Week 8 - GLM - ANOVA part 1

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Week	Topic
1	Introduction, Open Science, and Power
2	Introduction to R
3	Data Wrangling and Visualisation
4	General Linear Model - Regression
5	General Linear Model - Regression
6	No Timetabled Lecture - Reading Week
7	Consolidation Lab
8	General Linear Model - ANOVA
9	General Linear Model - ANOVA
10	Tidy Thursday Data Wrangling & Visualisation Challenge
11	Reproducing your Computational Environment using Binder
12	Dynamic, Reproducible Presentations Using xaringan

Semester 1 Assignments

Data wrangling and visualisation – Due December 5th

ANOVA/ANCOVA – Due January 17th

- We're going to have our first look at the Analysis of Variance (ANOVA).
- This week we'll look at ANOVA for within-subjects, between-subjects and mixed designs.
- ANOVA is an important statistical test and (in various forms) is used widely across many areas of psychology.

Assessment

- The second assessment will be on the ANOVA lectures. It will again require you to conduct an ANOVA and to produce a report using R Markdown.

- The assessment question will be of a similar type to the ones we'll look at in the lab classes over the next couple of weeks.

Reporting ANOVA

- Say what type of ANOVA it was, say what factors you had (and with labels for each level).
- Report the results of main effects first, then interactions.
- Report F values, exact *p*-values and effect size values.
- Remember to interpret interactions further - such as with contrasts or pairwise comparisons.
- When you have main effects, say which direction the effect goes.
- Avoid sillies - e.g., mixing up < and > or saying p = .000

Why ANOVA, why not t-tests?

- So, t-tests are fine if we're just comparing two means.

- In the real world of psychology, we often have more than two conditions.

- How could we analyse our data ?

- One possibility could be that we do multiple t-tests – but there's a problem with that.

- With one t-test, at $p < 0.05$ alpha level there is a 5% chance of falsely rejecting our null hypothesis (type I error).

- If we have three conditions, then we have three pairs of means to compare (condition 1 vs condition 2, condition 2 vs condition 3 and condition 1 vs condition 3).

- For each test, there is 0.95 probability of not having a type I error.

- But when we do three tests the probability is 0.95 x 0.95 x 0.95 which equals 0.857.

- So that means there is a 14.3% chance of us falsely rejecting the null hypothesis (1-0.857) x 100 = 14.3

The familywise error rate

- This is known as the <u>familywise</u> error rate.

$$\text{familywise error} = 1 - (0.95)^n$$

- If we had 5 conditions, and hence 10 t-tests to conduct, our error rate would be 0.4 – which means there is a 40% chance of having made at least one Type I error (i.e., thinking we have an effect when none is present).

Similarities between t-tests and the ANOVA

- t-tests tell us whether or not two samples have the same mean.
- ANOVA tells us whether two or more samples have the same mean.
- As the t-test produced the t-statistic, the ANOVA gives us an F-statistic or F-ratio which compares the amount of systematic variance with the amount of unsystematic variance.

- ANOVA can tell us that there is a difference between means – so for three samples it tells us that $\overline{X}_1 = \overline{X}_2 = \overline{X}_3$ is not true.

- But it doesn't tell us where the difference is.

- It doesn't tell us whether $\overline{X}_1$ differs from both $\overline{X}_2$ and $\overline{X}_3$ or whether $\overline{X}_2$ differs from $\overline{X}_3$ but not $\overline{X}_2$ etc.

ANOVA

- Imagine we're interested in the impact of caffeine consumption on an individual's motor performance.
- It's a between-subjects design with 3 conditions:
  - low amount of caffeine (single espresso)
  - large amount of caffeine (double espresso)
  - placebo group (water)

- We conduct an ANOVA and find a significant F-ratio.
- What does it mean?
- The single espresso people could have performed better from the double espresso and water group.
- Or maybe they performed the same as the water group but better than the double espresso group.
- Or maybe (unexpectedly) they performed worse than both the double espresso and water groups.
- To know what is the case we need to do planned contrasts (similar to 1 tailed tests) or post hoc tests (similar to 2 tailed tests).

- We know that at least one of our means differs from at least one of our other means but (so far) we don't know where that difference lies…..
- Luckily things easy for us as we can conduct what are known as post hoc tests. These will tell us which means differ from which other means (and allow us to begin to tell a story….)

Post hocs tests

- Work by doing pairwise comparisons on all the different combinations of experimental groups…..
- They control for the familywise error rate though to get round that problem.
- Bonferroni method divides our critical p value (0.05) by the number of tests. If we are conducting ten tests, then for each test the critical p is 0.005 – but this increases our chances of a Type II error – missing an effect when it's there.

When deciding which post hoc test to use :

- Does it control the Type I error rate ?
- Does it control the Type II error rate ?
- Is it reliable when ANOVA assumptions have been violated ?

LSD, Bonferroni, and Tukey tests.

- The least significant differences test (LSD) doesn't control the Type I error and is like doing multiple t-tests on the data (but only if the ANOVA is significant).
- Bonferroni and Tukey both control for Type I errors but are conservative. Bonferroni works by dividing the critical alpha level by the number of tests conducted.
- Tukey is less conservative than Bonferroni.

The Packages

```
library(tidyverse) #load the tidyverse packages

library(afex) #load afex for running ANOVA

library(emmeans) #load emmeans for running pairwise comparisons
```

ANOVA

We have 45 participants, a between participants condition with 3 levels (Water *vs*. Single Espresso *vs*. Double Espresso), and Ability as our DV measured on a continuous scale.

```
cond <- read_csv("data_files/cond.csv")

cond$Condition <- as.factor(cond$Condition)
```

```
> cond
# A tibble: 45 x 3
   Participant Condition Ability
         <dbl> <fct>       <dbl>
 1           1 Water        4.82
 2           2 Water        5.41
 3           3 Water        5.73
 4           4 Water        4.36
 5           5 Water        5.47
 6           6 Water        5.50
 7           7 Water        5.07
 8           8 Water        5.08
 9           9 Water        5.07
10          10 Water        4.94
# … with 35 more rows
```

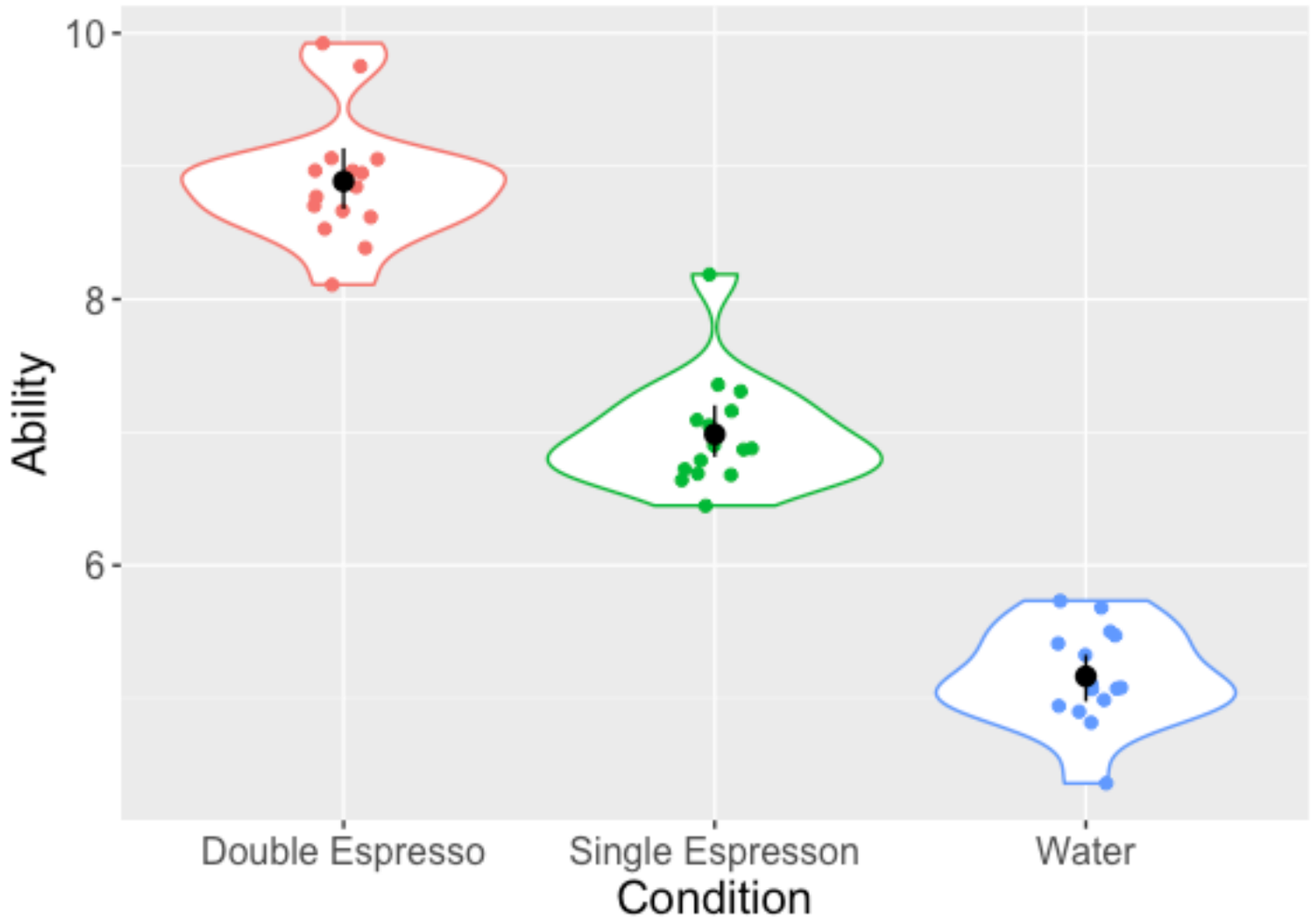
We have three columns - Participant number, Condition, and Ability. Condition is our IV, and Ability our DV. Note, our data are in tidy or long format with one observation per row.

Generating Descriptives and Visualising the Data

```
cond %>%
  group_by(Condition) %>%
  summarise(mean = mean(Ability), sd = sd(Ability))
```

```
# A tibble: 3 x 3
  Condition          mean    sd
  <fct>             <dbl> <dbl>
1 Double Espresso    8.89 0.467
2 Single Espresson   6.99 0.419
3 Water              5.17 0.362
```

```
cond %>%
  ggplot(aes(x = Condition, y = Ability, colour = Condition)) +
  geom_violin() +
  geom_jitter(width = .1) +
  guides(colour = FALSE) +
  stat_summary(fun.data = "mean_cl_boot", colour = "black") +
  theme(text = element_text(size = 15))
```



```
library(afex)

model <- aov_4(Ability ~ Condition + (1 | Participant),
data = cond)
```

This is our DV **This is our IV** **This is our random effect**

```
> summary(model)
Anova Table (Type 3 tests)

Response: Ability
          num Df den Df     MSE      F     ges    Pr(>F)
Condition      2     42 0.17484 297.05 0.93397 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

To determine what's driving the effect we can use the `emmeans::emmeans()` to run pairwise comparisons (note, default is Tukey correction).

```
> emmeans(model, pairwise ~ Condition)
$emmeans
 Condition         emmean    SE df lower.CL upper.CL
 Double Espresso     8.89 0.108 42     8.67     9.10
 Single Espresson    6.99 0.108 42     6.77     7.20
 Water               5.17 0.108 42     4.95     5.38

Confidence level used: 0.95

$contrasts
 contrast                                estimate    SE df t.ratio
p.value
 Double Espresso - Single Espresson          1.90 0.153 42 12.453  <.0001
 Double Espresso - Water                     3.72 0.153 42 24.372  <.0001
 Single Espresson - Water                    1.82 0.153 42 11.920  <.0001

P value adjustment: tukey method for comparing a family of 3
estimates
```

Measure of Effect Size

- The effect size is measured by *ges* which stands for generalised effect size or generalised eta squared (η_G^2).
- For designs with more than one factor it can be a useful indicator of how much variance in the dependent variable can be explained by each factor (plus any interactions between factors).

```
> summary(model)
Anova Table (Type 3 tests)

Response: Ability
          num Df den Df     MSE      F     ges    Pr(>F)
Condition      2     42 0.17484 297.05 0.93397 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

So, to make sense of our output

- We found a significant effect of Beverage type ($F(2,42) = 297.05$, $p < .001$, generalised $\eta 2 = .93$). Tukey comparisons revealed that the Water group performed significantly worse than the Single Espresso Group ($p < .001$), that the Water group performed significantly worse than the Double Espresso Group ($p < .001$), and that the Single Espresso Group performed significantly worse than the Double Espresso Group ($p < .001$).

- In other words, drinking some coffee improves motor performance relative to drinking water, and drinking a lot of coffee improves motor performance even more.

ANOVA for repeated measures designs

- Let's imagine we have an experiment where we asked 32 participants to memorise words of differing levels of spelling complexity - Very Easy, Easy, Hard, and Very Hard.

- They were presented with these words in an initial exposure phase. After a 30 minute break we tested them by asking them to write down all the words. We scored them as number correct for each condition.

- We want to know whether there is a difference in the number of words they remembered for each level of spelling complexity.

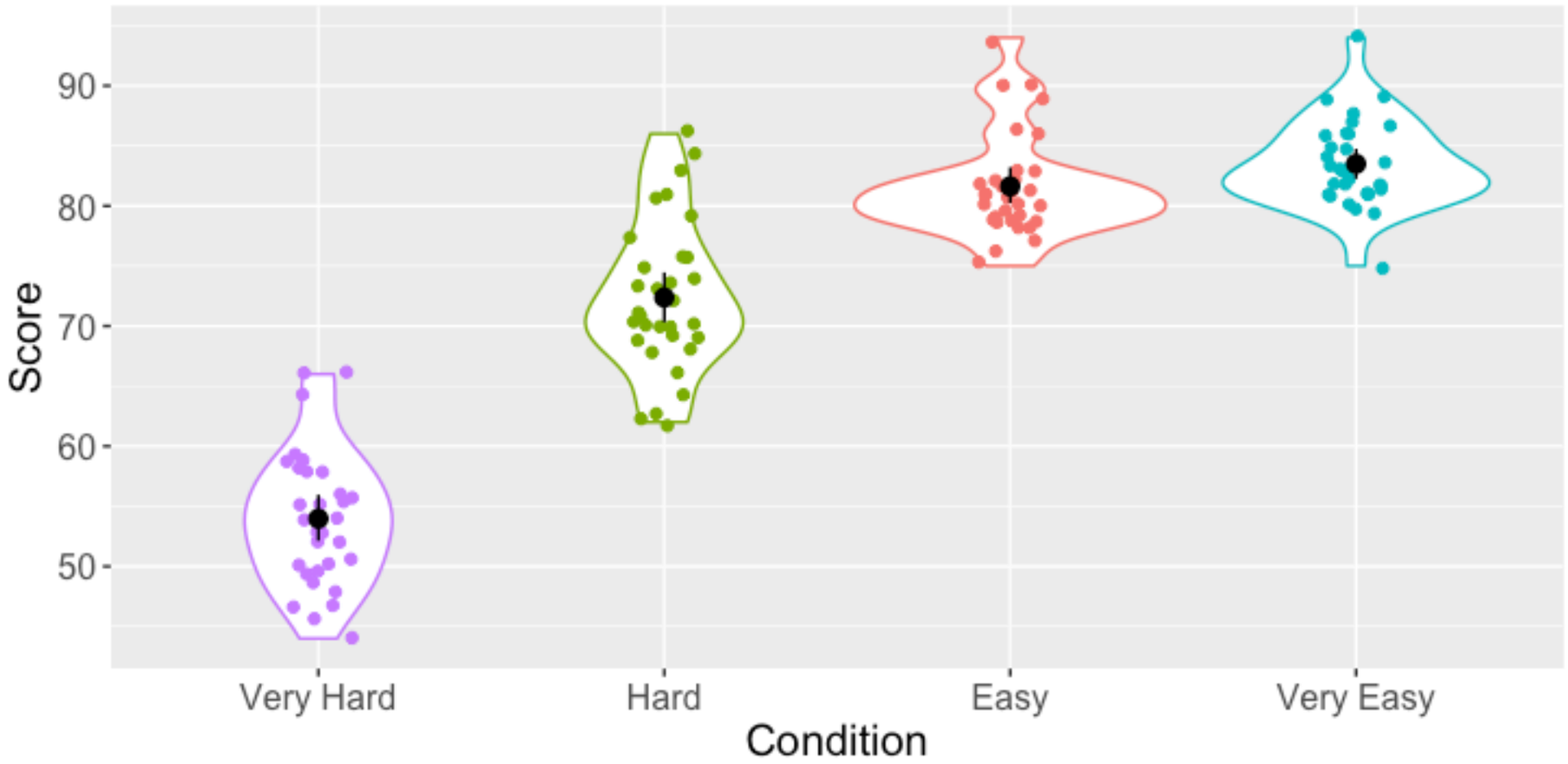
```
rm_data <- read_csv("data_files/rm_data.csv")
rm_data$Condition <- as.factor(rm_data$Condition)
rm_data
```

```
# A tibble: 128 x 3
   Participant Condition Score
         <dbl> <fct>     <dbl>
 1           1 Very Easy    80
 2           2 Very Easy    86
 3           3 Very Easy    89
 4           4 Very Easy    75
 5           5 Very Easy    86
 6           6 Very Easy    87
 7           7 Very Easy    82
 8           8 Very Easy    82
 9           9 Very Easy    82
10          10 Very Easy    81
# … with 118 more rows
```

Generating Descriptives and Visualising the Data

```
rm_data %>%
  group_by(Condition) %>%
  summarise(mean = mean(Score), sd = sd (Score))

# A tibble: 4 x 3
  Condition  mean    sd
  <fct>     <dbl> <dbl>
1 Easy       81.6  4.28
2 Hard       72.4  6.24
3 Very Easy  83.5  3.62
4 Very Hard  54.0  5.50
```

```
rm_data %>%
  ggplot(aes(x = fct_reorder(Condition, Score), y = Score, colour =
Condition)) +
  geom_violin() +
  geom_jitter(width = .1) +
  guides(colour = FALSE) +
  stat_summary(fun.data = "mean_cl_boot", colour = "black") +
  theme(text = element_text(size = 15)) +
  labs(x = "Condition")
```



This is the our ANOVA model - we have a significant effect of Condition.

```
> model <- aov_4(Score ~ Condition + (1 + Condition | Participant), data = rm_data)
> summary(model)

Univariate Type III Repeated-Measures ANOVA Assuming Sphericity

                SS num Df Error SS den Df         F    Pr(>F)
(Intercept) 679632      1   936.49     31 22497.36 < 2.2e-16 ***
Condition    17509      3  2179.48     93   249.04 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Mauchly Tests for Sphericity

          Test statistic p-value
Condition        0.90603 0.71042


Greenhouse-Geisser and Huynh-Feldt Corrections
 for Departure from Sphericity

          GG eps Pr(>F[GG])
Condition 0.9401  < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

            HF eps   Pr(>F[HF])
Condition 1.043895 2.615157e-44
```

```
> anova(model)
Anova Table (Type 3 tests)

Response: Score
          num Df den Df    MSE      F     ges    Pr(>F)
Condition 2.8203  87.43 24.928 249.04 0.84892 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

The effect size is measured by *ges* which stands for generalised effect size (η_G^2) - this is the recommended effect size measure for repeated measures designs (Bakeman, 2005). We get this by using the `anova()` function on our model. Note the dfs in this output are always corrected as if there is a violation of sphericity - to be conservative (and to avoid Type I errors) we might be better off to always choose these corrected dfs.

Where does the difference lie?

```
> emmeans(model, pairwise ~ Condition, adjust = "Bonferroni")
$emmeans
 Condition emmean    SE  df lower.CL upper.CL
 Easy        81.6 0.886 122     79.9     83.4
 Hard        72.4 0.886 122     70.6     74.1
 Very.Easy   83.5 0.886 122     81.7     85.3
 Very.Hard   54.0 0.886 122     52.2     55.7

Warning: EMMs are biased unless design is perfectly balanced
Confidence level used: 0.95

$contrasts
 contrast                estimate   SE df t.ratio p.value
 Easy - Hard                9.25 1.21 93  7.643  <.0001
 Easy - Very.Easy          -1.88 1.21 93 -1.549  0.7483
 Easy - Very.Hard          27.66 1.21 93 22.852  <.0001
 Hard - Very.Easy         -11.12 1.21 93 -9.192  <.0001
 Hard - Very.Hard          18.41 1.21 93 15.209  <.0001
 Very.Easy - Very.Hard     29.53 1.21 93 24.401  <.0001

P value adjustment: bonferroni method for 6 tests
```

So far we have looked at ANOVA for designs when we have one factor which is between subjects (i.e., each participant appears in one condition), and for designs when we have one factor that is repeated measures (each participant appears in all conditions). These are examples of 1-way ANOVA.

Now we're going to look at factorial ANOVA - this is for cases where we have more than one factor and we might be interested in how the factors interact with each other. If we have two factors, we have a 2-way ANOVA, three factors a 3-way ANOVA etc.

- Imagine we have 2 factors. Factor 1 with two levels, Factor 2 with three. Our analysis might reveal a main effect of Factor 1 (i.e., a difference between the two levels), a main effect of Factor 2 (i.e., a difference between the three levels) or an interaction between the two…..

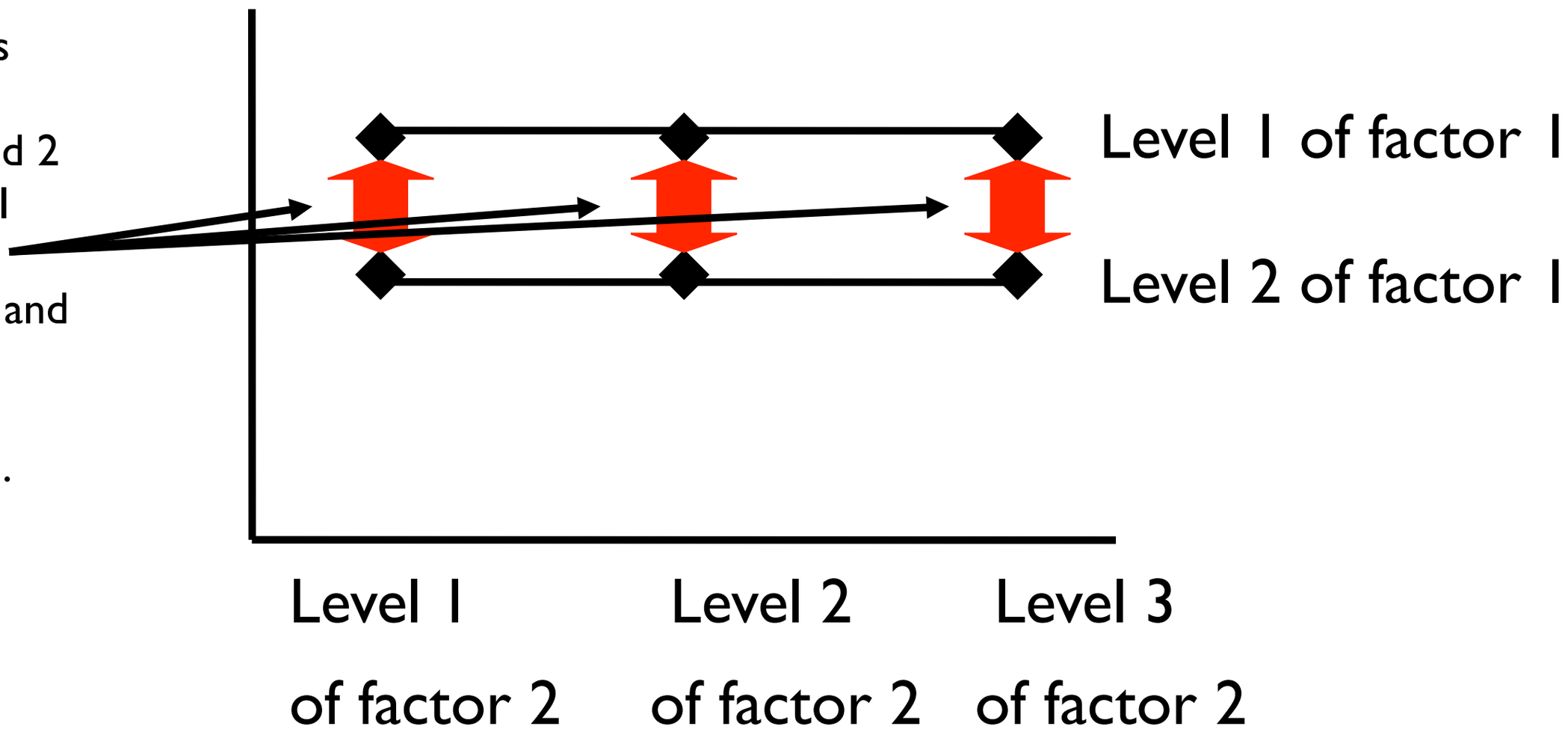
- This is a 2 x 3 ANOVA

Corresponds to Factor 1 – it has two levels.

Corresponds to Factor 2 – it has three levels.

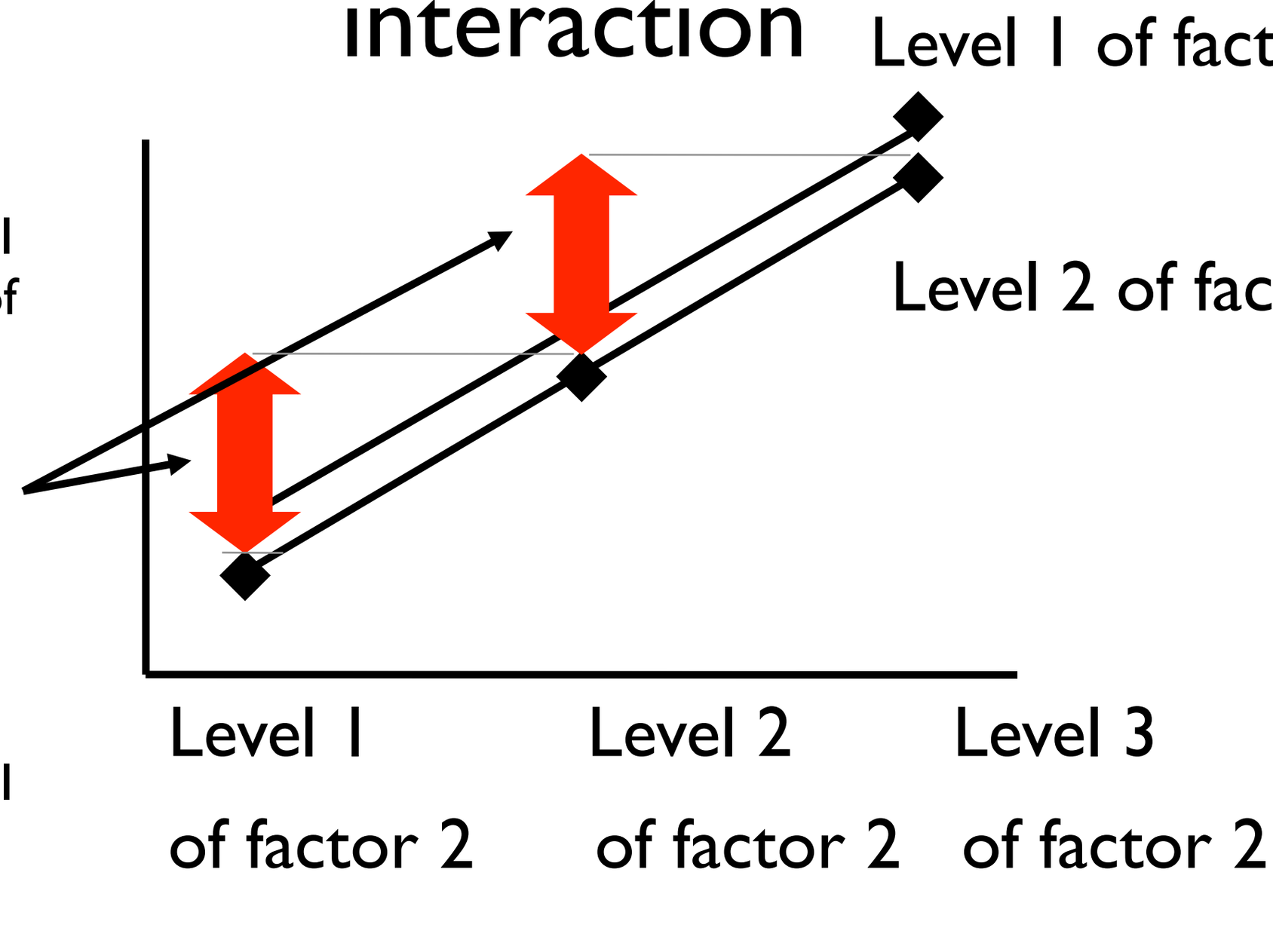
Main effect of Factor 1, no main effect of Factor 2 and no interaction

The differences between levels 1 and 2 of Factor 1 are all significant and are of the same magnitude.



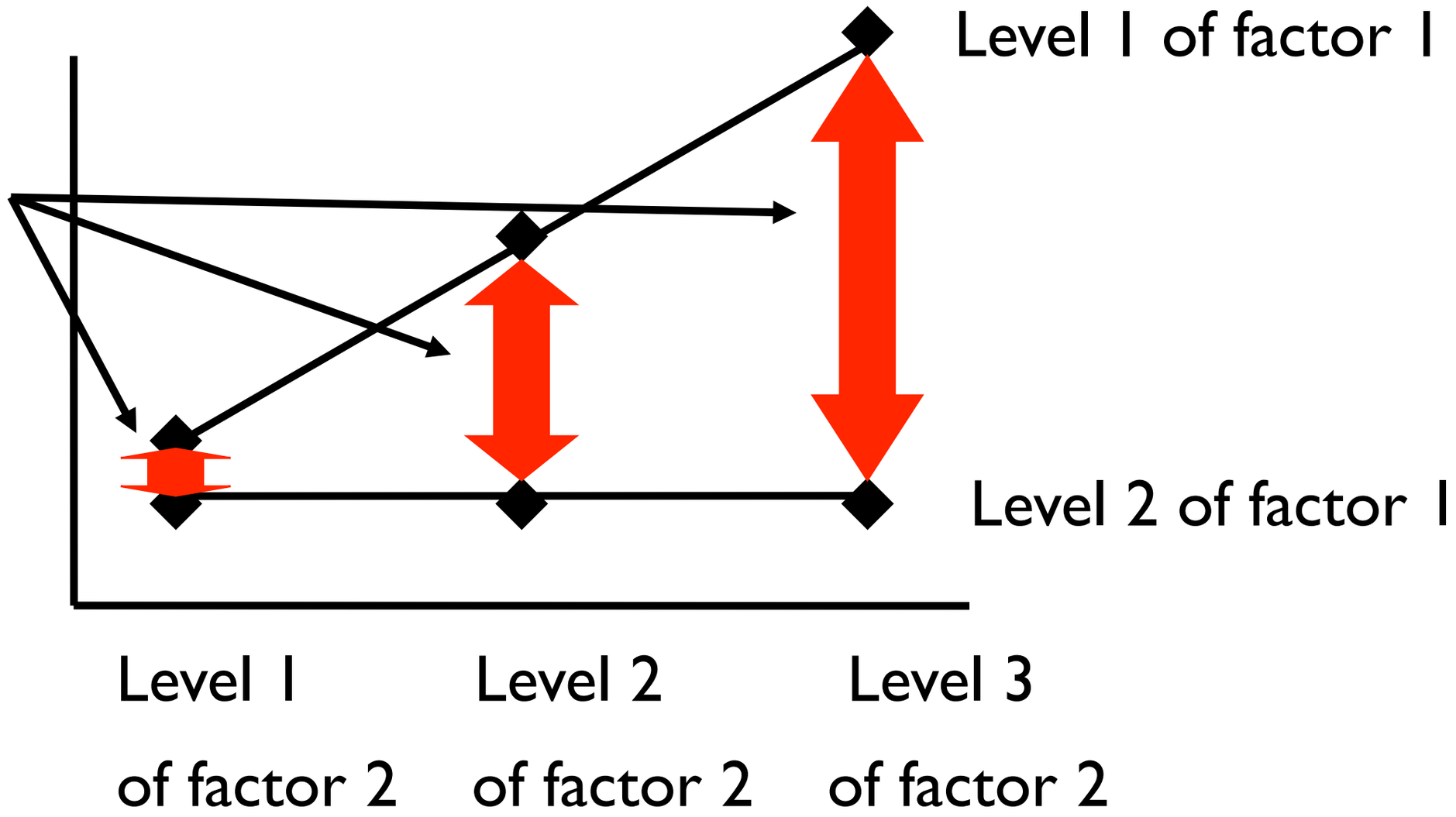
No main effect of Factor 1, main effect of Factor 2 and no interaction

The differences between levels 1 & 2 and 2 & 3 of Factor 2 are all significant and are of the same magnitude. There are no significant differences between levels 1 and 2 of Factor 1.



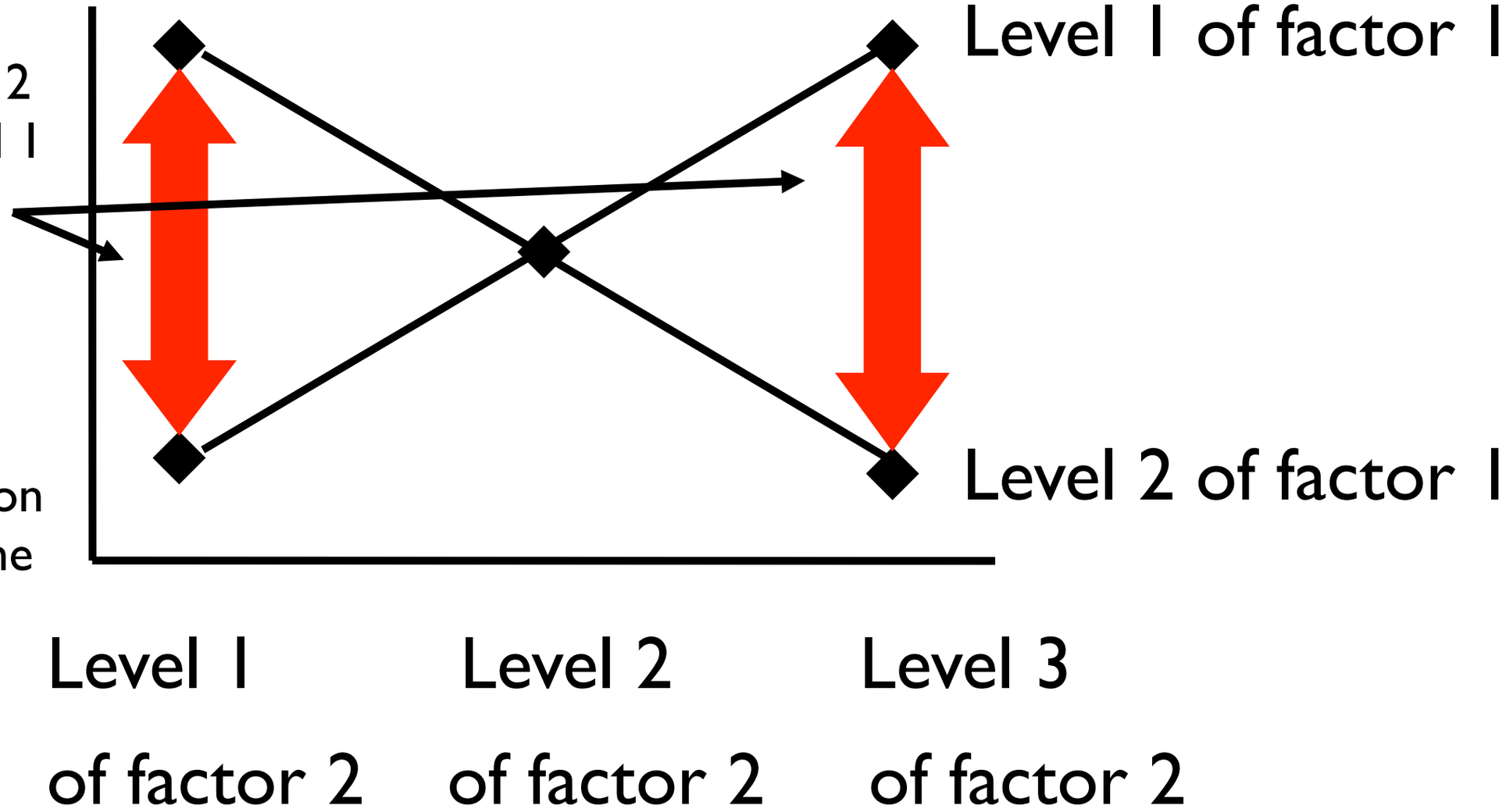
Main effect of Factor 1, main effect of Factor 2 and an interaction

The differences between the two levels of factor 1 change as a function of factor 2.



No main effect of Factor 1, no main effect of Factor 2 but an interaction

The difference between levels 1 & 2 of Factor 1 at Level 1 of Factor 2 is different from the same difference at Levels 2 and 3 of Factor 2. This is a crossover interaction as the polarity of the difference flips.



Factorial ANOVA

- Imagine the case where we're interested in the effect of positive vs. negative contexts on how quickly (in milliseconds) people respond to positive vs negative sentences. We think there might be a priming effect (i.e., people are quicker to respond to positive sentences after positive contexts vs. after negative contexts - and vice versa).

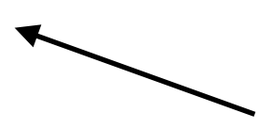
- So, we have two factors, each with two levels. This is what's known as a full factorial design where every subject participates in every condition.

```
fact_data <- read_csv("data_files/fact_data.csv")
fact_data$Sentence <- as.factor(fact_data$Sentence)
fact_data$Context <- as.factor(fact_data$Context)

fact_data

# A tibble: 1,680 x 5
   Subject  Item    RT Sentence Context
     <dbl> <dbl> <dbl> <fct>    <fct>
 1       1     3  1270 Positive Negative
 2       1     7   739 Positive Negative
 3       1    11   982 Positive Negative
 4       1    15  1291 Positive Negative
 5       1    19  1734 Positive Negative
 6       1    23  1757 Positive Negative
 7       1    27  1052 Positive Negative
 8       2     4  1706 Positive Negative
 9       2     8   533 Positive Negative
10       2    12  1009 Positive Negative
# … with 1,670 more rows
```

Generating Descriptives and Visualising the Data

```
fact_data %>%
   group_by(Context, Sentence) %>%
   summarise(mean = mean(RT), sd = sd(RT))
```

```
# A tibble: 4 x 4
# Groups:   Context [2]
  Context  Sentence  mean    sd
  <fct>    <fct>    <dbl> <dbl>
1 Negative Negative 1474.  729.
2 Negative Positive   NA    NA
3 Positive Negative   NA    NA
4 Positive Positive 1579.  841.
```

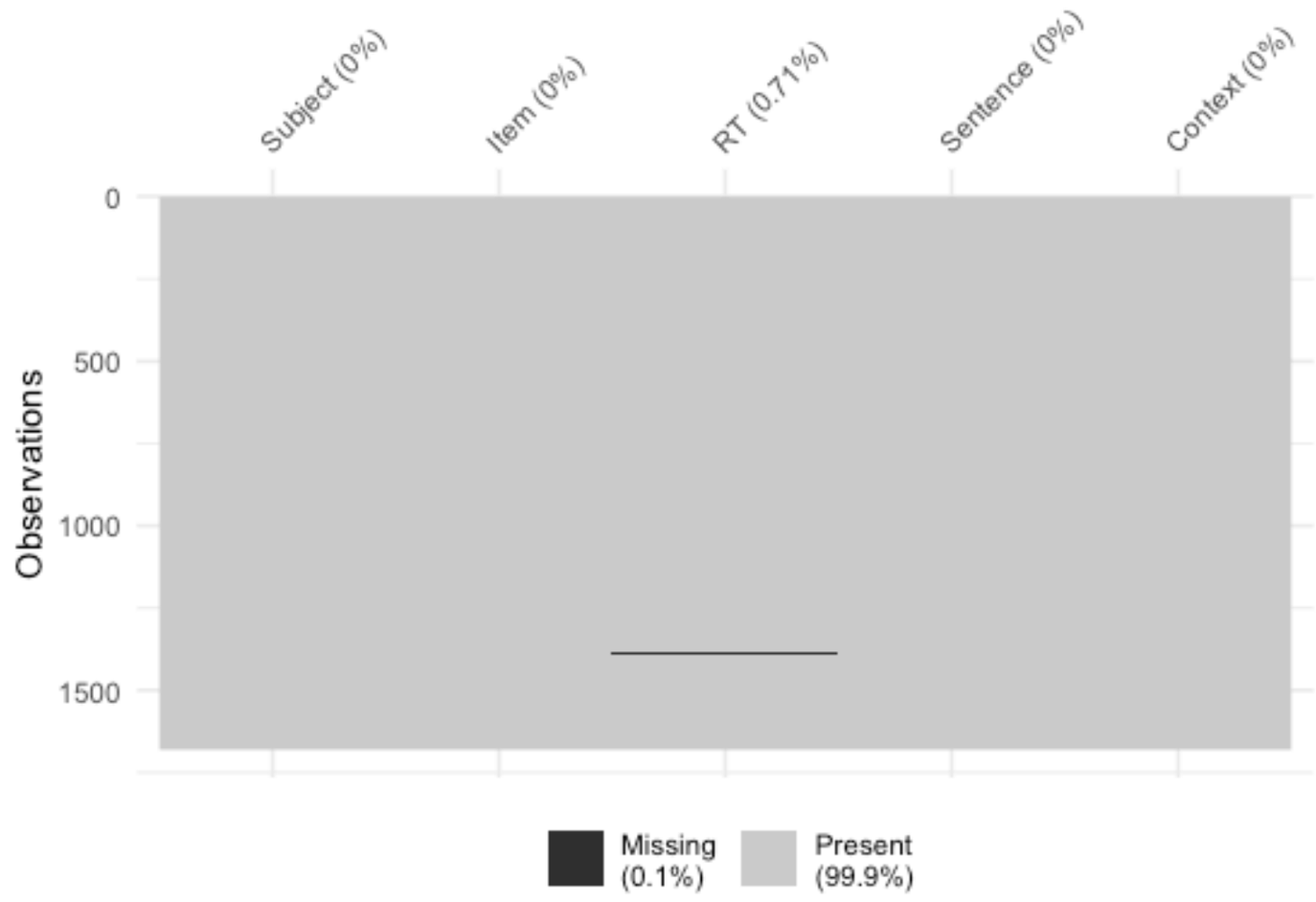


What's going on here?

Let's visualise the whole dataset...

We can use a function in a package without loading it into our library (although we still need to install it) using `package_name::function_name` like this:

```
visdat::vis_miss(fact_data)
```



Let's ignore the missing data (NAs) - one way:

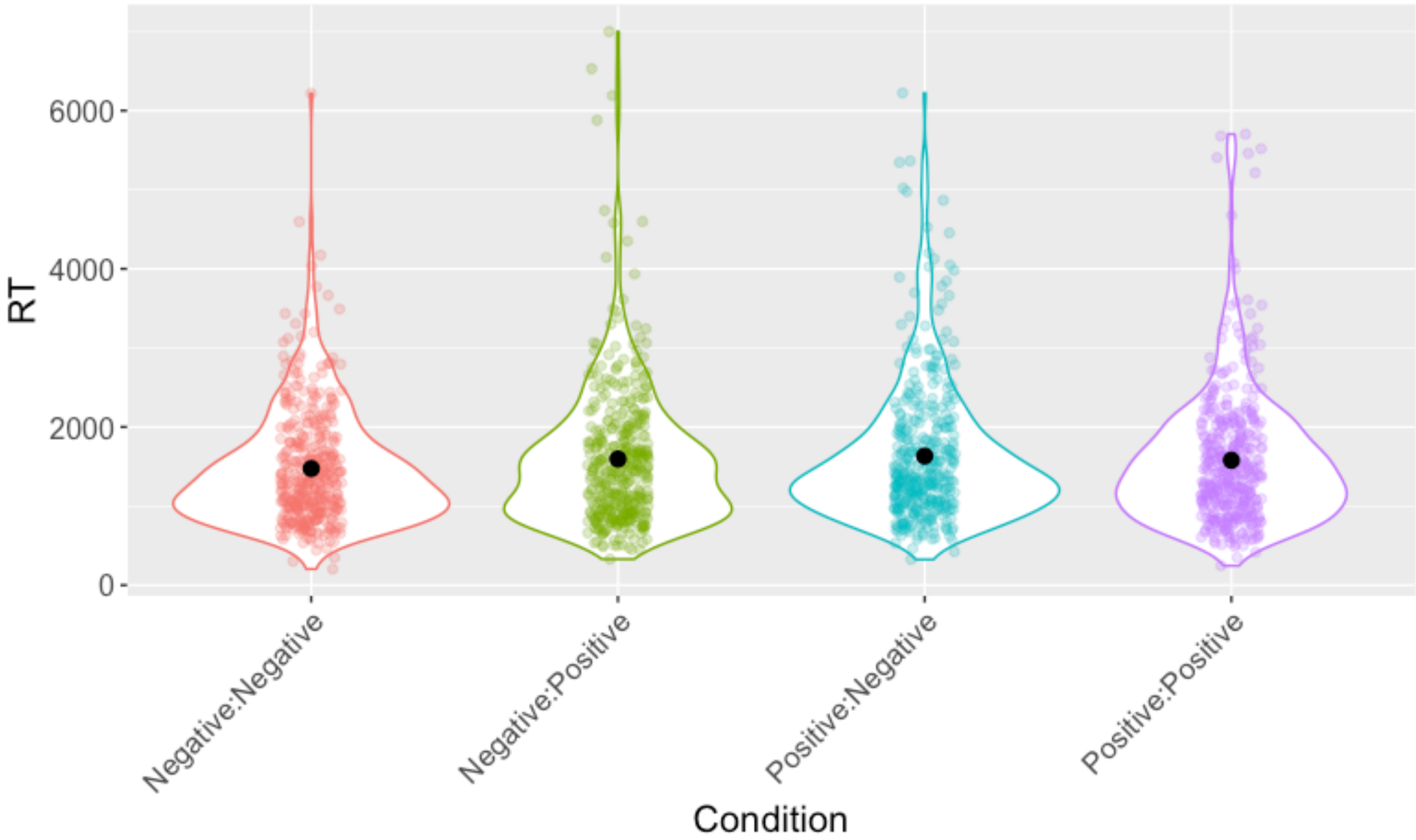
```
fact_data %>%
  filter(!is.na(RT)) %>%
  group_by(Context, Sentence) %>%
  summarise(mean = mean(RT), sd = sd(RT))
```

```
# A tibble: 4 x 4
# Groups:   Context [2]
  Context  Sentence  mean    sd
  <fct>    <fct>    <dbl> <dbl>
1 Negative Negative 1474.  729.
2 Negative Positive 1595.  887.
3 Positive Negative 1633.  877.
4 Positive Positive 1579.  841.
```

or another way...

```
fact_data %>%
  group_by(Context, Sentence) %>%
  summarise(mean = mean(RT, na.rm = TRUE),
      sd = sd(RT, na.rm = TRUE))

# A tibble: 4 x 4
# Groups:   Context [2]
  Context  Sentence  mean    sd
  <fct>    <fct>    <dbl> <dbl>
1 Negative Negative 1474.  729.
2 Negative Positive 1595.  887.
3 Positive Negative 1633.  877.
4 Positive Positive 1579.  841.
```

```
fact_data %>%
  ggplot(aes(x = Context:Sentence, y = RT, colour = Context:Sentence)) +
  geom_violin() +
  geom_jitter(width = .1, alpha = .25) +
  guides(colour = FALSE) +
  stat_summary(fun.data = "mean_cl_boot", colour = "black") +
  theme(text = element_text(size = 15), axis.text.x = element_text(angle =
45, hjust = 1)) +
  labs(x = "Condition")
```



By Subjects

```
model_subjects <- aov_4(RT ~ Context * Sentence + (1 + Context
* Sentence | Subject), data = fact_data, na.rm = TRUE)
```

- Syntax corresponds to RT being predicted by the two factors (`Context * Sentence`) corresponds to two main effects plus the interaction) plus the random effect by Subjects using the datafile called `fact_data`. By setting `na.rm` to be TRUE, we are telling the analysis to ignore individual trials where there might be missing data - effectively this calculates the condition means over the data that is present (and ignores trial where it is missing).

- aov_4 aggregates over the grouping term in the random effect. Simply change to (1 + `Context * Sentence` | Item) for by-item (i.e., F2) analysis. This requires the data to contain the individual observations (not aggregated as means).

```
> anova(model_subjects)
Anova Table (Type 3 tests)

Response: RT
                  num Df den Df    MSE      F        ges  Pr(>F)
Context               1     59  90195 3.1767 0.0060231 0.07984 .
Sentence              1     59 124547 0.6283 0.0016524 0.43114
Context:Sentence      1     59  93889 4.5967 0.0090449 0.03616 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

- The output contains the main effect of Sentence, the main effect of Context, and the interaction between the two. Associated with each are the dfs, the Mean Squared Error, the F ratio, the generalized eta-squared, and p-value. Note, you can ask for partial eta-squared as effect size measure too.

By Items

```
> model_items <- aov_4(RT ~ Context * Sentence + (1 + Context * Sentence | Item),
data = DV, na.rm = TRUE)

> anova(model_items_

Anova Table (Type 3 tests)

Response: RT
                  num Df den Df    MSE      F        ges  Pr(>F)
Context               1     27  39844 4.0013 0.0080150 0.05561 .
Sentence              1     27 203164 0.1221 0.0012553 0.72951
Context:Sentence      1     27  40168 5.7687 0.0116070 0.02346 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

- With the same datafile and just by changing *one* word in the analysis code.

Interpreting Interactions

We can build the model as before and pass the model to the function `emmeans` (remember to load the `emmeans` package) and ask for pairwise comparisons with no correction - we need to work out the Bonferroni corrected values ourselves…

```
> emmeans(model_subjects, pairwise ~ Context * Sentence, adjust = "none")
$emmeans
 Context  Sentence emmean   SE  df lower.CL upper.CL
 Negative Negative   1474 57.8 138     1360     1588
 Positive Negative   1628 57.8 138     1514     1742
 Negative Positive   1595 57.8 138     1481     1709
 Positive Positive   1579 57.8 138     1465     1693

Warning: EMMs are biased unless design is perfectly balanced
Confidence level used: 0.95

$contrasts
 contrast                                estimate   SE  df t.ratio p.value
 Negative,Negative - Positive,Negative    -153.9 55.4 118 -2.779  0.0064
 Negative,Negative - Negative,Positive    -120.9 60.3 116 -2.004  0.0474
 Negative,Negative - Positive,Positive    -105.2 59.8 115 -1.759  0.0813
 Positive,Negative - Negative,Positive      33.0 59.8 115  0.551  0.5824
 Positive,Negative - Positive,Positive      48.7 60.3 116  0.807  0.4213
 Negative,Positive - Positive,Positive      15.7 55.4 118  0.284  0.7772
```

Results

We conducted a 2 (Context: Positive vs. Negative) x 2 (Sentence: Positive vs. Negative) repeated measures ANOVA to investigate the influence of context valence on reaction times to words of the same or different valence. The ANOVA revealed no effect of Sentence ($F < 1$), no effect of Context ($F(1, 59) = 3.18$, $p = .080$, $\eta_G^2 = .006$), but an interaction between Sentence and Context ($F(1, 59) = 4.60$, $p = .036$, $\eta_G^2 = .009$).

The interaction was interpreted by conducting Bonferroni-corrected pairwise companions. These comparisons revealed that the interaction was driven by Negative sentences being processed faster in Negative vs. Positive contexts (1,474 ms. vs. 1,628 ms., $t(118) = 2.78$, $p = .006$) while Positive sentences were read equivalently in Negative vs. Positive contexts (1,595 ms. vs. 1,579 ms., $t(118) = .284$, $p = .777$).

Now for the lab…