ENGS 199.12

GEOPHYSICAL FLUID DYNAMICS

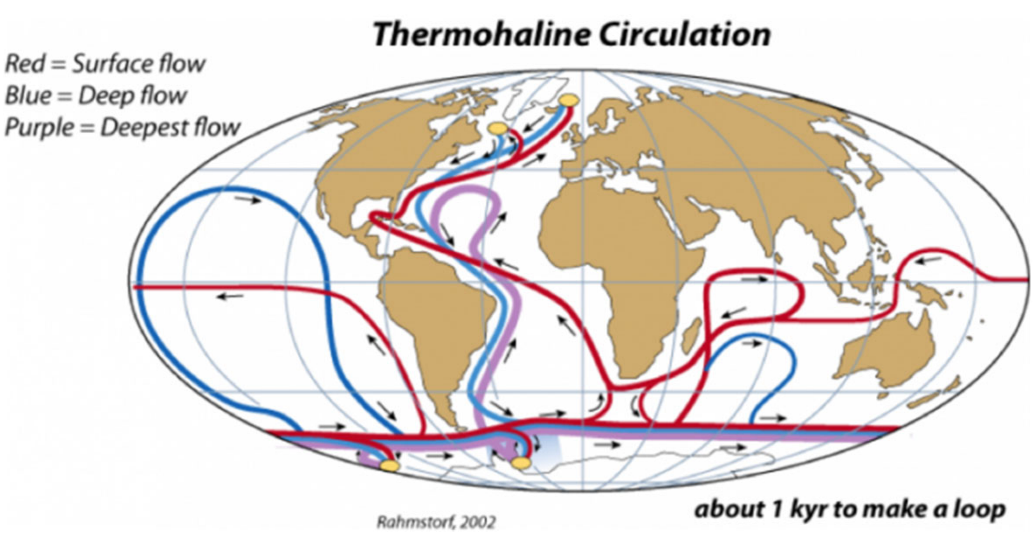
Chapter 20 – Oceanic General Circulation

The driving mechanisms of the
ocean circulation: mechanical and thermal.
Why is the Gulf Stream so narrow.

Benoit Cushman-Roisin

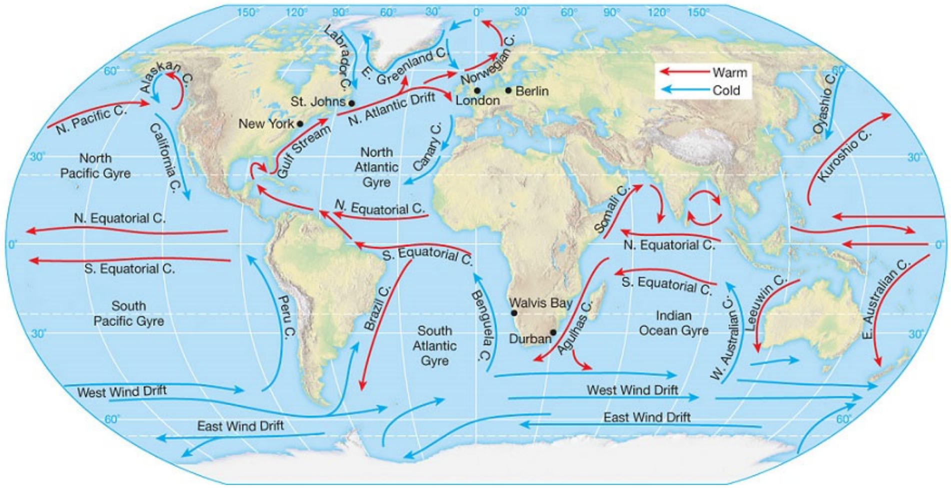
17 May 2023

First, schematic of 3D circulation

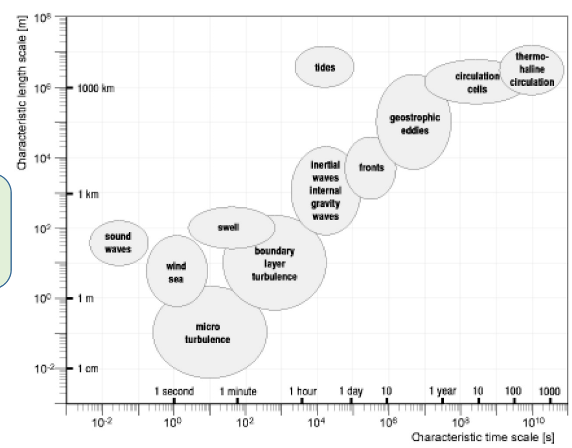


The Global Conveyor Belt system of surface and deep currents.

Focus on the surface circulation



What drives the ocean circulation



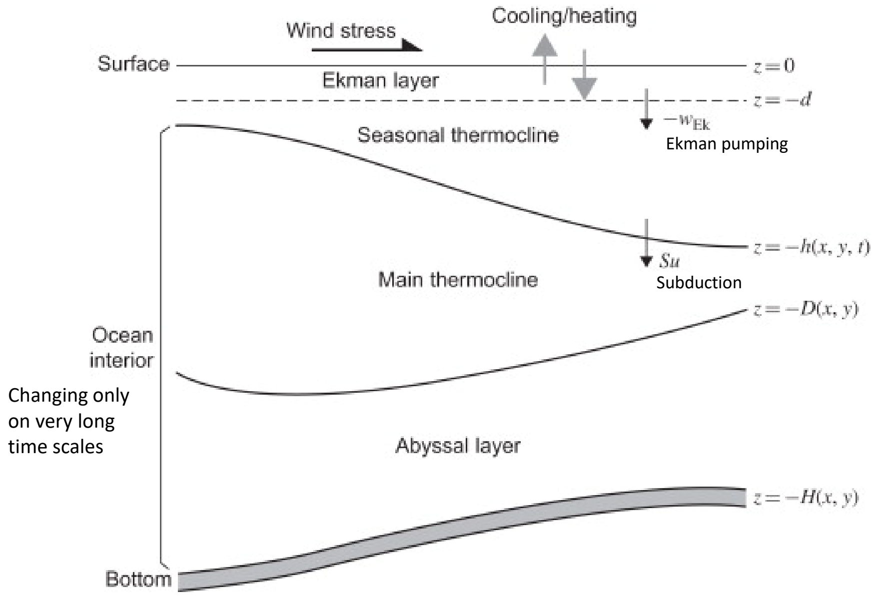
1. Gravitational attraction of moon and sun
   → Tides

Not important for large-scale circulation

2. Mechanical forcing
   → Surface wind stress driving surface currents

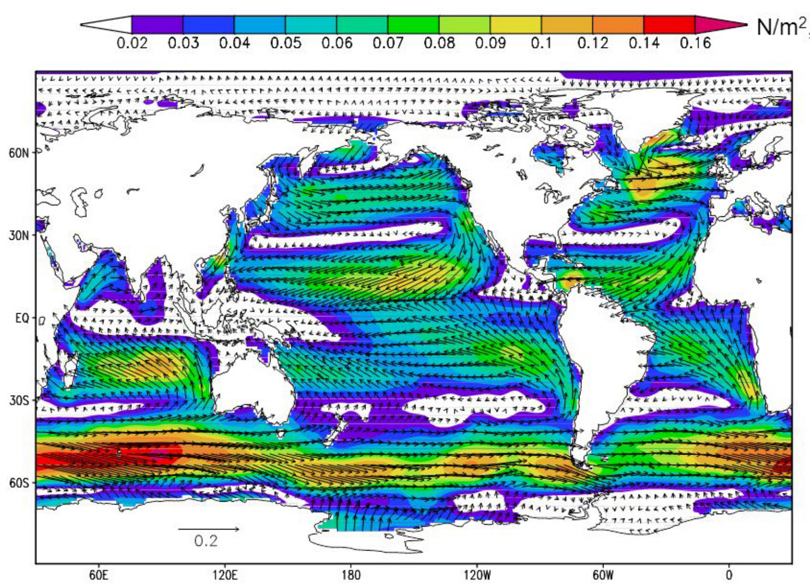
3. Thermal forcing
   → Cooling, evaporation and ice formation producing dense waters

Vertical Structure of the Deep Ocean



Cushman-Roisin & Beckers, 2011

Wind Distribution over Oceanic Basins



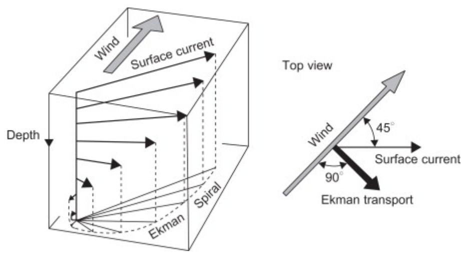
Surface Marine Data (NODC)

Annual mean surface distribution of the wind stress

Note:

- Clockwise structures in the northern mid-latitude basins,
- Counterclockwise structures in the southern mid-latitude basins.

Recall: Ekman Dynamics



Wind-driven currents near the surface in the Northern Hemisphere

Net transport:

$$Q_x = \int_{-\infty}^{0} u(z)\,dz = +\frac{\tau_y}{\rho_0 f}$$

$$Q_y = \int_{-\infty}^{0} v(z)\,dz = -\frac{\tau_x}{\rho_0 f}$$

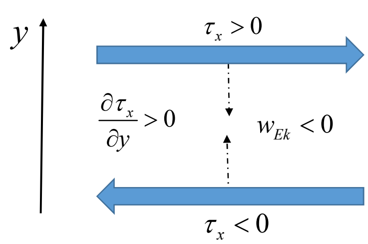
In the Southern Hemisphere, Ekman currents are directed to the LEFT of the wind stress.

What happens to the Ekman drift where the wind stress varies spatially:

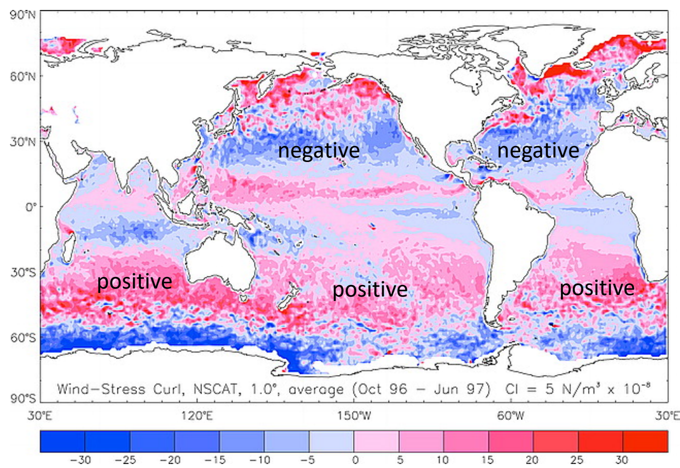
Volume conservation $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$ integrated over the vertical extent of the Ekman layer ($\sim$10m):

$$\frac{\partial}{\partial x}\left(\frac{\tau_y}{\rho_0 f}\right)-\frac{\partial}{\partial y}\left(\frac{\tau_x}{\rho_0 f}\right)-w_{Ek}=0 \quad\rightarrow\quad w_{Ek}=\frac{1}{\rho_0}\left[\frac{\partial}{\partial x}\left(\frac{\tau_y}{f}\right)-\frac{\partial}{\partial y}\left(\frac{\tau_x}{f}\right)\right]=\frac{1}{\rho_0}\,curl\left(\frac{\vec{\tau}_{wind}}{f}\right)$$

This is **Ekman pumping**.



Wind-Stress Curl Distribution over Oceanic Basins

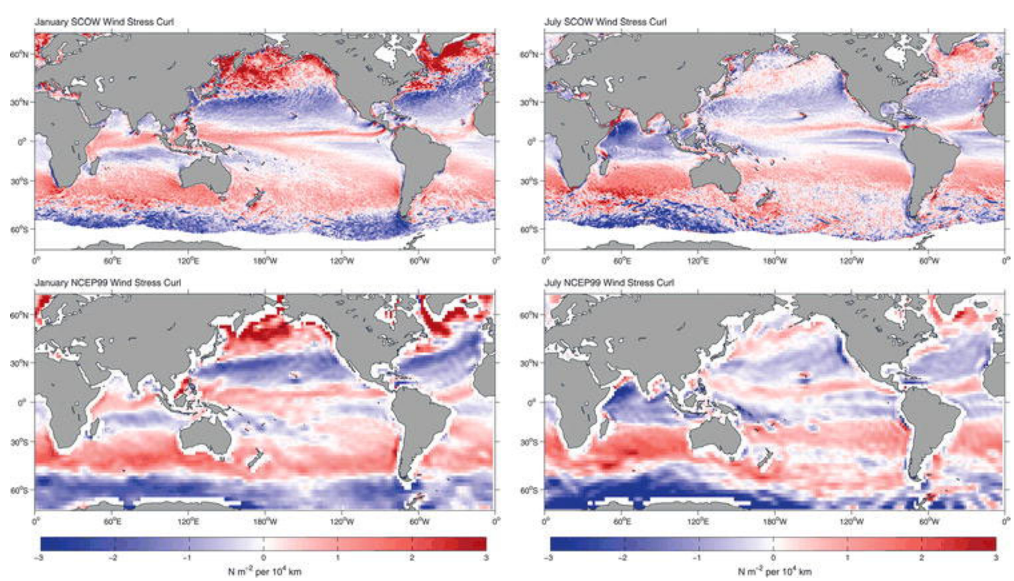


$$\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}$$ in 10^{-8} N/m^3

At mid-latitudes,
the wind stress curl is

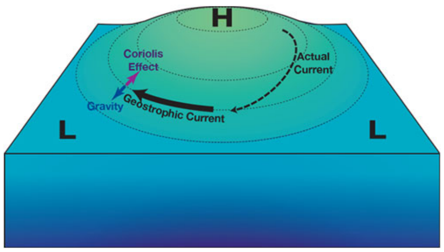
- negative in the Northern Hemisphere,
- positive in the Southern Hemisphere.

The Coriolis parameter f is > 0 in Northern Hemisphere and < 0 in the Southern Hemisphere.
This makes the Ekman pumping be downward in both mid-latitude bands.



Global wind stress curl maps from two different data sources (SCOW top & NCEP99 bottom) for January (left) and July (right).
Source: Journal of Physical Oceanography 38, 11; [10.1175/2008JPO3881.1](10.1175/2008JPO3881.1)

Recall: Geostrophy



Geostrophic current in balance between forces of gravity and Coriolis effect

[In the southern hemisphere, geostrophic currents are directed to the LEFT of the down pressure gradient.]

Equations:

$$\frac{\partial u}{\partial t} + \text{nonlinear terms} - fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} + \text{nonlinear terms} + fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y}$$

At the largest basin scale, the Rossby number is very small because L is very large:

$$\mathrm{Ro} = \frac{U}{f\,L} = \frac{(\sim 0.5\,m/s)}{(\sim 10^{-4}/s)(\sim 5\times 10^{6}\,m)} \sim 10^{-3} \ll 1$$

The smallness of the Rossby number allows us to use the geostrophic approximation:

$$u = -\frac{1}{\rho_0 f}\frac{\partial p}{\partial y}$$

$$v = +\frac{1}{\rho_0 f}\frac{\partial p}{\partial x}$$

with $f = f_0 + \beta_0 y$

Caution: We may not do this near the Equator where the Coriolis vanishes.
So, our focus here is on mid-latitude basins.

Now consider the conservation of volume in the interior, below the Ekman layer:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \rightarrow \quad -\frac{\partial}{\partial x}\left(\frac{1}{\rho_0 f}\frac{\partial p}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\rho_0 f}\frac{\partial p}{\partial x}\right) + \frac{\partial w}{\partial z} = 0$$

$$\cancel{-\frac{1}{\rho_0 f}\frac{\partial^2 p}{\partial x \partial y}} + \cancel{\frac{1}{\rho_0 f}\frac{\partial^2 p}{\partial x \partial y}} - \underbrace{\boxed{\frac{1}{\rho_0 f^2}\frac{\partial p}{\partial x}}}_{=\frac{v}{f}}\frac{df}{dy} + \frac{\partial w}{\partial z} = 0 \quad \rightarrow \quad -\frac{v}{f}\frac{df}{dy} + \frac{\partial w}{\partial z} = 0 \quad \rightarrow \quad \beta_0 v = f\frac{\partial w}{\partial z}$$

We fully recognize that the ocean basins are density-stratified (with seasonal and main thermoclines), but for the sake of simplicity and in retracing some historical development, let us assume temporarily that the oceanic interior has a homogeneous density.

This allows us to consider the whole interior (whole ocean minus thin Ekman layer at top) to be described by a single horizontal velocity field.

Integrating over the thickness of this homogeneous ocean interior, from the ocean bottom at $z = -H$ to ceiling at $z = -d$, and assuming a flat bottom along which the vertical velocity must vanish, we obtain:

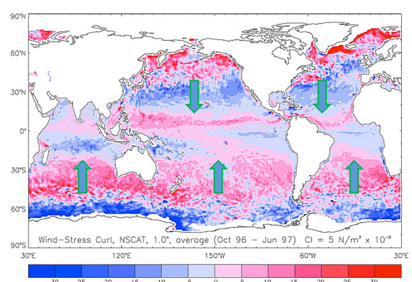
$$\beta_0 \int_{-H}^{-d} v \, dz = f \int_{-H}^{-d} \frac{\partial w}{\partial z} dz \quad \rightarrow \quad \beta_0 Q_y = f \, w_{Ek}$$

$$\boxed{Q_y = \frac{f}{\beta_0} w_{Ek} = \frac{f}{\rho_0 \beta_0} curl \left(\frac{\vec{\tau}_{wind}}{f} \right)}$$

This is called the **Sverdrup relation**.

Harald Sverdrup (1888-1957)
This work in 1947.

Now, recall that in mid-latitude basins, the wind-stress curl
- is negative (clockwise) in the Northern Hemisphere,
- is positive (counterclockwise) in the Southern Hemisphere.



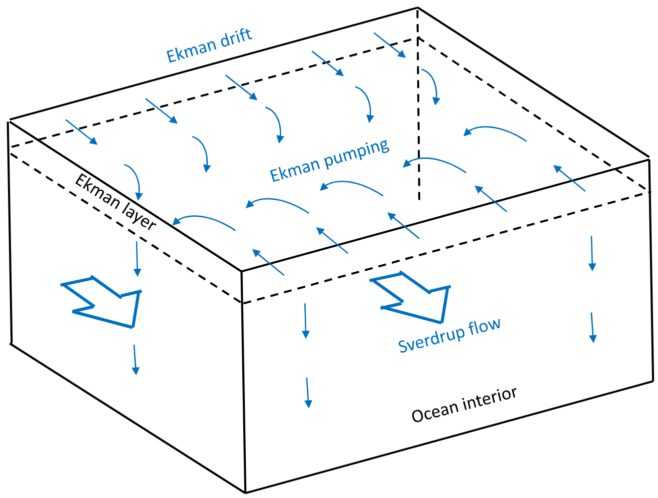
This implies that the volumetric transport in the meridional direction is negative (southward) in the Northern Hemisphere and positive (northward) in the Southern Hemisphere.

Thus, in all mid-latitude basins, the meridional transport (per unit width, in m^2/s) is everywhere directed toward the Equator.

$$Q_y = \frac{f}{\beta_0} w_{Ek} < 0 \quad \text{for} \quad y > 0$$

$$Q_y = \frac{f}{\beta_0} w_{Ek} > 0 \quad \text{for} \quad y < 0$$

Summing up, we arrive at the following flow depiction



But where is the Gulf Stream, which is clearly flowing north in the Northern Hemisphere?

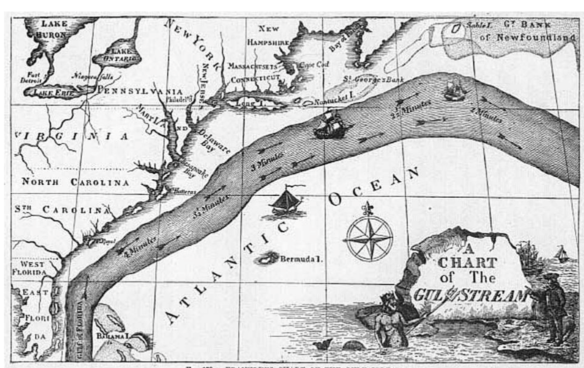


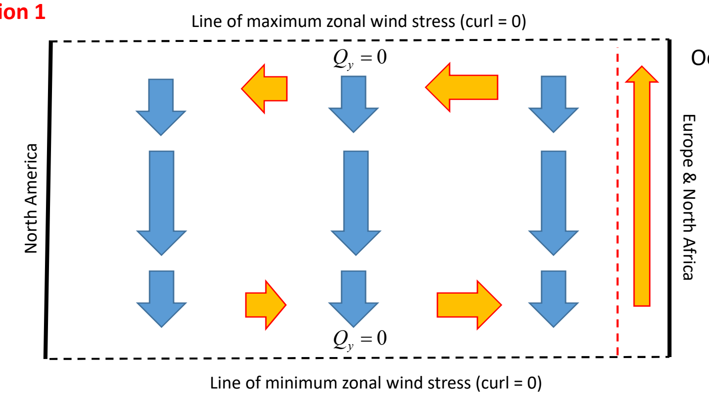
Fig. 173.—FRANKLIN'S CHART OF THE GULF STREAM.

First map
of the Gulf Stream,
by Benjamin Franklin
in 1769.

Our reduction of the dynamics has been excessive.

The way to close the circulation is to add zonal flows
and their gathering in a boundary layer on one side of the ocean basin.
There are two options depending on where we place the boundary layer.

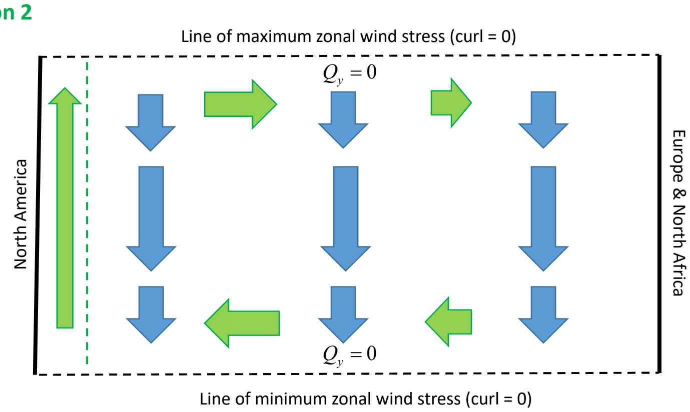
Option 1



Ocean interior seen from above

Zonal transport Q_x in the interior from volume conservation: $\dfrac{\partial Q_x}{\partial x} + \dfrac{\partial Q_y}{\partial y} = -w_{Ek}$

Option 2

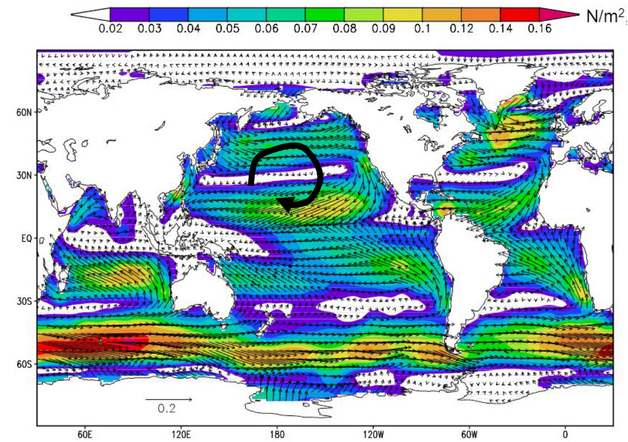


Zonal transport Q_x in the interior from volume conservation: $\dfrac{\partial Q_x}{\partial x} + \dfrac{\partial Q_y}{\partial y} = -w_{Ek}$

We must have different dynamics in the return boundary layer (on east or west side).

We know from the existence of the Gulf Stream that Option 2 is it.
But how can we prove it?

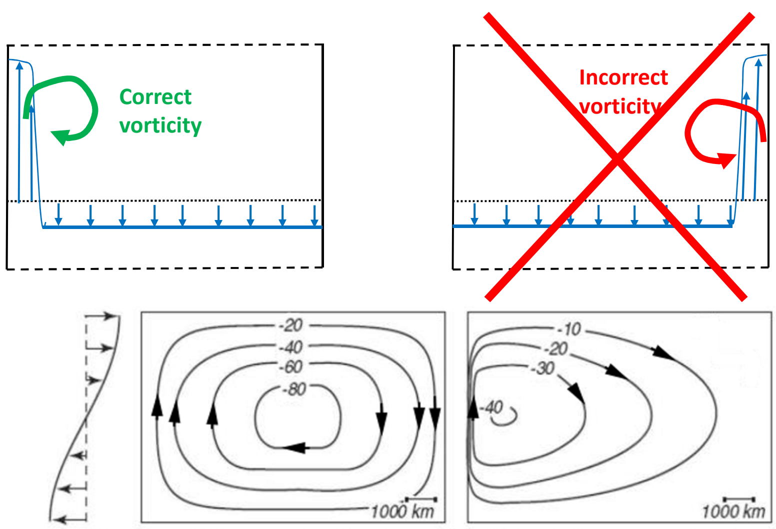
So far, we have invoked mostly volume conservation. Additional considerations may be drawn from vorticity consideration.



Surface Marine Data (NODC)

There is clockwise vorticity in the wind stress.

This imparts a clockwise vorticity to the ocean.



Wind Stress

With single-valued Coriolis (basin on a rotating plane)

With latitude-dependent Coriolis (basin on a sphere)

First correct theory for the existence of the Gulf Stream

Henry Stommel (1920-1992)
This work in 1948.

Volumetric Flowrate in the Gulf Stream

Measurements estimate the volumetric transport in the Gulf Stream reveal that it emerges from the Gulf of Mexico at the Florida Strait with about **30 x 10⁶ m³/s** and then gathers more waters as it flows northeastward to reach values exceeding **85 x 10⁶ m³/s**.

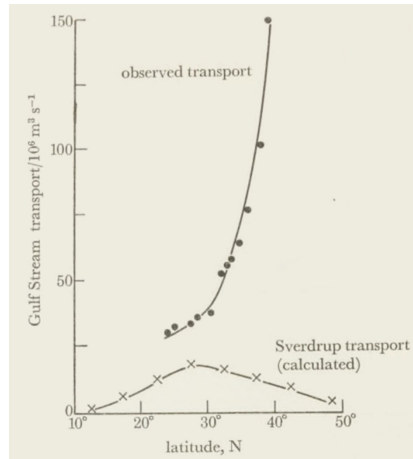
Let's check the theoretical prediction. Sverdrup dynamics suggest

$$Q_y = \frac{1}{\rho_0 \beta}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right)$$
$$\approx \frac{-5\times10^{-8}\,N/m^3}{(1030\,kg/m^3)(2.0\times10^{-11}\,m^{-1}s^{-1})} = 2.4\frac{m^2}{s}$$

Q_y is per unit meridional width.
So, let's multiply by the averaged width of the Atlantic Ocean:

$$Q_y L_x \approx \left(2.4\frac{m^2}{s}\right)(6,000\,km) = 14\times10^6\,\frac{m^3}{s}$$

Conclusion: Correct order of magnitude but falling short…



(Gill, 1971)

Shortcomings of Stommel's Theory for the Gulf Stream

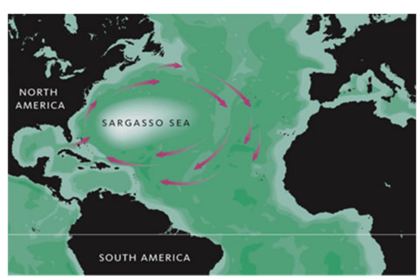
1. The theory is based on the assumption of a homogeneous ocean interior, something that is far from being the case.
2. The added dynamics to create the western boundary layer is bottom friction (to dissipate the infused vorticity), but the Gulf Stream does not touch the bottom; it slips rather easily on layers of water below it; observations indicate a frictionless, inertial current.
3. Its prediction for the volumetric flow falls short.

In Stommel's defense, it must be noted that Stommel did not present his theory as one for the Gulf Stream but merely as an argument for "western intensification".

Then, we also note this strange feature called the **Sargasso Sea**, a sea within the ocean.

It is named from the Sargasso weeds that float on it.

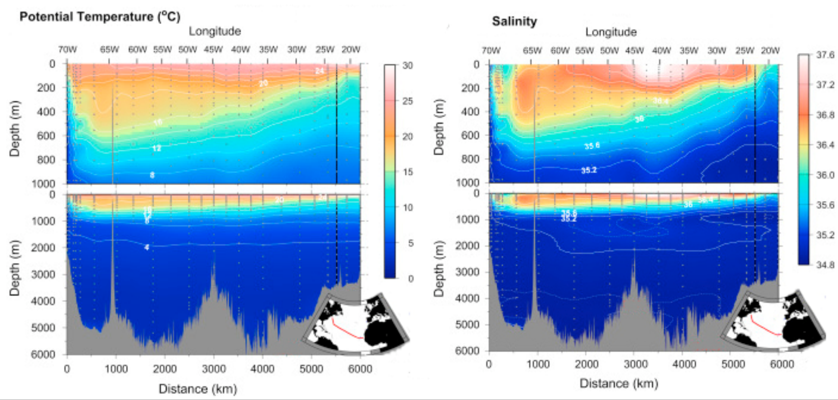
It is an anomalously warm pool of water for that latitude. It appears to be a recirculation of a major part of the Gulf Stream.



Sargasso Sea

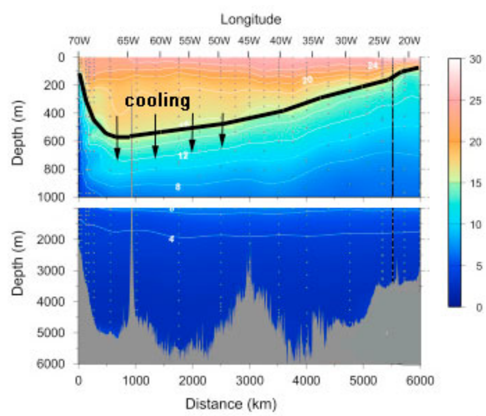
From a physical (not biological) point of view, the Sargasso Sea is characterized by

1. Deep warm-water pool, significantly deeper than the upper ocean layer in the rest of the North Atlantic;
2. Intense clockwise circulation, contributing to augmentation of the Gulf Stream's volumetric flow;
3. Intense heat loss to the atmosphere.



Jenkins, Smethie, Boyle & Cutter, 2015

This realization suggests that we use a model with at least two distinct density layers below the Ekman layer, thus making a distinction between the main thermocline and the abyssal layer.



The cooling of the upper layer in the warm Sargasso Sea is modeled by a (partial) transfer of water from the upper, warm layer to the lower, cold layer.

In the process of losing water through its floor, the upper layer water column gets vertically stretched, which decreases its counterclockwise spin.

Long story short

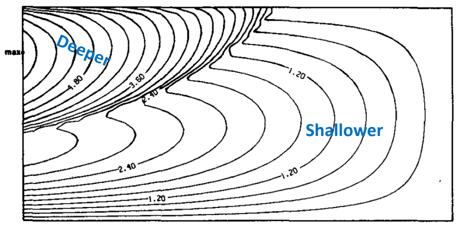


FIG. 4. Upper-interface displacement, η, for the typical parameters of the North Atlantic ($\delta = 5$, $r = 1.2$, $\alpha = 0.5$) and $\eta_0 = 0$. Positive values indicate a downward interfacial displacement. Note the clear distinction between the shallower interior and the deeper recirculation. Cooling occurs everywhere but more so where displacements are larger.

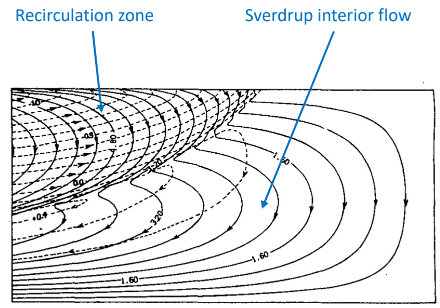


FIG. 5. Upper-layer streamfunction, ψ_1, (solid lines) and lower-layer streamfunction, ψ_2, (dashed lines) for the typical parameter values of the North Atlantic. Note the recirculation from western boundary back to western boundary in the upper layer. No recirculation is present in the lower layer.

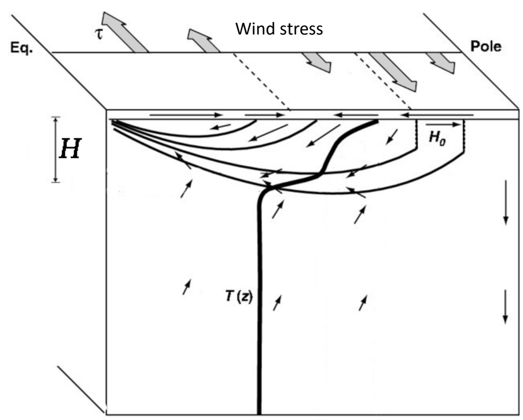
(Cushman-Roisin, 1987)

In this model, the Gulf Stream is a western boundary current (not represented in the figures) with same potential vorticity at the entrance and exit of the same streamline (but varying from streamline to streamline). In other words, the Gulf Stream is frictionless and purely inertial.

For realistic parameters values ascribed to the model, the prediction of the Gulf Stream maximum strength at its highest level of recirculation in 102 x 10^6 m^3/s, which has been observed.

Ventilated Thermocline

Why stop at two layers?



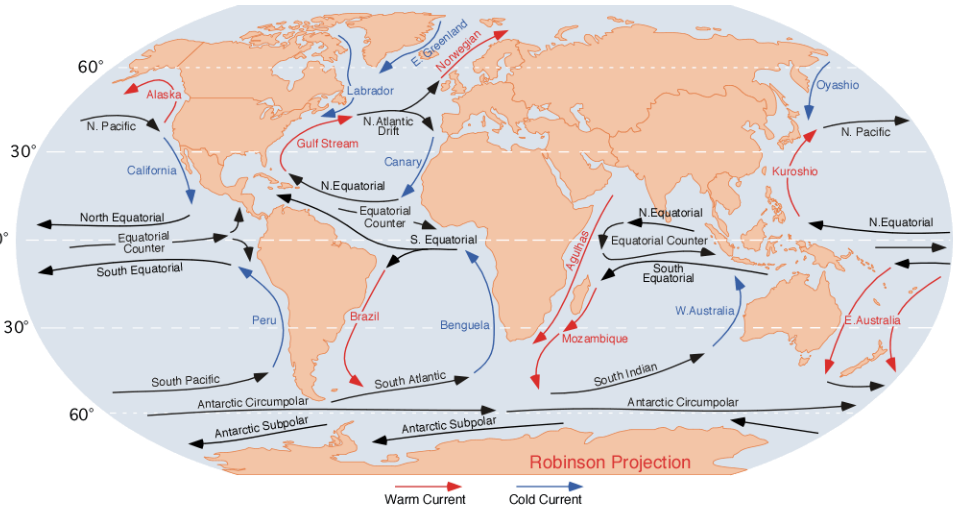
Vertical scale for the thickness of the thermocline:

Sverdrup relation: $\beta_0 v = f\frac{\partial w}{\partial z} \approx f\frac{w_{Ek}}{H}$

Geostrophic flow with hydrostatic pressure based on density difference: $fv = \frac{1}{\rho_0}\frac{\partial p}{\partial x} \approx \frac{1}{\rho_0}\frac{gH\Delta\rho}{L_x}$

$$\Longrightarrow \quad H \approx \sqrt{\frac{f^2 L_x w_{Ek}}{\beta_0 g\left(\Delta\rho/\rho_0\right)}} \approx 500\,m$$

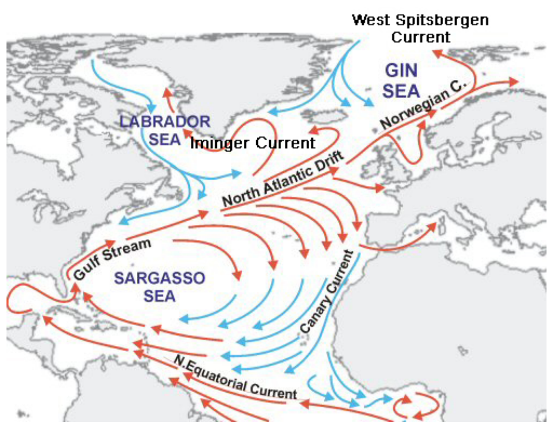
Western Intensification in Oceanic Basins: Gulf Stream not alone!



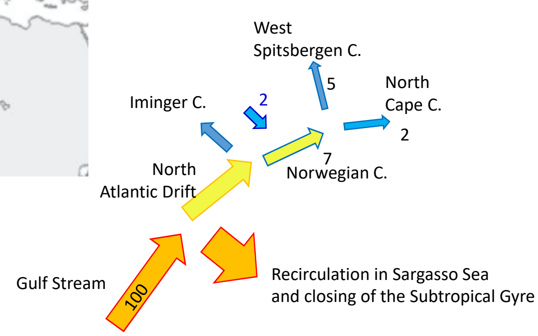
North Atlantic: Gulf Stream
North Pacific: Kuroshio

South Atlantic: Brazil Current
South Pacific: East Australia Current
Indian Ocean: Agulhas Current

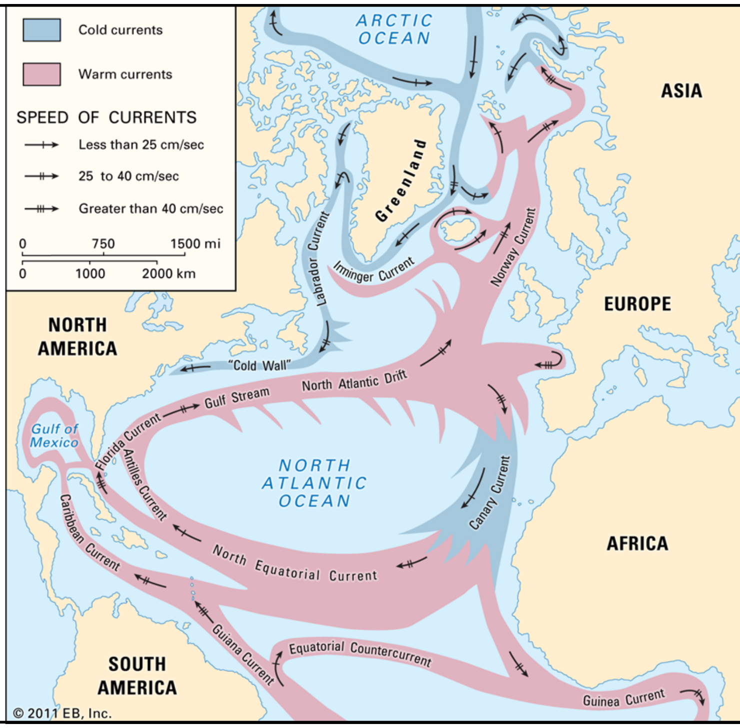
The Branching of the Gulf Stream



“GIN” Sea = Greenland, Icelandic & Norwegian Sea



Units: 10^6 m^3/s



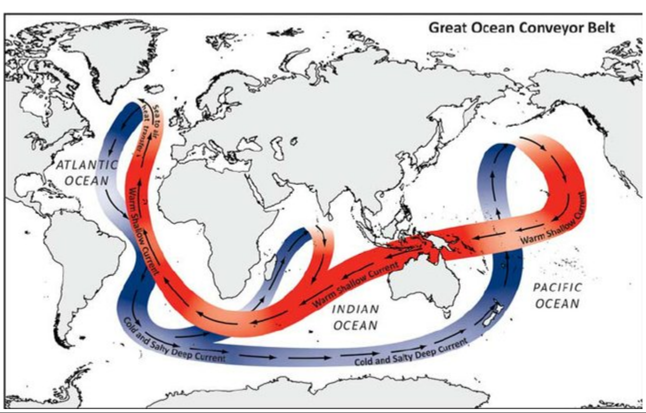
Melting of Arctic ice by climatic change causes a reduction of inflow from the West Spitsbergen Current, thus threatening the intensity of the latter.

Some pundits have jumped erroneously to the conclusion that this threatens the Gulf Stream itself.

Thermal drive of the ocean: Deep-water formation and abyssal Circulation

The circulation in the abyssal layer, that is the circulation below the main thermocline, is not driven by the surface wind stress but by the formation of dense waters in the high latitudes.

These dense waters sink into the abyss and proceed from their place of formation in a pattern that has been dubbed the **conveyor belt**.



The oceanic conveyor belt as proposed by Wallace Broecker in 1987.

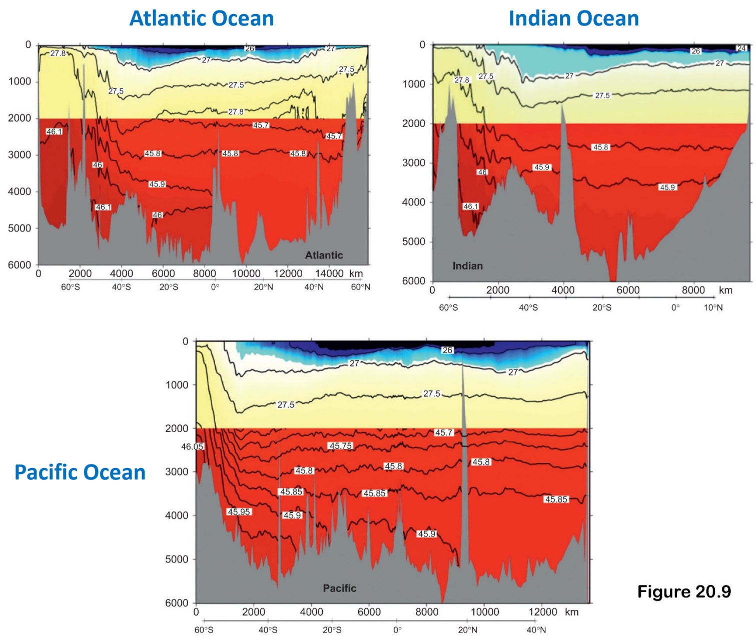


Figure 20.9

Meridional structure of density in the major oceanic basins.

The density anomalies quoted include a correction for compressibility under high pressure.

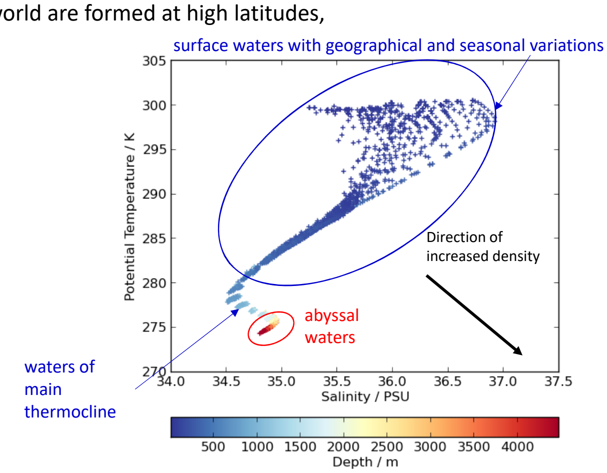
Data from the World Ocean Circulation Experiment (WOCE, 1990-2002) analyzed by Talley *et al*. (2007)

Dense Water Formation

Dense water formation proceeds by a combination of
- Cooling, causing a density increase by thermal contraction,
- Evaporation, causing an increase in salinity in the remaining water, and/or
- Sea ice formation, causing salt rejection into the remaining water.

The coldest and most saline waters in the world are formed at high latitudes, both north (Arctic) and south (Antarctica).

These waters are so dense that they penetrate to the deepest parts of the oceanic basins, defining their water characteristics.



Direction of Dense Water Currents

Large-scale Sverdrup dynamics hold in the abyss as in the main thermocline

$$u = -\frac{1}{\rho_0 f}\frac{\partial p}{\partial y}, \quad v = +\frac{1}{\rho_0 f}\frac{\partial p}{\partial x} \quad + \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \Longrightarrow \quad -\frac{1}{\rho_0 f}\frac{\partial^2 p}{\partial x \partial y} + \frac{1}{\rho_0 f}\frac{\partial^2 p}{\partial x \partial y} - \frac{1}{\rho_0 f^2}\frac{\partial p}{\partial x}\frac{df}{dy} + \frac{\partial w}{\partial z} = 0$$

$$\Downarrow$$

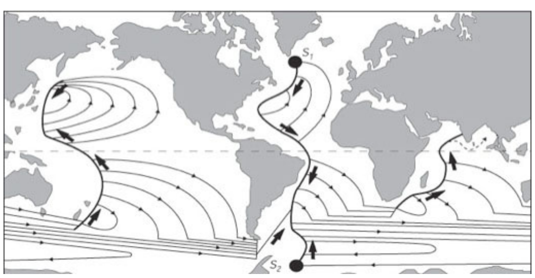
$$\beta_0 v = f\frac{\partial w}{\partial z} \quad \Longrightarrow \quad Q_y = \frac{f}{\beta_0} w_{top}$$

The areas of dense water sinking, where $w_{top} < 0$, are small and highly localized.
Dense waters slowly rise where $w_{top} > 0$, which is most everywhere.

Since $\beta_0 > 0$ everywhere, this makes Q_y have the same sign as $f = 2\Omega\sin(\text{latitude})$, which is positive in the Northern Hemisphere and negative in the Southern Hemisphere.
This means that Q_y is everywhere directed toward, not away from, the polar regions where the dense waters are being formed.

There seems to be a paradox!

The paradox is resolved by invoking, once again, western boundary currents (now deep ones).



Stommel & Arons, 1958-1960