configuration of *nodes* connected by *edges*.



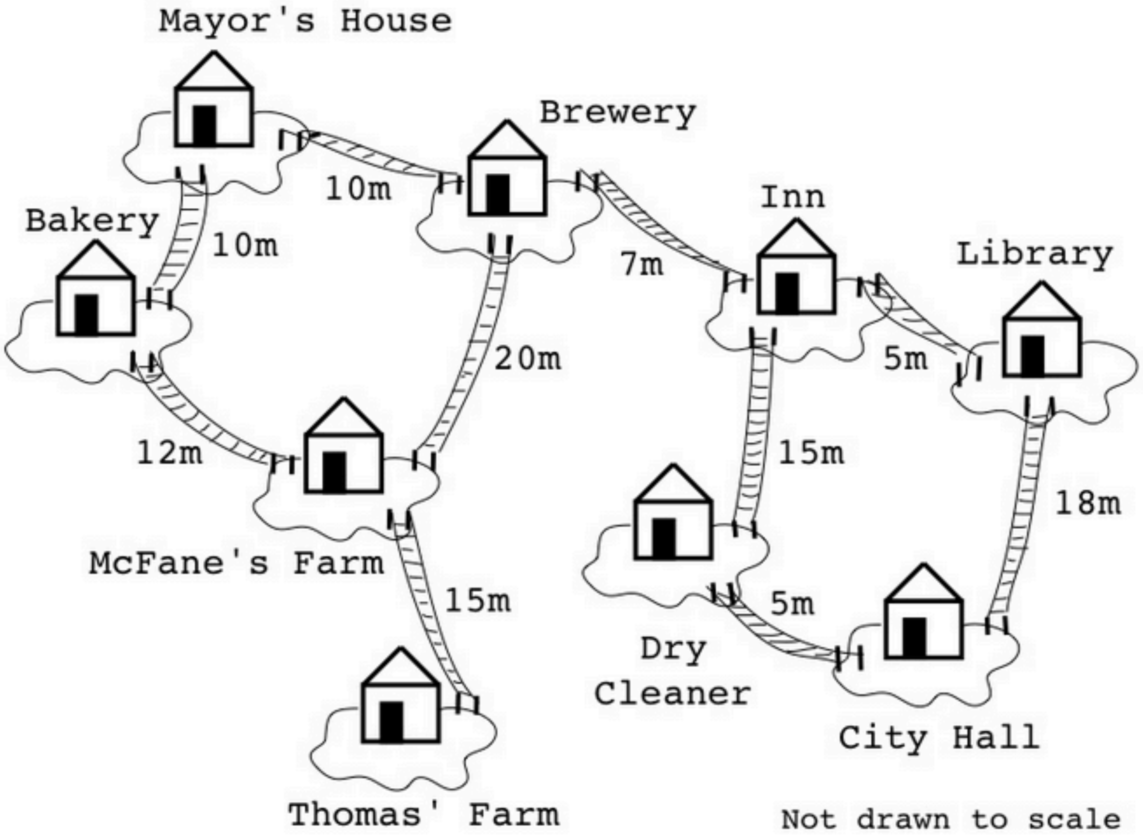
Nodes represent *actors* in the graph, and edges represent the *relationships* between actors.





NOTE

A *directed graph* has edges that point from one node to another.



NOTE

A *weighted graph* contains edges associated with real-valued numbers, e.g., to measure distance or importance.

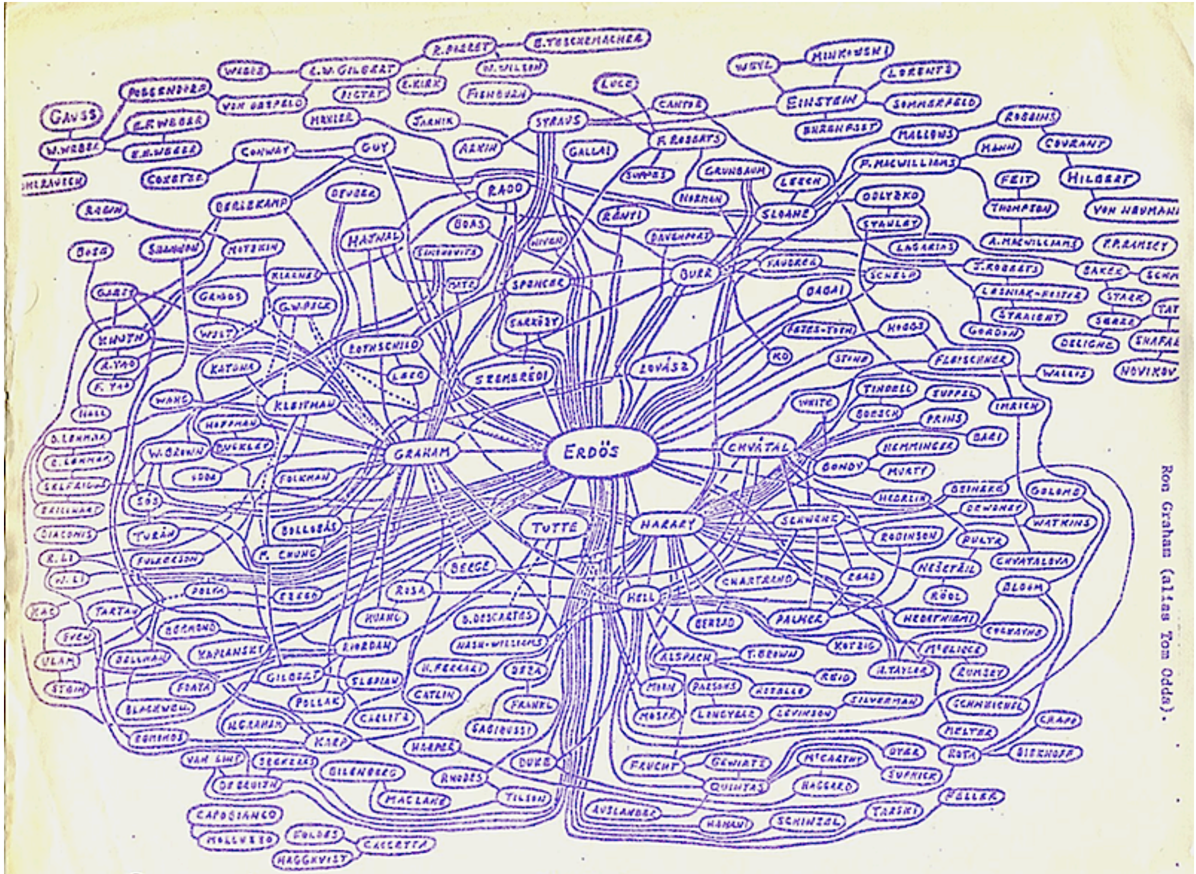
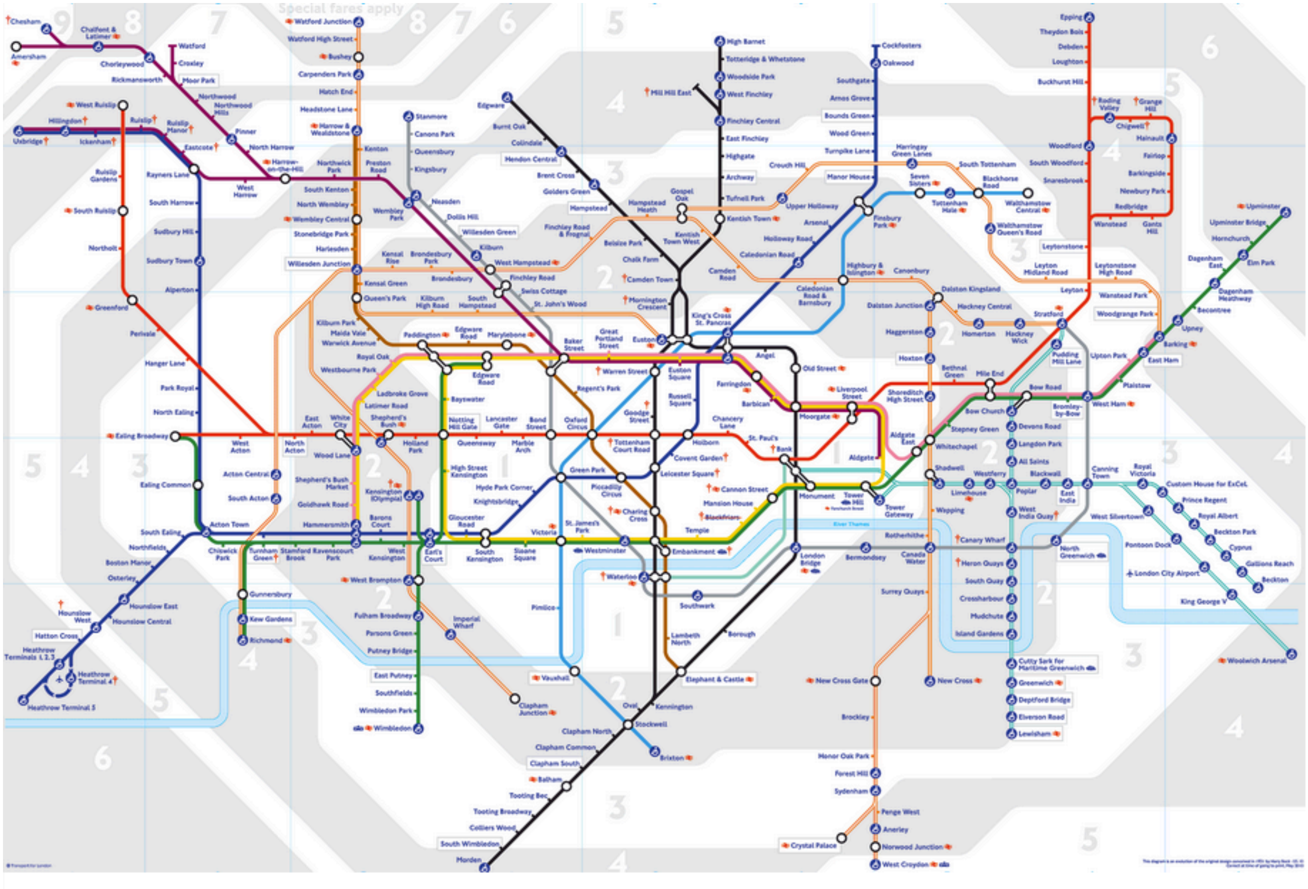


Figure 1

To appear in Topics in Graph Theory (F. Harary, ed.) New York Academy of Sciences (1979).

NETWORK REPRESENTATION



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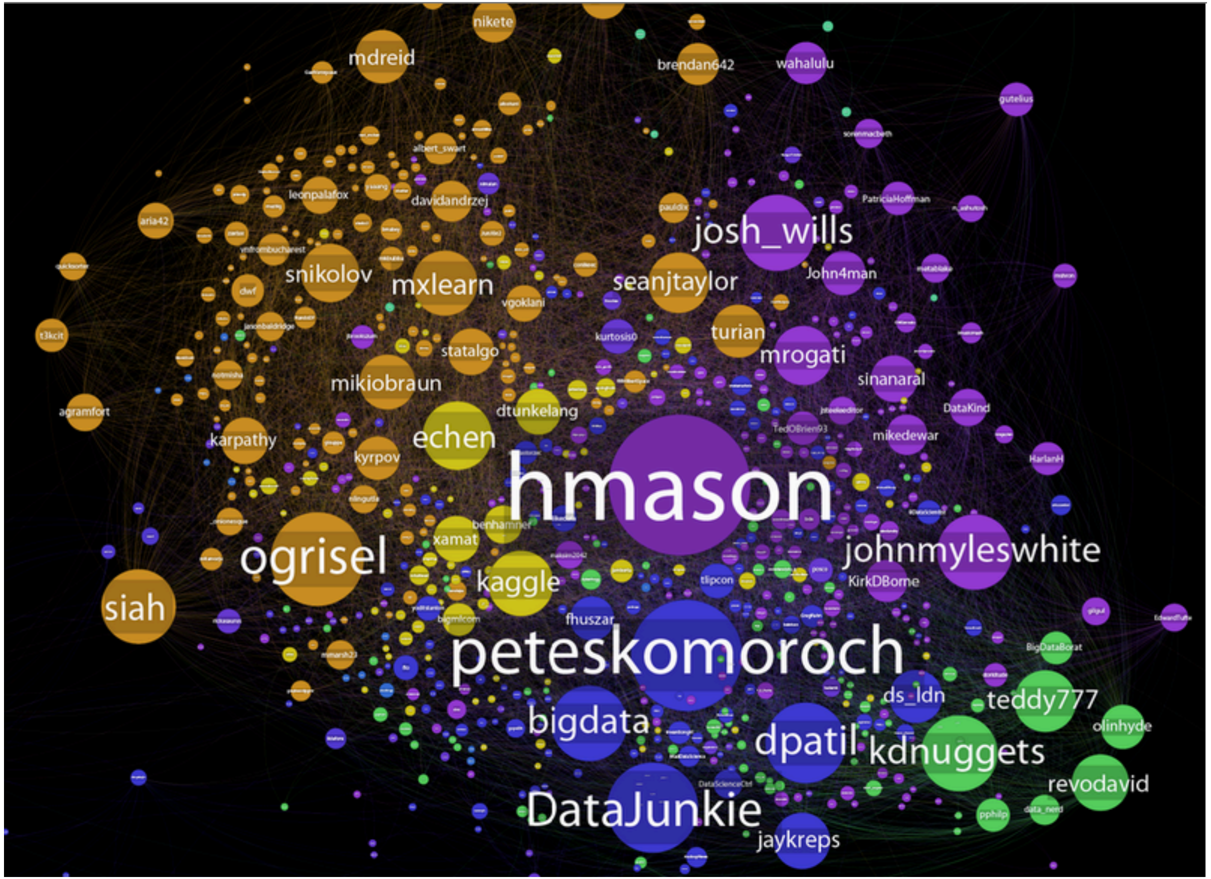
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vgoklani
aria42
t3kcit
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John4man
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HarlanH
BigDataBorat
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In practical terms, we need some data structures to represent and manipulate our network data.

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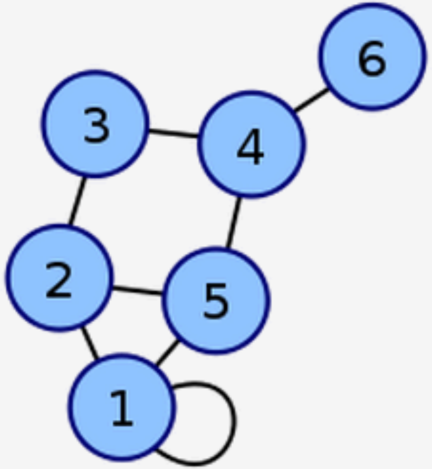
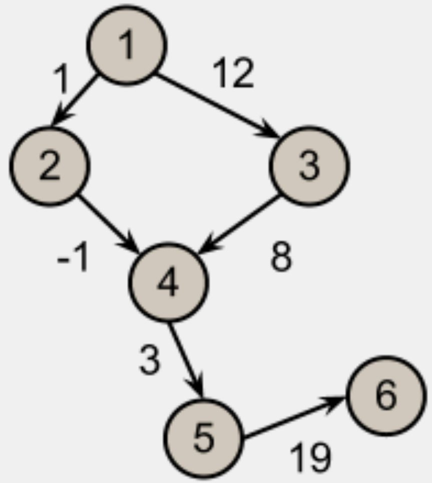
One common graph representation is the **adjacency matrix**. An n-node undirected graph can be represented by a symmetric n x n adjacency matrix A whose nonzero off-diagonal entries A_{ij} represent an edge between nodes i and j.

NETWORK REPRESENTATION

Labeled graph	Adjacency matrix
	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$ Coordinates are 1-6.

Weighted Directed Graph & Adjacency Matrix



Weighted Directed Graph

	1	2	3	4	5	6
1	0	1	12	0	0	0
2	-1	0	0	-1	0	0
3	-12	0	0	8	0	0
4	0	1	-8	0	3	0
5	0	0	0	-3	0	19
6	0	0	0	0	-19	0

Adjacency Matrix

NOTE

A directed graph has an *asymmetric* adjacency matrix.

Can you see why?

Another useful tool is the **adjacency list**. The dict here is related:

```
graph = {'A': ['B', 'C'],
         'B': ['C', 'D'],
         'C': ['D'],
         'D': ['C'],
         'E': ['F'],
         'F': ['C']}
```

Does this adjacency dict represent a directed or undirected graph?
How could you generalize this to represent a weighted graph?

II. NETWORK STATICS

One key concept in the study of network structure is centrality. The **centrality** of a node is a measure of its importance in the network.

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The simplest centrality measure is the **degree** of a node, which is simply the number of edges connected to it. Using the adjacency matrix notation for an undirected graph, we can express the degree k_i of node *i* as:

$$k_i = \sum_{j=1}^{n} A_{ij}.$$

A more sophisticated measure called **eigenvector centrality** allows important edges to give larger contributions to centrality:

$$x_i = \frac{1}{\lambda} \sum_{j=1}^{n} A_{ij} x_j,$$

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To see where the name eigenvector centrality comes from, try to imagine this relation expressed in vector notation!

Another useful centrality measure is based on the idea of shortest-distance (or *geodesic*) paths through the graph.

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If σ_{st} is the number of geodesic paths from node s to node t, and $\sigma_{st}(v)$ is the number of these paths that cross node v, then the **betweenness centrality** of node v is given by:

$$g(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

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NOTE

Betweenness centrality measures the proportion of geodesic paths passing through a node.

This gives an idea of the node's *influence* in the network.

Geodesic paths form the basis of another well-known property of networks called the *small-world effect*.

Specifically, most networks have a mean geodesic distance between nodes that is small compared to the network size as a whole.

A famous study in the 1960s asked participants to try to get a letter to a particular individual by passing it from one acquaintance to another, and found that the mean geodesic distance in this case was about 6.

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This is where the phrase “six degrees of separation” comes from.

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III. NETWORK DYNAMICS

Suppose we're interested in the idea of how information (or behavior) spreads through a network:

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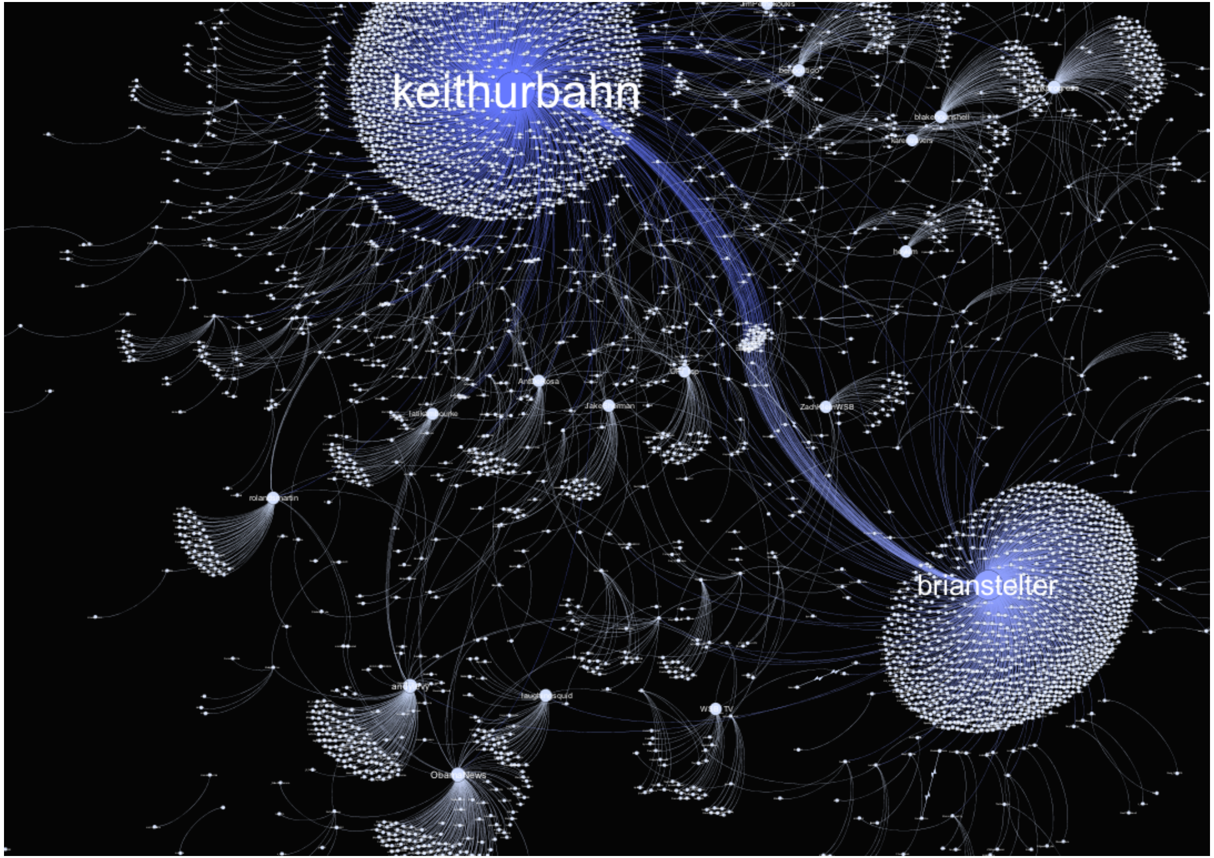
- How do members of a social network influence each other to adopt a new technology/product/behavior?
- How did information about the bin Laden raid spread over Twitter?
- What's the best way to use a social network to market your product?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informational effects – people observe the decisions of their network neighbors & gain indirect information that lead them to try the innovation themselves

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direct benefit effects – people may have incentives to use the same products/technology/etc. as their network neighbors

Studies of informational effects have shown that while initial lack of information makes innovations risky to adopt, adopters ultimately benefit.

Furthermore, *early adopters* share certain common traits (e.g., higher socio-economic status, wider travel experience), and they influence their neighbors by providing indirect information about the innovation.

Consider the following model of information **diffusion** for two nodes *v*, *w* and two behaviors *A*, *B* (with payoffs *a, b*):

		w: A	w: B
v	A	a, a	$0, 0$
	B	$0, 0$	b, b

Figure 19.1: A-B Coordination Game

The question we'd like to answer is, how can *v* maximize its payoff given that some of its neighbors adopt *A* & some adopt *B*?

To start modeling this problem, suppose first that the proportion of *v*'s neighbors selecting *A* is *p*, and the proportion selecting *B* is *(1-p)*.

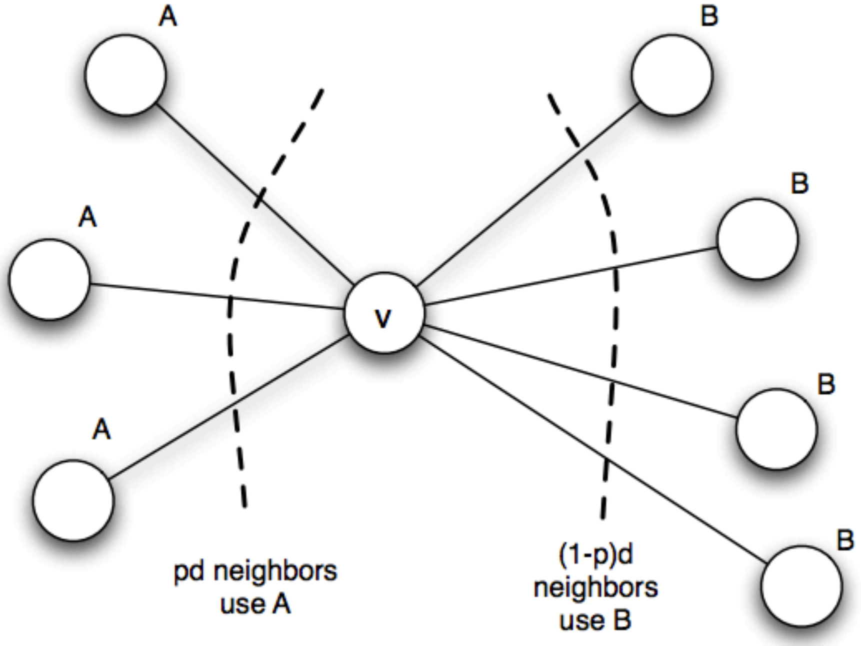

Figure 19.2: v must choose between behavior A and behavior B, based on what its neighbors are doing.

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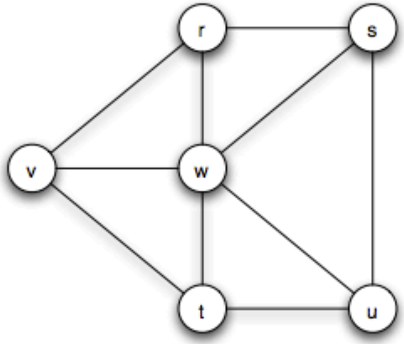
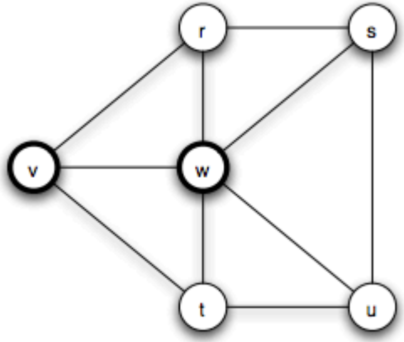
And thus v will adopt A if p (meets or) exceeds a *threshold* q:

$$pda \geq (1-p)db \quad \leftrightarrow \quad p \geq b/(a+b) = q$$

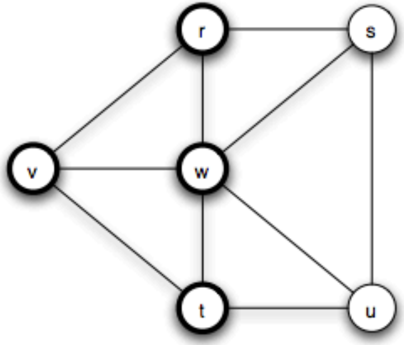
Adoption depends not only on the relative payoffs, but also on the structure of the network (eg, how many neighbors v has, and which particular nodes these neighbors are).



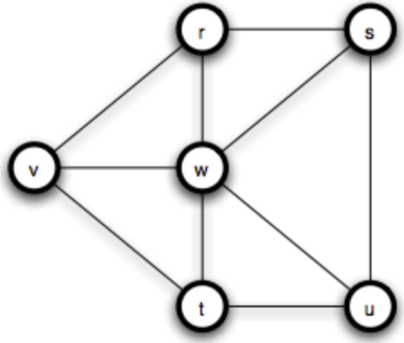
(a) *The underlying network*



(b) *Two nodes are the initial adopters*



(c) *After one step, two more nodes have adopted*

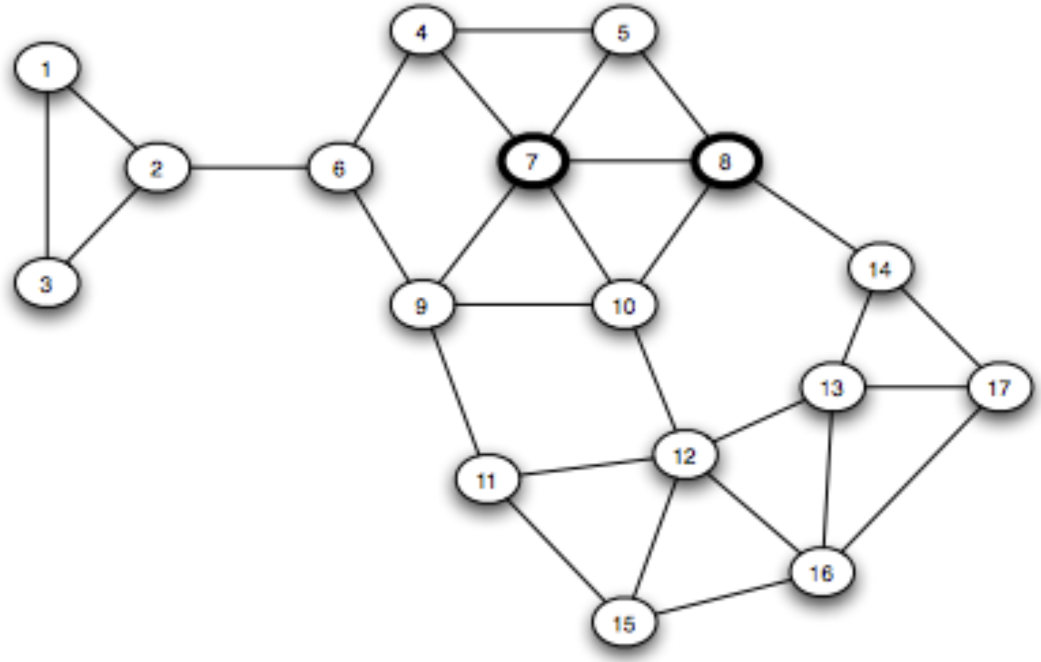


(d) *After a second step, everyone has adopted*

NOTE

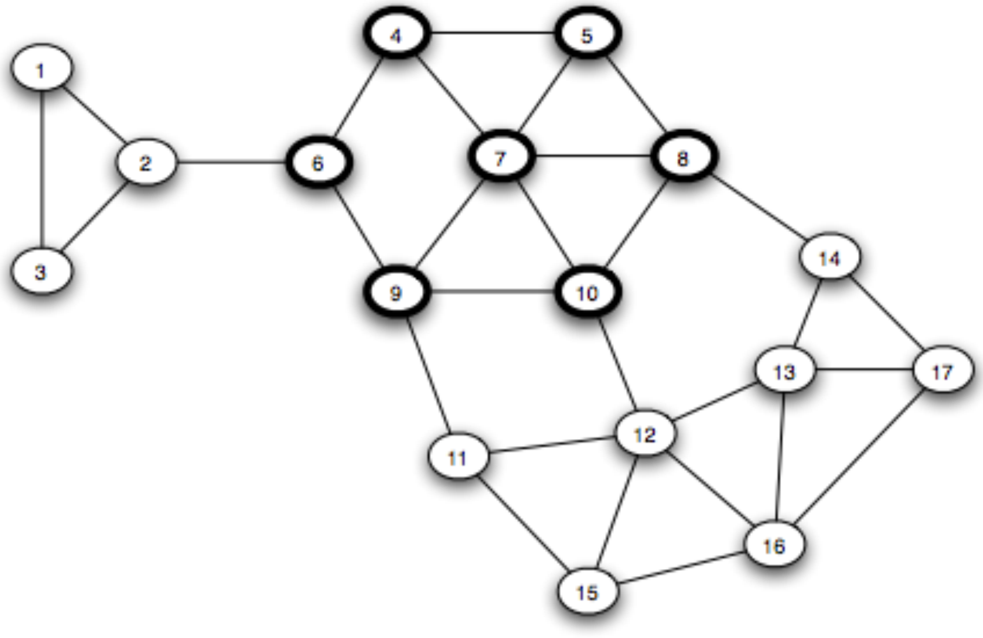
Since all nodes have adopted, this is called a *complete cascade* (at threshold q).

Consider the same diffusion now on another graph.



(a) *Two nodes are the initial adopters*

Since not all nodes adopt, this is called a *partial cascade*.



(b) *The process ends after three steps*

Here’s an interesting question: how can you identify which (non-adopting) nodes are most important to allowing the cascade to continue?

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Answering this question effectively is the idea behind *viral marketing*.

IV. EXERCISE