

#15) $y = e^{ax} \sin x$, $a \in \mathbb{R}$

$$y'' - 2y' + 2y = 0.$$

$$y' = ae^{ax} \sin x + e^{ax} \cos x = e^{ax} (a \sin x + \cos x)$$

$$y'' = a^2 e^{ax} \sin x + ae^{ax} \cos x + ae^{ax} \cos x + e^{ax} (-\sin x) = e^{ax} (a^2 \sin x - \sin x + 2a \cos x)$$

$$e^{ax} (a^2 \sin x - \sin x + 2a \cos x) - 2e^{ax} (a \sin x + \cos x) + 2e^{ax} \sin x = 0$$

$$\Rightarrow e^{ax} \neq 0 \Rightarrow a^2 \sin x - \sin x - 2a \sin x + 2 \sin x = 0.$$

$$\Rightarrow (a^2 - 2a + 1) \sin x = 0, \forall x \in \mathbb{R}$$

$$\Rightarrow a^2 - 2a + 1 = (a-1)^2 = 0$$

$$\therefore a = 1.$$