

# Week 9 - GLM - ANOVA part 2

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Week	Topic
1	Introduction, Open Science, and Power
2	Introduction to R
3	Data Wrangling and Visualisation
4	General Linear Model - Regression
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12	Dynamic, Reproducible Presentations Using xaringan

## **Semester 1 Assignments**

Data wrangling and visualisation – Due December 5th

ANOVA/ANCOVA – Due January 17th

# More ANOVA...

- Last week we looked at 1-way between participants ANOVA, 1-way repeated measures ANOVA, and 2-way repeated measures ANOVA.
- We used the `afex` package for building our models as it uses Type III Sums of Squares with effect coding of contrasts (allowing us to more easily interpret our results when we have interactions).

# A slightly more complex study

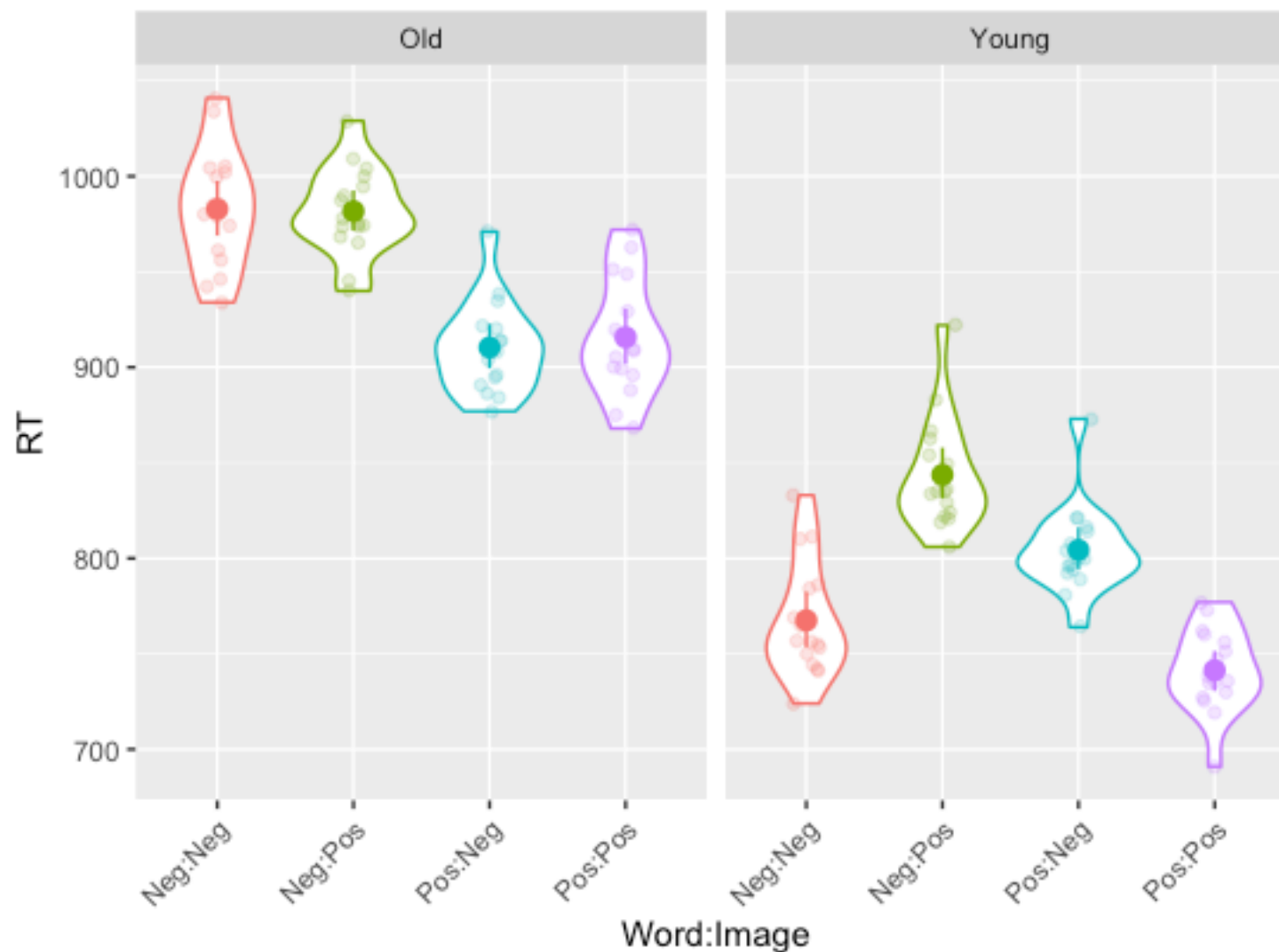
- Similar to our 2 x 2 ANOVA from last time, but let's say we now have Age as an additional factor. It's a between subjects factor. We might think that priming effects might be different for young vs old people.
- So we need to run a 2 x 2 x 2 ANOVA. The first two factors are still within subjects (i.e., repeated measures), but our new one (age) is between subjects and has two levels.

	Participant	Image	Word	Age	RT
1	1	Pos	Pos	Young	719
2	2	Pos	Pos	Young	756
3	3	Pos	Pos	Young	777
4	4	Pos	Pos	Young	691
5	5	Pos	Pos	Young	760
6	6	Pos	Pos	Young	762
7	7	Pos	Pos	Young	735
8	8	Pos	Pos	Young	736
9	9	Pos	Pos	Young	735
10	10	Pos	Pos	Young	727
11	11	Pos	Pos	Young	738
12	12	Pos	Pos	Young	725
13	13	Pos	Pos	Young	730
14	14	Pos	Pos	Young	751
15	15	Pos	Pos	Young	773
16	16	Pos	Pos	Young	747
17	1	Pos	Neg	Young	834
18	2	Pos	Neg	Young	822

Showing 1 to 18 of 128 entries

Remember, for the `aov_4` function we need each factor to be in its own column and for each row to be one observation - this is long or tidy format data.

```
ggplot(my_data, aes(x = Word:Image, y = RT, colour = Word:Image)) +
  geom_violin() +
  geom_jitter(width = .1, alpha = .2) +
  stat_summary(fun.data = "mean_cl_boot") +
  theme(axis.text.x = element_text(angle = 45, hjust = 1)) +
  facet_wrap(~ Age) +
  guides(colour = FALSE)
```



We can see it looks like the Young and Old groups are behaving a little differently.

We need to build our model with two repeated and one between participants factor...

```
model <- aov_4(RT ~ Word * Image * Age + (1 + Word * Image |  
Participant), data)
```

We are asking for the model to be built using the three factors - this will give us three possible main effects, 3 possible 2-way interactions, and a possible 3-way interaction...

The `aov_4` function knows which factors are repeated and which are between from the model structure - note that between participant factors shouldn't appear in the random effects term - if you have only between factors then the term should be something like `(1 | Participant)`...

```

> anova(model)
Anova Table (Type 3 tests)

Response: RT

```

	num	Df	den	Df	MSE	F	ges	Pr(>F)	
Age		1		30	788.86	1017.8790	0.90328	< 2.2e-16	***
Word		1		30	750.85	110.7147	0.49157	1.380e-11	***
Age:Word		1		30	750.85	14.1946	0.11029	0.0007202	***
Image		1		30	752.62	0.8022	0.00697	0.3775568	
Age:Image		1		30	752.62	0.2152	0.00188	0.6460346	
Word:Image		1		30	573.74	61.4309	0.29074	9.553e-09	***
Age:Word:Image		1		30	573.74	73.9247	0.33033	1.363e-09	***

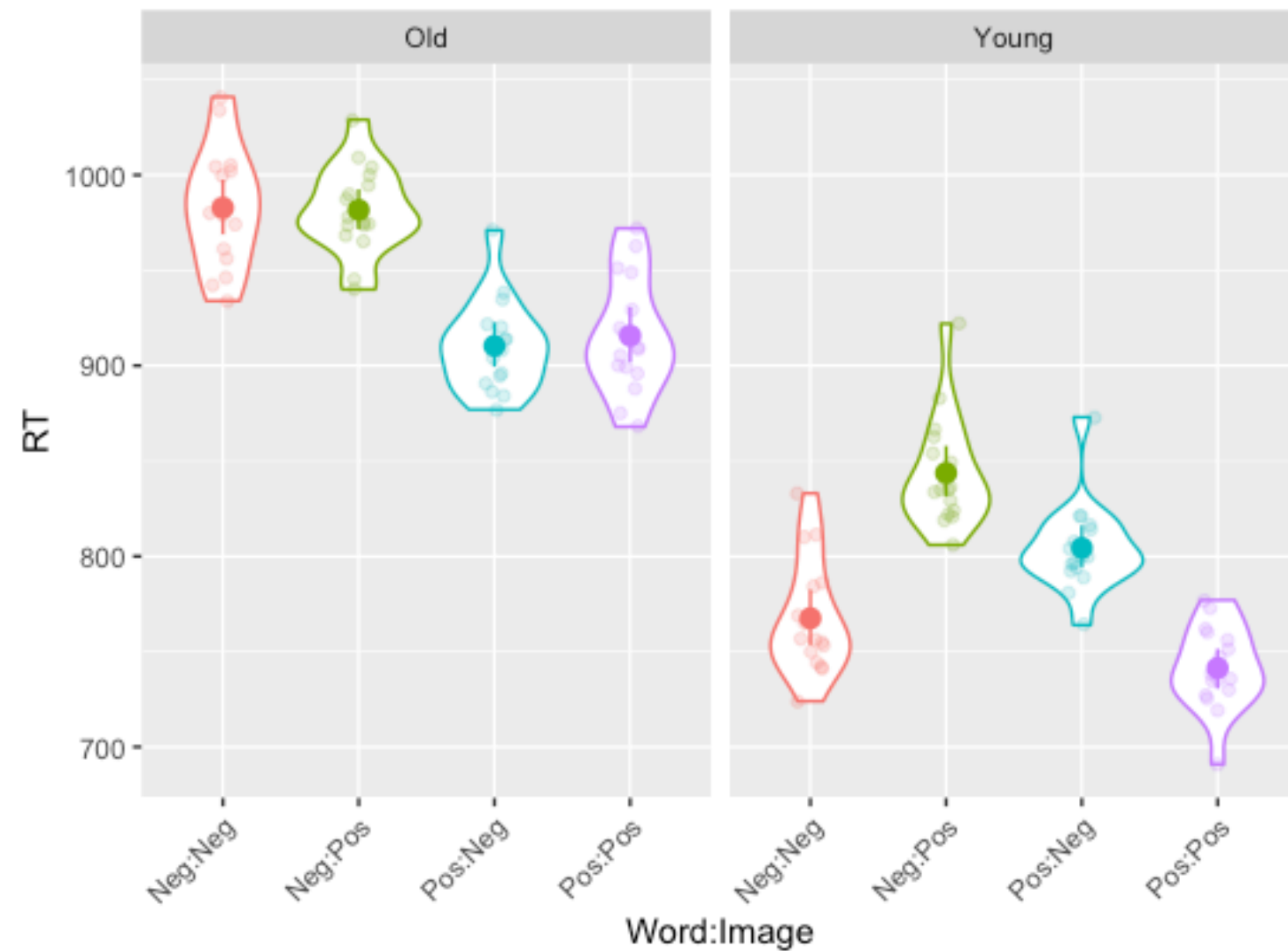
```

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

From this we can see we have main effects of Age and Word, no main effect of Image, significant 2-way interactions of Age x Word, and of Word x Image and, crucially, a 3-way interaction between all three factors - Age x Word x Image...





The 3-way interaction suggests that the Word x Image interaction is different for Young vs. Old people (which is supported by what we see in the graph...)

To interpret this 3-way, we should examine the Word x Image interaction separately for Young and Old people...

We can do this by filtering our dataset:

```
> young_filter <- filter(data, Age == "Young")
> model_young <- aov_4(RT ~ Word * Image + (1 + Word * Image | Participant), data =
young_filter)
> anova(model_young)
Anova Table (Type 3 tests)
```

Response: RT

	num	Df	den	Df	MSE	F	ges	Pr(>F)	
Word		1		15	681.87	25.1197	0.29236	0.0001547	***
Image		1		15	918.44	0.7574	0.01650	0.3978527	
Word:Image		1		15	560.77	138.1887	0.65146	5.725e-09	***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1



**Significant  
interaction**

# We need to follow this up with pairwise comparisons.

```
> emmeans(model_young, pairwise ~ Word * Image, adjust = "Bonferroni")
```

```
$emmeans
```

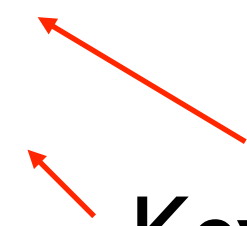
Word	Image	emmean	SE	df	lower.CL	upper.CL
Neg	Neg	768	6.57	57.7	754	781
Pos	Neg	804	6.57	57.7	791	818
Neg	Pos	844	6.57	57.7	831	857
Pos	Pos	741	6.57	57.7	728	755

Warning: EMMs are biased unless design is perfectly balanced  
Confidence level used: 0.95

```
$contrasts
```

contrast	estimate	SE	df	t.ratio	p.value
Neg,Neg - Pos,Neg	-36.9	8.81	29.7	-4.184	0.0014
Neg,Neg - Neg,Pos	-76.2	9.62	28.3	-7.924	<.0001
Neg,Neg - Pos,Pos	26.1	10.00	29.4	2.612	0.0842
Pos,Neg - Neg,Pos	-39.3	10.00	29.4	-3.931	0.0028
Pos,Neg - Pos,Pos	63.0	9.62	28.3	6.552	<.0001
Neg,Pos - Pos,Pos	102.3	8.81	29.7	11.610	<.0001

P value adjustment: bonferroni method for 6 tests



**Key pairwise  
comparisons  
are  
significant**


## But what about the Old group?

```
> old_filter <- filter(data, Age == "Old")
> model_old <- aov_4(RT ~ Word * Image + (1 + Word * Image | Participant), data =
old_filter)
> anova (model_old)
Anova Table (Type 3 tests)
```

Response: RT

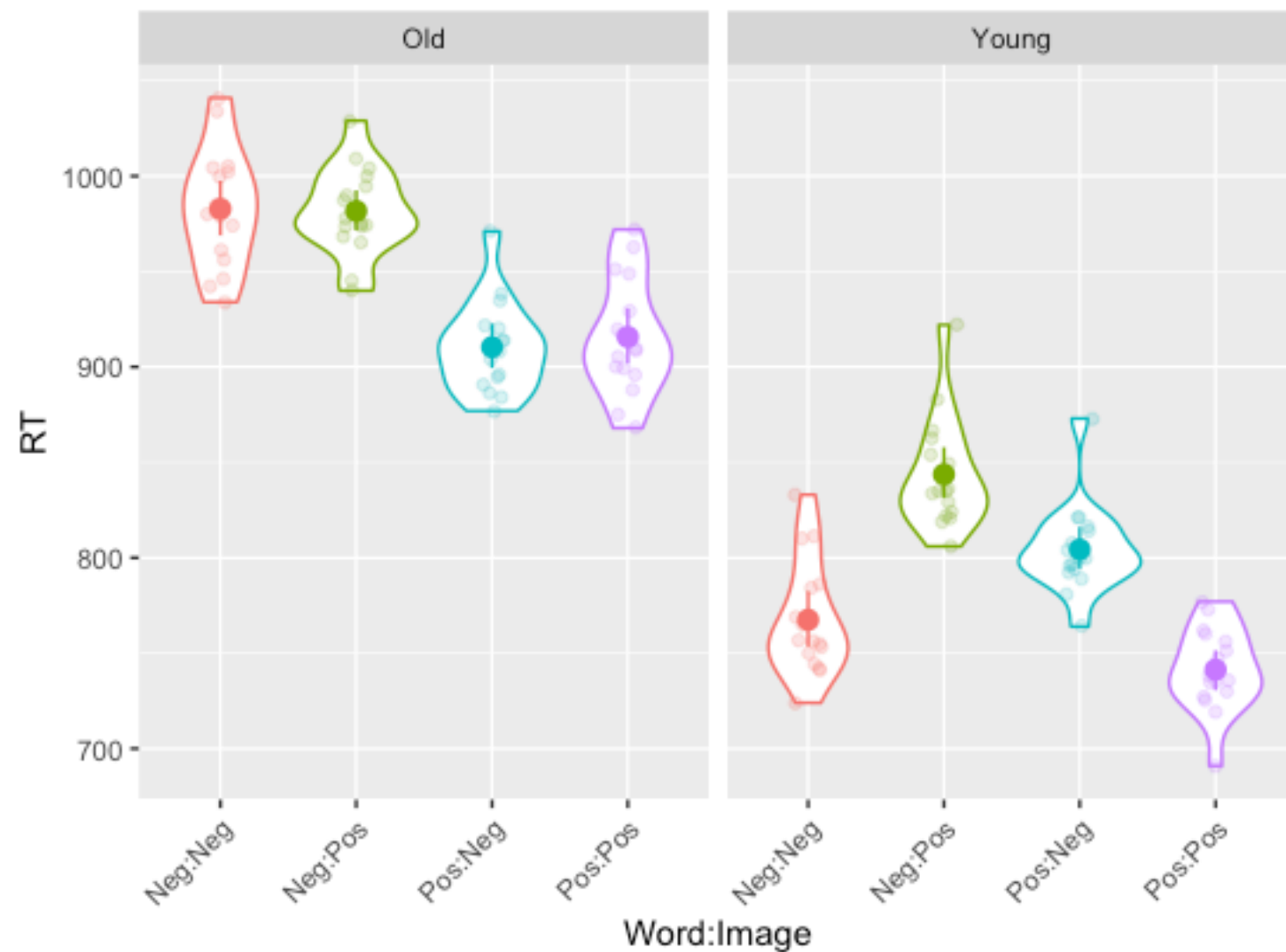
	num	Df	den	Df	MSE	F	ges	Pr(>F)	
Word		1		15	819.83	93.5066	0.63260	7.757e-08	***
Image		1		15	586.81	0.1195	0.00157	0.7343	
Word:Image		1		15	586.70	0.2825	0.00371	0.6028	

---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1



**Interaction  
not significant**

So we have found a 2-way interaction of Word x Image that *\*differs\** between our two groups. The 2-way interaction is significant for our Young group, but not significant for our Old group. For our Old group, we simply have a main effect of Word...

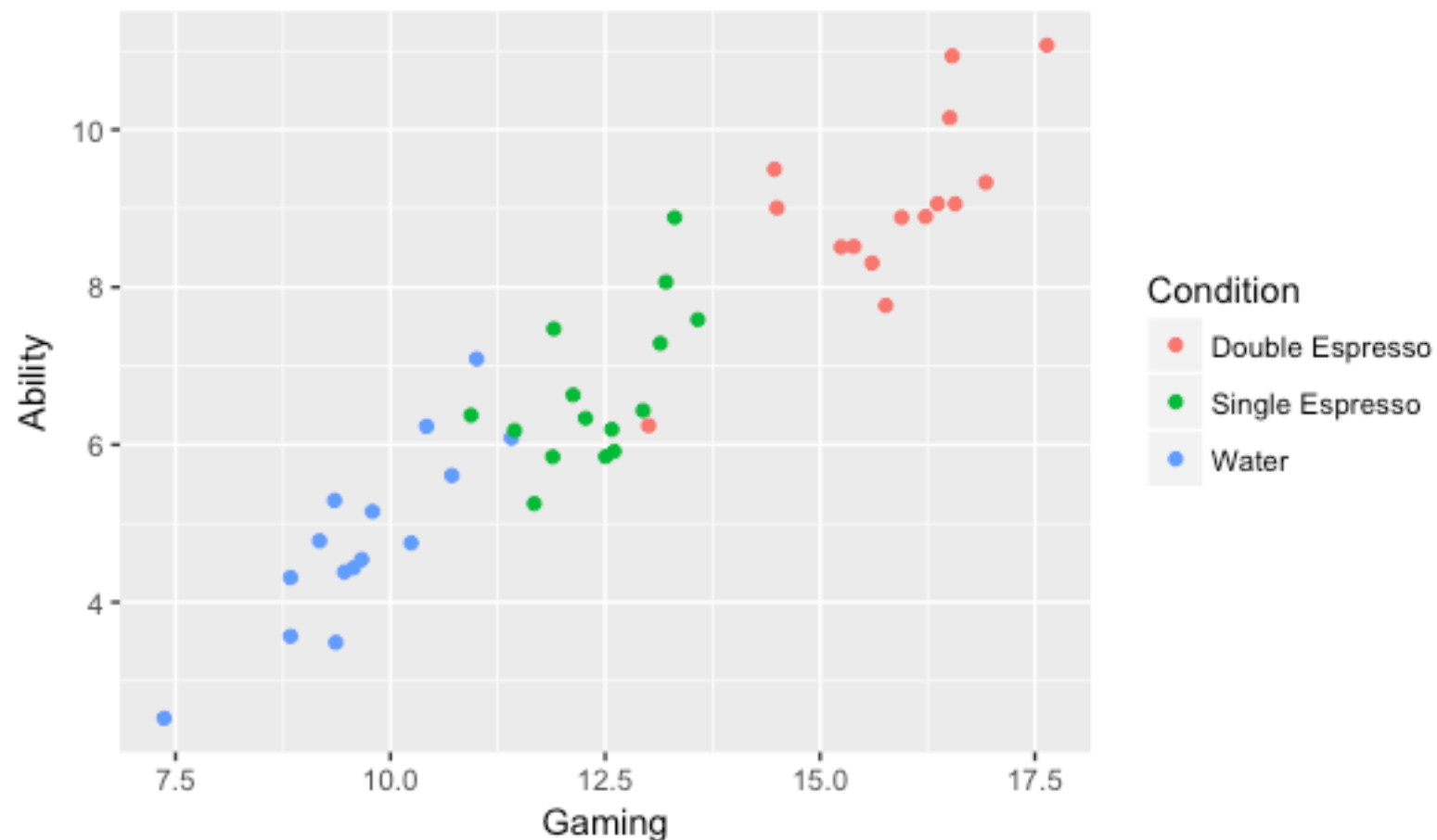


# ANCOVA

- One of our examples from last week looked at how double espresso vs. single espresso vs. water drinking (our IV) might influence motor performance (our DV).
- Imagine we sampled from a new group of participants - and we think other factors that we are not manipulating might also influence the DV – e.g., practice with computer games.
- What we want is to be able to see the effect on our DV of our IV after we have removed the effects of other things (computer gaming frequency in this case).

- Now, imagine we have a measure of computer games frequency - perhaps hours per week people play computer games...
- So, in addition to manipulating the type of beverage we're giving people (i.e., double espresso vs. single espresso vs. water) we also measure how often they play computer games...
- Let's do a plot first with our DV (Ability) on the y-axis, and our covariate (Gaming Frequency) on the x-axis...

```
> ggplot(cond, aes(x = Gaming, y = Ability, colour = Condition)) + geom_point()
```

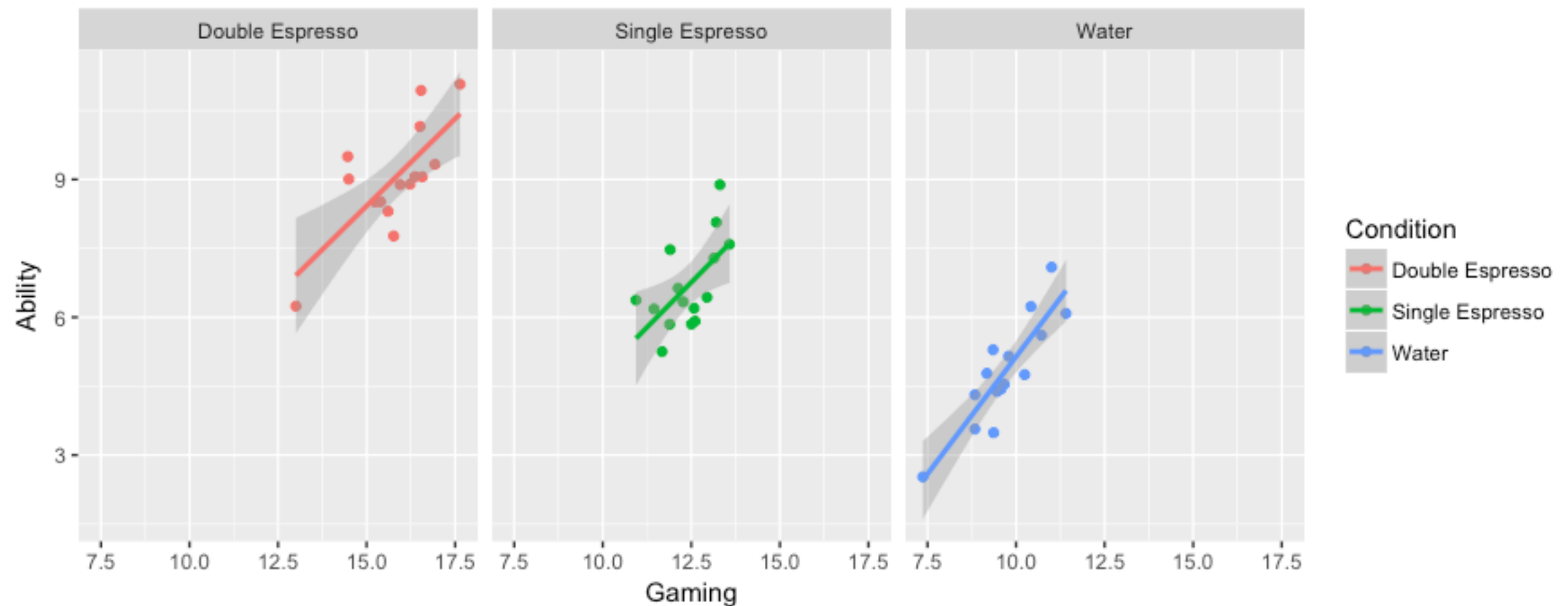


- So we can see there's a relationship between our DV (Ability) and our covariate (Gaming Frequency)...
- We can also see our Gaming Ability groups appear to be clustering in our data by Condition...



We can look at the data separately by condition using the *facet\_wrap()* function:

```
> ggplot(cond, aes(x = Gaming, y = Ability, colour =  
Condition)) + geom_point() + facet_wrap(~ Condition) +  
geom_smooth(method = "lm")
```



# Running a 1-way between participants ANOVA (and ignoring the covariate)...

```
> model1 <- aov_4(Ability ~ Condition + (1 | Participant), data = cond)
Contrasts set to contr.sum for the following variables: Condition
> anova(model1)
Anova Table (Type 3 tests)
```

Response: Ability

	num	Df	den	Df	MSE	F	ges	Pr(>F)
Condition	2		42		1.2422	53.432	0.71786	2.882e-12 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

The factor Condition is significant with an  $F = 53.432$ . We would erroneously conclude that our manipulation has had an effect...

But now let's control for the effect of our co-variate (which we first need to scale and centre)...

```
> cond$Gaming <- scale(cond$Gaming)
> model_ancova <- aov_4(Ability ~ Gaming + Condition + (1 | Participant),
data = cond, factorize = FALSE)
Contrasts set to contr.sum for the following variables: Condition
> anova(model_ancova)
Anova Table (Type 3 tests)
```

Response: Ability

	num	Df	den	Df	MSE	F	ges	Pr(>F)	
Gaming		1		41	0.55171	53.5636	0.56643	5.87e-09	***
Condition		2		41	0.55171	0.8771	0.04103	0.4236	

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

The factor Condition is now not significant with an  $F < 1$ . However, our covariate *Gaming Frequency* is significant. Adding it means a lot of the variance we previously attributed to our experimental factor is actually explained by our covariate. Note, the F values are calculated using Type III Sum of Squares by the `aov_4()` function - more on that in a bit...

Rather than calculating over the raw means which are:

Water Group = 4.82

Double Espresso Group = 9.02

Single Espresso Group = 6.69

```
> cond %>%  
  group_by(Condition) %>%  
  summarise(mean_ability = mean(Ability), sd_ability = sd(Ability))  
# A tibble: 3 x 3  
  Condition      mean_ability sd_ability  
  <chr>          <dbl>      <dbl>  
1 Double Espresso    9.02      1.19  
2 Single Espresso    6.69      0.977  
3 Water              4.82      1.16
```

The calculation is performed over the *adjusted* means (which take into consideration the influence of the covariate):

Water Group = 7.33

Double Espresso Group = 6.32

Single Espresso Group = 6.87

```
> emmeans(model_ancova, pairwise ~ Condition, adjust = "none")
```

```
$emmeans
```

Condition	emmean	SE	df	lower.CL	upper.CL
Double Espresso	6.319464	0.4152816	41	5.480786	7.158142
Single Espresso	6.871614	0.1934303	41	6.480974	7.262255
Water	7.327960	0.3931110	41	6.534056	8.121864

```
Confidence level used: 0.95
```

If our experimental factor in the ANCOVA *had* been significant, we could have looked at the pairwise comparisons reported by *emmeans* to determine what condition was different from what other condition...

```
$contrasts
contrast
Double Espresso - Single Espresso -0.5521505 0.4779448 41 -1.155 0.2547
Double Espresso - Water -1.0084959 0.7614421 41 -1.324 0.1927
Single Espresso - Water -0.4563454 0.4179276 41 -1.092 0.2812
```

But once we take account of the influence of our covariate we found no effect of Condition...

Note, if we had used the `aov()` function the F-tests would have been conducted using Type I (sequential) Sums of Squares. For Type III, we need to use the `aov_4()` function.

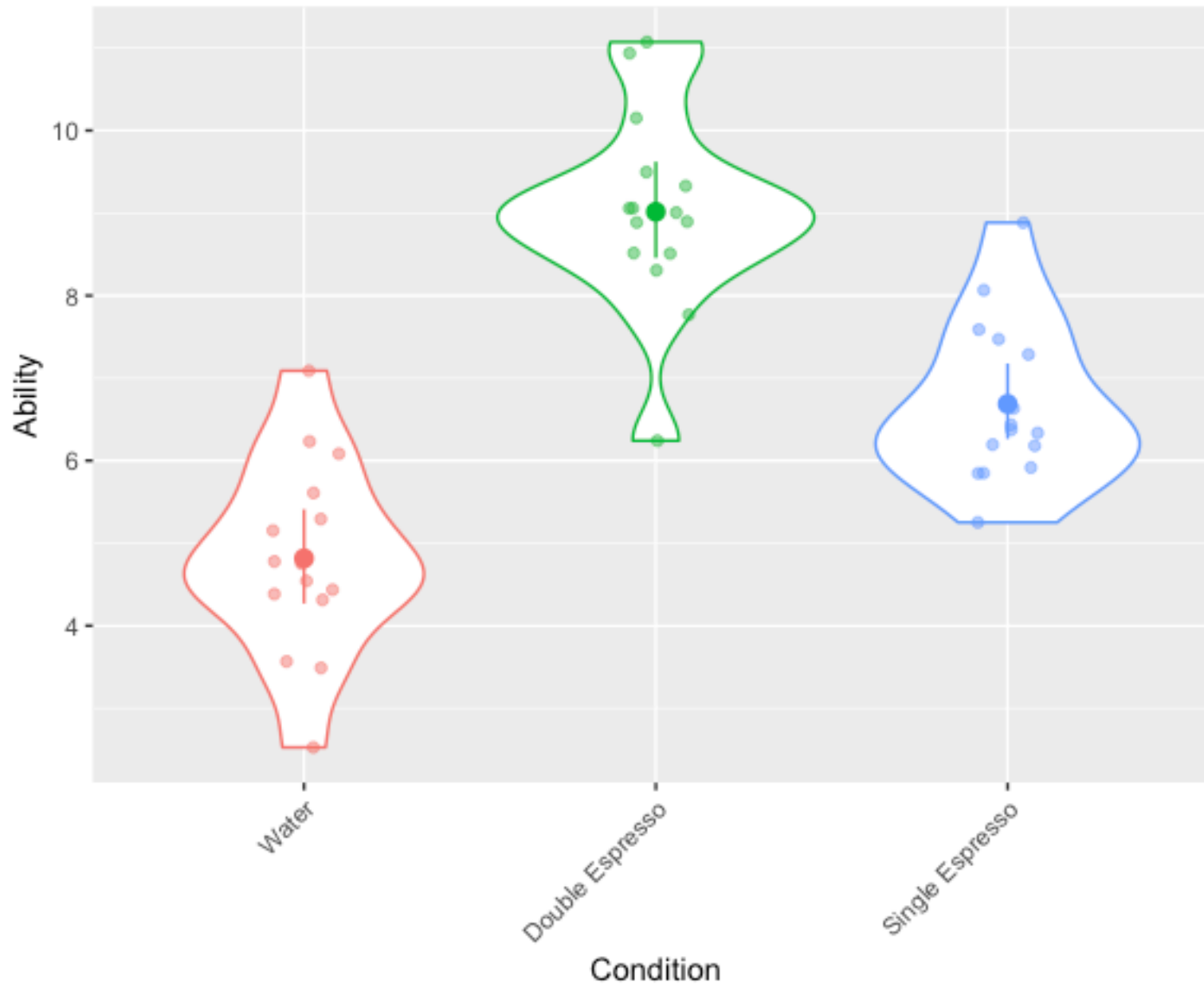
# Type I vs. II vs. III Sums of Squares

- Type I Sum of Squares is calculated sequentially - e.g., first for Factor A main effect, then for Factor B main effect, then for the interaction. The order in which they are calculated matters and can be misleading for unbalanced design or cases where predictors are correlated. Total SS is the sum of the individual effect SS.
- Type II Sum of Squares assumes no interaction(s) when testing main effects or higher order interaction(s) when testing lower order interaction(s).
- Type III Sum of Squares tests for effects adjusted for the presence of the other effects (so does not depend on the order of terms).
- Much debate about which one is 'correct' - each has their own purpose - for factorial designs where you're interested in testing an interaction (or when your predictors correlate), Type III is most commonly used.

# AN(C)OVA as a special case of regression...

- Let's return to the example we looked at for ANCOVA - and let's forget the co-variate for a moment...
- We looked at how double espresso vs. single espresso vs. water drinking (our IV) might influence people's gaming ability (our DV).





Water mean = 4.82

Double Espresso mean = 9.02

Single Espresso mean = 6.69

- First we need to use dummy coding of the levels of our experimental factor - which is the default coding in R for factors...

```
> # Set up the Water level as the reference level and check the contrasts
> cond$Condition <- relevel(cond$Condition, ref = 3)
> contrasts(cond$Condition)
```

	Double Espresso	Single Espresso
Water	0	0
Double Espresso	1	0
Single Espresso	0	1

$$\text{Ability} = \text{Intercept} + \beta_1(\text{Double Espresso}) + \beta_2(\text{Single Espresso}) + \varepsilon$$

The Intercept is our reference category (Water) with coding (0, 0), while the dummy coding for Double Espresso is (1, 0) and for Single Espresso (0, 1)

$$\text{Ability} = \text{Intercept} + \beta_1(\text{Double Espresso}) + \beta_2(\text{Single Espresso}) + \varepsilon$$

We want to calculate  $\beta_1$  and  $\beta_2$

```
> lm1 <- lm(Ability ~ Condition, data = cond)
> lm1
```

Call:

```
lm(formula = Ability ~ Condition, data = cond)
```

Coefficients:

(Intercept)	ConditionDouble Espresso	ConditionSingle Espresso
4.817	4.199	1.871

The intercept is 4.817 (which is the mean of our Water group),  $\beta_1$  is 4.2, and  $\beta_2$  is 1.87

# To work out the mean Ability of our Double Espresso Group:

$$\text{Ability} = \text{Intercept} + \beta_1(\text{Double Espresso}) + \beta_2(\text{Single Espresso}) + \varepsilon$$

$$\text{Ability} = 4.82 + 4.2(1) + 1.87(0) + \varepsilon$$

$$\text{Ability} = 4.82 + 4.2 + \varepsilon$$

$$\text{Ability} = 9.02 + \varepsilon$$

# To work out the mean Ability of our Single Espresso Group:

$$\text{Ability} = \text{Intercept} + \beta_1(\text{Double Espresso}) + \beta_2(\text{Single Espresso}) + \varepsilon$$

$$\text{Ability} = 4.82 + 4.2(0) + 1.87(1) + \varepsilon$$

$$\text{Ability} = 4.82 + 1.87 + \varepsilon$$

$$\text{Ability} = 6.69 + \varepsilon$$

Which are the exact same means generated by the ANOVA...

Water mean = 4.82

Double Espresso mean = 9.02

Single Espresso mean = 6.69



We can do ANCOVA like this too - let's consider our co-variate of Gaming frequency...

The *adjusted* means from the ANCOVA (which take into consideration the influence of the covariate) were:

Water Group = 7.33

Double Espresso Group = 6.32

Single Espresso Group = 6.87

$$\text{Ability} = \text{Intercept} + \beta_1(\text{Gaming}) + \beta_2(\text{Double Espresso}) + \beta_3(\text{Single Espresso}) + \varepsilon$$

Add the covariate to our model *before* the experimental factor:

```
> lm2 <- lm(Ability ~ Gaming + Condition, data = cond)
> lm2
```

```
Call:
lm(formula = Ability ~ Gaming + Condition, data = cond)
```

```
Coefficients:
            (Intercept)              Gaming  ConditionDouble Espresso  ConditionSingle Espresso
                -3.4498                0.8538                -1.0085                -0.4563
```



$$\text{Ability} = \text{Intercept} + \beta_1(\text{Gaming}) + \beta_2(\text{Double Espresso}) + \beta_3(\text{Single Espresso}) + \varepsilon$$

The  $\beta_2$  and  $\beta_3$  coefficients tell us the difference between each group mean (i.e., the adjusted mean) compared to the reference Group (Water) when taking into account the covariate of Gaming frequency:

$\beta_2$  is the difference between the Double Espresso and Water group adjusted means (= -1.01) while  $\beta_3$  is the difference between the Double Espresso and Water group adjusted means (= -0.46)...

Let's check - the following are the adjusted means output by the ANCOVA model:

Water Group = 7.33

Double Espresso Group = 6.32

Single Espresso Group = 6.87

Difference between the Water and Double Espresso Group is 1.01 and the difference between the Water and Single Espresso Group is 0.46...

We can work out the mean of our reference group (Water) by plugging in the values to our equation - note that Gaming is not a factor and we need to enter the mean of this variable (which is 12.62296). So,...

$$\text{Ability} = \text{Intercept} + \beta_1(\text{Gaming}) + \beta_2(\text{Double Espresso}) + \beta_3(\text{Single Espresso}) + \varepsilon$$

$$\text{Ability} = -3.4498 + 0.8538(12.62296) + (-1.0085)(0) + (-0.4563)(0) + \varepsilon$$

$$\text{Ability} = -3.4498 + 10.777 + \varepsilon$$

$$\text{Ability} = 7.33 + \varepsilon$$

7.33 is the adjusted mean for the Water group...which is what we had from calling the `emmeans` function following the ANCOVA...

You can now build ANOVA models in R for different kinds of designs, add between participant co-variates, factor out the influence of these co-variates, and you also know why AN(C)OVA is a special case of regression (with dummy coding of variables)...



**To the computer lab...**