Point Size and Correlation Perception in Scatterplots

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ABSTRACT

Place abstract here.

Index Terms: Human-centered computing—Visualization—Empirical studies in visualization—; Human-centered computing—Human computer interaction (HCI)—Empirical studies in HCI—

1 INTRODUCTION

Scatterplots, utilized in scientific communication for a variety of tasks, are some of the most widely used and widely studied data visualizations. Viewers mostly interpret them in similar ways [11], and they are simple enough to be easily studied while providing important insights into visualization design, human-computer interaction, and human perception. In a previous study [24], we showed that a novel point contrast manipulation, in which the contrast of a certain scatterplot point was reduced as the size of that point's residual increased, could be used to partially correct for a systematic correlation underestimation bias present in the literature [4–6, 13, 14, 17, 23]. We suggested that this was due to a narrowing of the width of the perceived probability distribution of a particular plot brought on by the lower contrast (and therefore lower point-salience) in those outer areas. We tested linear, non-linear, and non-linear inverted functions relating point contrast to residual size, finding that the non-linear function produced the most accurate estimates of correlation, and that the non-linear inverted produced the least accurate.

1.1 Scatterplots and Correlation

Scatterplots have been widely studied, especially as mediums for the communication of correlation (see [24] for a detailed review of the history of this work). Previous literature has found evidence for a pronounced underestimation in judgements of correlation in positively correlated plots, especially between $0.2 < r < 0.6$. The nature of this investigation has varied, ranging from direct estimation, to discriminative judgement, bisection, and staircase tasks. As in our previous work, we choose to use the direct estimation paradigm owing to its simplicity and its suitability to online experimentation. The use of this paradigm renders the judgements we collect comparative by nature, although such work does allow us to inform design guidelines as well as human perception. It is our duty as visualization designers to ensure that the messages we are trying to communicate are being interpreted as accurately as possible by viewers. To do this, we must understand human perception, apply that understanding to design, and test those designs in rigorous empirical studies.

1.2 Point Size

Point contrast is not the only available visual feature that might be used to influence viewer's perceptions of the width of a probability distribution, neither is it the only visual feature of a scatterplot that we can exploit. While contrast adjustments have been used extensively to solve issues of overplotting and clutter in scatterplots [3, 16], there is no established use for varying point size. Common sense dictates that scatterplots visualizing larger datasets inherently require their points to be smaller to prevent obfuscation of the data, but to our knowledge there is little testing of the impact of point size on correlation perception. Studies have found invariance in the bias and variability of correlation perception with regards to changing point sizes [21, 22], but these have been low-powered. From the wider literature there is evidence that larger points are more salient [9], can bias judgements of point position [10] more strongly than point contrast can, and can result in faster reaction times [8] to peripherally presented stimuli. In addition, smaller stimuli are associated with greater levels of spatial uncertainty [1], and if this is driving the reduction in error we saw in our previous work [24], we would expect a similar effect when point size is used instead of point contrast.

1.3 Hypotheses

We hypothesize that correlation estimates will be most accurate when viewers are presented with the non-linear size decay condition, and will be least accurate when presented with the non-linear inverted size decay condition. We thereby present a single experiment study in which we demonstrate that the use of a non-linear size decay function relating to the residuals of points on scatterplots can be employed to correct for a systematic underestimation of correlation by viewers in scatterplots. We find no evidence for an effect of graph literacy or training. The effect we observe here is much stronger, both with regards to effect size and in terms of the observed reduction in error, than the effect we observed in our previous study [24]. We suggest that this function can be used to facilitate more accurate correlation perception in scatterplots, and provide exciting future avenues for the continuation and refinement of these techniques. Ethical approval was granted by the University of Manchester's Computer Science Departmental Panel (Ref: 2022-14660-24397).

2 METHODOLOGY

2.1 Open Research Statement

The experiment was conducted according to the principles of open and reproducible research. All data and analysis code are available at `https://github.com/gjpstrain/size_and_scatterplots`. This repository contains instructions for building a docker image to fully reproduce the computational environment used, allowing for full replications of stimulus generation, analyses, and the paper itself. The experiment was pre-registered with the OSF (`https://osf.io/k4gd8`).

2.2 Participants

150 participants were recruited using the Prolific.co platform. Normal to corrected-to-normal vision and English fluency were required for participation. As in [24], and in accordance with previously published guidelines [19], participants were required to have completed at least 100 studies on Prolific, and were required to have a Prolific score of at least 100, indicating acceptance on at least 100/101 previously completed studies. Participants who took part in any of our previous studies were prevented from participating, and

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participants were only permitted to complete the experiment on a desktop or laptop computer.

Data were collected from 164 participants. 14 failed more than 2 out of 6 attention check questions, and, as per pre-registration stipulations, were rejected from the study. Data from 150 participants was included in the analysis (48% male, 50% female, and 2% non-binary). Mean age of participants was 29.56 (SD = 8.54). Mean graph literacy score was 21.77 (SD = 4.29) out of 30. The average time taken to complete the experiment was 39 minutes (SD = 14 minutes).

2.3 Stimuli

The data used to generated the scatterplots in the current study was identical to that used previously [24]. Scatterplots were generated based on 45 uniformly distributed r values between 0.2 and 0.99. Scatterplot points were generated based on bivariate normal distributions with standard deviations of 1 in each direction. Each scatterplot had a 1:1 aspect ratio, was generated as a 1200 x 1200 pixel .png image, and was scaled up or down according to the participant's monitor. See Sect. 2.4 for a more detailed discussion of precise point sizes and dot pitch in crowd-sourced experiments.

As in our previous study [24], we used equation 1 to map residuals to point sizes in three of our conditions. We used a scaling factor of 4 and a constant of 0.2 to achieve a minimum on-screen point size of 12 pixels, which is consistent with the point size on a 1920 x 1080 monitor for both experiments in [24]. Again, see Sect. 2.4 for a discussion of dot pitch. In our fourth condition, which we refer to as *standard size*, point size was uniformly set to be consistent with the point size in our previous studies. Scripts detailing scatterplot and mask generation can be found in the item preparation folder in the repository linked below.

$$size_{point} = 1 - b^{R} \tag{1}$$

2.4 Dot Pitch and Crowdsourced Experiments

In our previous study [24], we had no way of obtaining dot pitch or participant to monitor distance due to the online, crowdsourced nature of the experiments. Since then we have adopted a method for obtaining the height of a participant's monitor in inches [18]. Combining this with the monitor resolution fetched from Psychopy and assuming a widescreen 16:9 aspect ratio allows us to infer dot pitch and therefore the physical size of the points in our experiment. Mean dot pitch was 0.33mm, ($SD = 0.06$), corresponding to a physical size on the screen of 4.32mm for the smallest points displayed. While dot pitch is not necessary information for the present study, as we are only interested in the relative differences in correlation estimates between point size conditions, the fact that we can collect it at all is indicative of the gap being narrowed with regards to psychophysical testing between in-person and online experiments. Given that the latter are cheaper, easier, and immeasurably quicker, anything that can be done be done to narrow this gap in capability is a boon. See Sect. 3 for analyses including dot pitch as a predictor.

2.5 Visual Threshold Testing

It is key that our manipulation does not functionally remove data from the scatterplot, thus, in order to test that all our points were visible across a range of viewing contexts and on a range of apparatus, we included visual threshold testing prior to the experimental items in the study. Participants were shown six scatterplots with a number of points, and were asked to enter in a textbox how many points were being displayed. The points were the same size as the smallest points used in the experimental materials. 5% of participants were correct on 5 out of 6 visual threshold questions, while 95% were correct on 6 out of 6. It should be noted that those participants scoring 5/6 did not answer incorrectly, rather did not answer at all for this particular questions, which is more suggestive of either a mis-click or an initial misunderstanding of the task they were asked to complete. Regardless, we consider these results to be indicative of a sufficient level of point visibility for the current experiment.

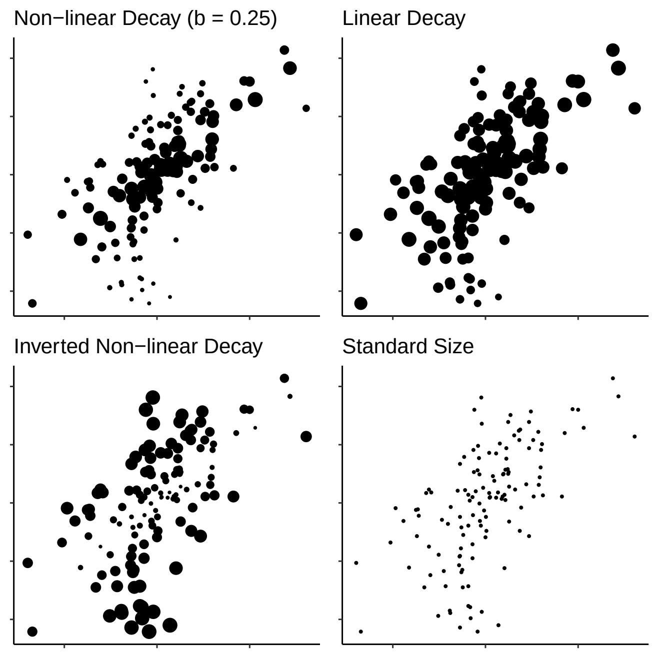

Figure 1: Four levels of the point size decay condition, demonstrated with an r value of 0.6

2.6 Design

The experiment used a fully repeated measures, within-participants design, with each participant seeing and responding to each of the 180 scatterplots in a randomized order. There were four scatterplots for each of the 45 r values corresponding to the four levels of the size decay condition, examples of which can be see in Figure 1. Everything needed to run the experiment, including code, materials, instructions, and scripts, is hosted at `https://gitlab.pavlovia.org/Strain/exp_size_only`.

2.7 Procedure

Each participants was shown the participants information sheet (PIS) and provided consent through key presses in response to consent statements. They were asked to provide their age in a free text box, and their gender identity. Participants then completed the 5-item Subjective Graph Literacy (SGL) test [7], followed by the visual threshold testing described above. Participants then completed the screen scaling task described in Sect. 2.4. Participants were given instructions, and then shown examples of r = 0.2, 0.5, 0.8, and 0.95, as in our previous work [24]. Sect. 4.1 includes a discussion of the potential effects of this training. Two practice trials were given before the experiment began. Participants worked through a series of 180 trials in which they were asked to use a slider to estimate the correlation shown in the scatterplot. Visual masks preceded each plot. Interspersed were six attention check trials which asked participants to set the slider to 1 or 0 and ignore the scatterplot.

3 Results

All analyses were conducted using R (version 4.3.0 [20]). Models were built using the **buildmer** (version 2.8 [25]) and **lme4** (version 1.1-32 [2]) packages, with size decay condition being set as the predictor for participants' errors in correlation estimates.

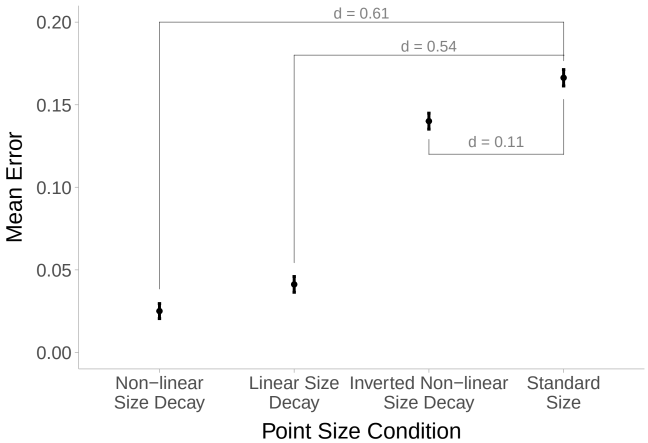

Figure 2: Mean error in correlation estimates across the four size decay conditions, with 95% confidence intervals shown. Brackets show the effect sizes between standard size and all other conditions in Cohen's d.

Table 1: Contrasts between each of the four levels of the size decay condition.

Contrast	Z.ratio	p.value
Non-linear Decay: Linear Decay	-3.99	<0.001
Non-linear Decay : Inverted Non-linear Decay	-20.57	<0.001
Non-linear Decay : Standard Size	-15.41	<0.001
Linear Decay : Inverted Non-linear Decay	-16.86	<0.001
Linear Decay : Standard Size	-11.96	<0.001
Inverted Non-linear Decay : Standard Size	-3.63	0.002

Mean errors in correlation estimates for the four size decay conditions can be seen in figure 2. A likelihood ratio test revealed that the model including size decay condition as a predictor explained significantly more variance than a null model ($\chi^2(3) = 205.35$, $p < .001$). This model has random intercepts for items and participants. The effect here is driven by participants' errors being lower for scatterplots with the non-linear size decay manipulation than for all other conditions, for error being lower for scatterplots with linear size decay than for plots with inverted non-linear decay or standard size, and for errors being higher for scatterplots with standard size than for plots with inverted non-linear decay.

Testing for contrasts between the four levels of the size decay condition were performed with the **emmeans** package (version 1.8.5 [15]), and of correlation estimates are shown in Figure 2. The **EMAtools** (version 0.1.4 [12]) package was used to calculate effects sizes in Cohen's d, the results of which can be seen in 2. The largest effect size we found was 0.61 when comparing the non-linear size decay and standard size decay conditions. This is significantly higher than any of the effects sizes we have found in our previous work.

In addition, we find no significant difference between the experimental model and another including graph literacy as a fixed effect ($\chi^2(1)$ 0.45, $p = .502$), suggesting the effect we found was not driven by differences in graph literacy.

Figure 4 shows how participants' mean errors in correlation estimates change with the objective Pearson's r value, plotted separately for each size decay condition. Note the close-to-zero errors present in the non-linear size decay condition.

As discussed previously, in the present study we employed a method for obtaining a measurement of dot pitch from each participant. While Sect. 2.5 provides evidence that participants had little to no problem perceiving all the dots on the scatterplots shown, there may be some other facet of using a larger or smaller monitor with a larger or smaller resolution that could have affected the estimates our participants gave. To check this, we built a model including the dot pitch measurement as a fixed effect. Comparing this to the experimental model revealed no significant effect of dot pitch ($\chi^2(1) = 4.44$, $p = .035$)

4 Discussion

We found support for our first hypothesis. As can be seen in Figure 4, participants' errors in correlation estimation were significantly lower when they were presented with the non-linear size decay condition (see Figure 1) compared to when they were presented with all other conditions. We found no support for our second hypothesis, that participants' estimates would be worse in the inverted non-linear size decay condition than all other conditions. We found that errors in this condition were indeed significantly higher than for the other two size decay conditions, but that this error was similar to, but significantly lower than, the error with the standard size condition.

The mean error in correlation estimation for the non-linear size decay condition used in the present study was 0.025, while the equivalent condition in the second experiment of our previous work, which used the same equations applied to contrast, resulted in a mean error of 0.086 [24]. Taken together, this is evidence that point size represents a much stronger channel for the manipulation of perceived correlation in scatterplots than contrast. If the effects we have found here and in our previous work are being driven by increased uncertainty in the outer regions of the plots, the fact we have found a large effect of size is congruent with previous research [1, 8, 10] showing clear influences of stimulus size on perception and uncertainty, both in data visualization and elsewhere. Contrary to this, there is little to no (revise) literature linking contrast to perceptual uncertainty. Unlike our previous work, in which the standard deviations of errors for most conditions became smaller as the objective r value increased, participants' distributions of standard deviations of correlation estimates remained mostly constant. This is unexpected, as previous work, including our own, finds precision in r estimation to increase as the objective r value increases. Given that we found this in our previous work [24] manipulating point contrast, and its robustness in the literature, this result is surprising. We suggest that this is due to the nature of the stimuli. At high values of r there is a large amount of clumping and overlap with the non-linear, non-linear inverted,(REVISE?) and linear size decay conditions (see examples in the item preparation folder in the repository linked above). It may be that this clumping is itself producing greater uncertainty and causing an absence of the increased precision we would expect to see at higher r values. While the visual character of the scatterplots in the aforementioned conditions can account for the absence of higher precision with higher r values, the same cannot be said for the standard size condition. Aside from the inverted non-linear decay condition in experiment two of our prior work [24], the finding that precision increased with r was robust. Its absence here is curious given that the standard size decay condition in the present study is identical to the full contrast conditions in our prior work. Taken together, this suggests that there is something particular about the scatterplots in the present study that is causing this. Relying on relative judgements as we are, the interplay between scatterplots with different visual features must be accounted for. Here this interplay seems to have resulted in the absence of this effect, although further testing would be required for a more concrete explanation.

The lack of support for our second hypothesis was, again, surprising, although it should be noted that the difference between the inverted non-linear condition and the standard size condition was small (see Figure 2). The shape of the errors in correlation estimates (see Figure 4) is also very similar to that of the standard size decay condition. It would seem then that despite the size channel being more powerful than contrast with regards to correcting for the under-

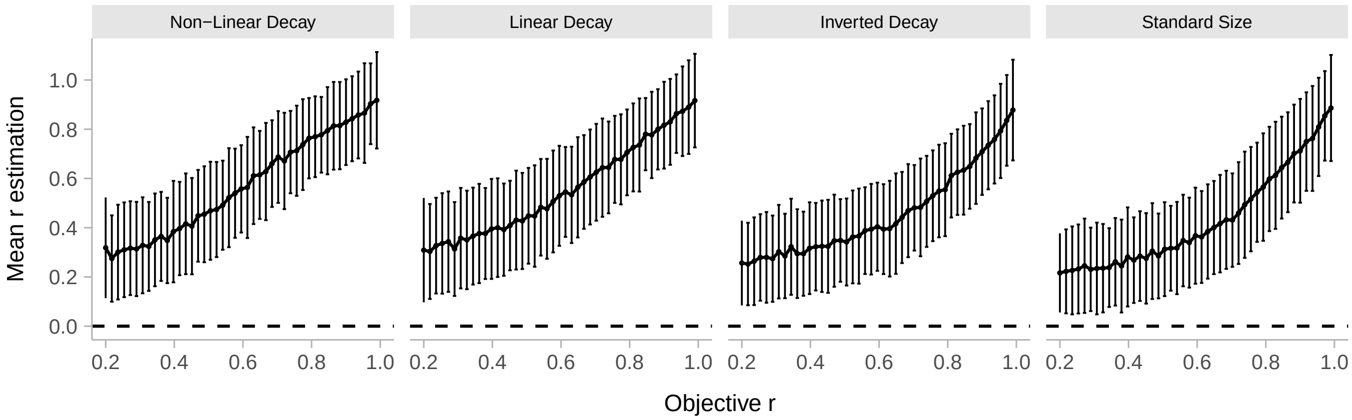

Figure 3: Participants' mean r estimates plotted against the objective r value separately for each size decay condition.

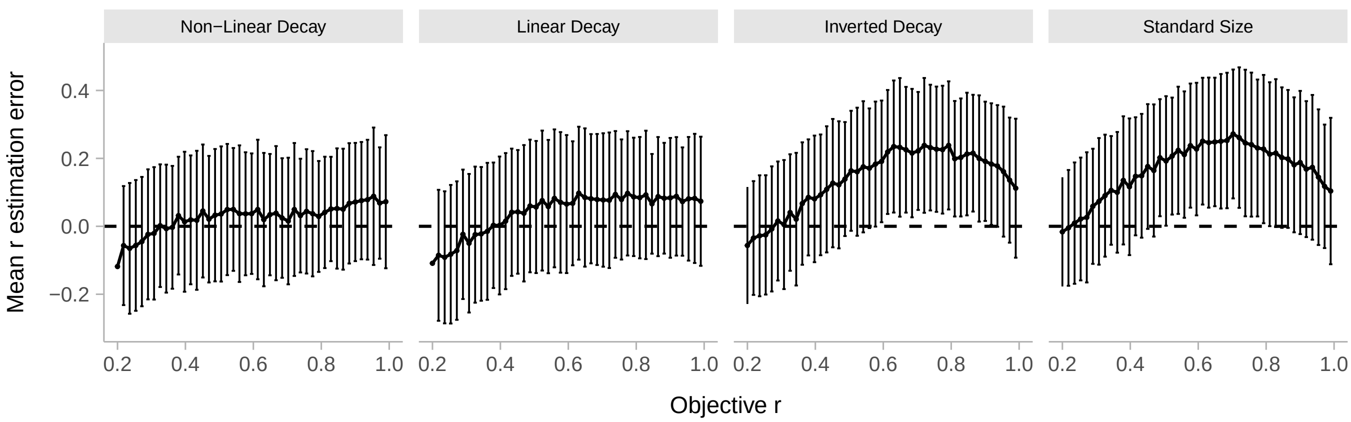

Figure 4: Participants' mean errors in r estimates plotted against the objective r value separately for each size decay condition. The dotted horizontal line represents perfectly accurate estimation.

estimation of correlation, it is weaker than the contrast channel at producing the opposing effect. In our previous work we suggested that contrast manipulations could be used to correct for the *over-estimation* of the correlation of negatively correlated scatterplots; we would not suggest the use of the size channel for this given the results here.

4.1 Training

Before the experiment began, participants viewed plots for a minimum of eight seconds with examples of $r = 0.2$, 0.5, 0.8, and 0.95. This was to account for any potential unfamiliarity with scatterplots present in the samples that we recruited; this risk is inherent in recruiting from lay populations, but we would argue is acceptable given it leads to more generalisable and broadly applicable findings. To test whether this training had an effect on correlation estimation, we built a model including session half as a predictor. Comparing this to the original model revealed no significant effect ($\chi^2(1) = 1.28$, $p = 0.26$), suggesting that training having more recently viewed the example plots did not have an effect on participants' estimates of correlation.

4.2 Limitations

The data we have gathered on correlation estimation is inherently comparative in nature. Despite confirming a method of calculating dot pitch for each participant, we still have no method of obtaining head-to-monitor distances, nor does it seem there is one on the horizon. Taken together, these aspects of our experiment prevent us from making rigorous psychophysical conclusions, but instead allow for findings that are rigorous to different viewing contexts that we argue are of particular importance for the HCI and design audiences.

4.3 Future Work

At present we have confirmed the potential for both point contrast and size manipulations to influence participants' perceptions of correlation in scatterplots, each to varying degrees. It is also clear that these manipulations are not necessary, and may even be making perception worse, at very low and high values of r. Our future work will therefore take two directions:

1. We will investigate the effect of manipulating both point contrast and point size. Will these effects be additive, and to what degree?

2. We will introduce a parameter to control the strength of the manipulation according to the objective r value in order to fine tune the effects we have found.

Acknowledgments

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