DATA440 Final Project Math

$$Var[X] = E[(X-\mu)^2]$$

$$= E[X^2 - 2X\mu + \mu^2]$$

$$= E[X^2] - E[2X\mu] + E[\mu^2]$$

$$= E[X^2] - 2\mu^2 + \mu^2$$

$$= E[X^2] - \mu^2$$

Scenario 1

Dice: [6, 4]

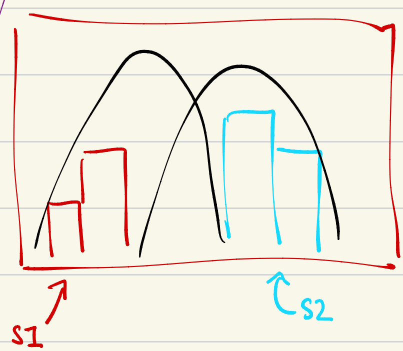
modifier: 1

Scenario 2

Dice: [8, 8]

modifier: 2

rolls: 100



if you roll once, get one answer. If you roll 100 times have something to actually compare.

$$E[X] = \sum p_i \cdot x_i$$

convolution:

Ex:

D3 $f(x) = \frac{1}{3}$

D2 $f(x) = \frac{1}{2}$

#1

1/3 1/3 1/3
1 2 3
1
2
1/2 1/2

$= \frac{1}{6}$

; Slide

$h(2) = 1/6$

#2

1/3 1/3 1/3
1 2 3
2 1
1/2 1/2

1/6 1/6

$1/6 + 1/6 = 2/6 = h(3)$

#3

1/6 1/6
1/3 1/3 1/3
1 2 3
2 1
1/2 1/2

$h(4) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

#4

1/3 1/3 1/3
1 2 3
2 1
1/2 1/2

$h(5) = \frac{1}{6}$

$h(z) = 1/6$	2
2/6	3
2/6	4
1/6	5

$$E[Z] = 1/6 \cdot 2 + 2/6 \cdot 3 + 2/6 \cdot 4 + 2/6 \cdot 5$$

NOTE:

mean def:

$E[X] = \mu$

variance def:

$Var(X) = E[(X-\mu)^2] = E[X^2] - \mu^2$

Expected value of a random variable * a constant:

$E[kX] = k\,E[X]$

Variance of a random variable * a constant (variance is quadratic operator)

$Var(kX) = k^2\,Var(X)$

Expected value of a random variable plus a constant

$E[X + k] = E[X] + E[k] = \mu + k$ mean gets effected by constant

Variance of a random variable plus a constant:

$Var(X + k) = Var(X) + Var(k) = Var(X) + 0$ since k is constant, it doesn't effect var.

$Var[kX] = k^2 \, Var[X]$

rely upon: $E[kX] = kE[X]$

$Var[X] = E[X^2] - (E[X])^2$

$\boxed{\text{proof: let } \underline{Y = kX}}$

$\underset{Var[kX]}{\underbrace{Var[Y]}} = E[Y^2] - (E[Y])^2$;

$= E[(kX)^2] - (E[kX])^2$;

$= E[k^2 X^2] - (E[kX])^2$;

$= k^2 E[X^2] - (kE[X])^2$;

$= k^2 E[X^2] - k^2 (E[X])^2$;

$= k^2 (\underbrace{E[X^2] - (E[X])^2}_{\text{variance}})$;

$= k^2 \, Var[X]$

* shown that variance of a constant * RV is the same as constant^2 * variance

- Dice rolls are random variables
- They are independent

☆ The distribution of a sum of <u>independent</u> random variables is the convolution of their distributions.

Ex: $E[X] = \sum p_i * x_i$

D4

$F(x) = 1/4$

D6

$F(x) = 1/6$

$1/6 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6$

$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$4 \quad 3 \quad 2 \quad 1$

$1/4 \quad 1/4 \quad 1/4 \quad 1/4$

$1/6 \cdot 1/4 = 1/24 = h(2)$

$1/24 \quad 1/24$

$1/6 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6$

$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$4 \quad 3 \quad 2 \quad 1$

$1/4 \quad 1/4 \quad 1/4 \quad 1/4$

$2/24 = 1/12 = h(3)$

$1/6 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6$

$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$4 \quad 3 \quad 2 \quad 1$

$1/4 \quad 1/4 \quad 1/4 \quad 1/4$

$3/24 = h(4)$

$1/6 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6$

$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$4 \quad 3 \quad 2 \quad 1$

$1/4 \quad 1/4 \quad 1/4 \quad 1/4$

$4/24 = 2/12 = 1/6 = h(5)$

1/6 1/6 1/6 1/6 1/6 1/6

1 2 3 4 5 6

4 3 2 1

1/4 1/4 1/4 1/4

$4/24 = 1/6 = h(6)$

1/6 1/6 1/6 1/6 1/6 1/6

1 2 3 4 5 6

4 3 2 1

1/4 1/4 1/4 1/4

$4/24 = 1/6 = h(7)$

1/6 1/6 1/6 1/6 1/6 1/6

1 2 3 4 5 6

4 3 2 1

1/4 1/4 1/4 1/4

$3/24 = h(8)$

1/6 1/6 1/6 1/6 1/6 1/6

1 2 3 4 5 6

4 3 2 1

1/4 1/4 1/4 1/4

$2/24 = 1/12 = h(9)$

1/6 1/6 1/6 1/6 1/6 1/6

1 2 3 4 5 6

4 3 2 1

1/4 1/4 1/4 1/4

$1/24 = h(10)$

$h(z) =$

1/24	2
1/12	3
3/24	4
1/6	5
1/6	6
1/6	7
3/24	8
1/12	9
1/24	10

$E[Z] =$

$1/24 * 2 + 1/12 * 3 + 3/24 * 4 + 1/6 * 5 +$

$1/6 * 6 + 1/6 * 7 + 3/24 * 8 + 1/12 * 9 +$

$1/24 * 10$

$= 6$

Scenario 1: [D6, D4]

- 1 roll of D6
- 1 roll of D4

★ The distribution of a sum of independent random variables is the convolution of their distributions.

$$E[Z] = \tfrac{1}{24}*2 + \tfrac{1}{12}*3 + \tfrac{3}{24}*4 + \tfrac{1}{6}*5 + \tfrac{1}{6}*6 + \tfrac{1}{6}*7 + \tfrac{3}{24}*8 + \tfrac{1}{12}*9 + \tfrac{1}{24}*10$$

μ:

$$E[X] = \mu$$

μ * constant

$$E[kX] = kE[X]$$

var * constant:

$$Var(kX) = k^2 Var(X)$$

μ + constant

$$E[X + k] = \mu + k$$

$$E[\mu] \quad E[k]$$

var + constant

$$Var(X + k) = Var(X) + 0$$

variance:

$$Var(X) = E[X^2] - \mu^2$$

convolution can be used as μ

Prob. of getting results	possible results
1/24	2
1/12	3
3/24	4
1/6	5
1/6	6
1/6	7
3/24	8
1/12	9
1/24	10

$$E[Z] = 6 \quad \therefore \quad E[X] = 6$$

$$var(X) = E[X^2] - \mu^2 ;$$

$$Var(x) = E[x^2] - 6^2 ;$$

mean:

$$\mu = \tfrac{1}{24}*2 + \tfrac{1}{12}*3 + \tfrac{3}{24}*4 + \tfrac{1}{6}*5 + \tfrac{1}{6}*6 + \tfrac{1}{6}*7 + \tfrac{3}{24}*8 + \tfrac{1}{12}*9 + \tfrac{1}{24}*10$$
$$= 6$$

$$E[X^2] = \sum X^{2} \cdot P(X = x)$$

$$= \left(2^2 * \tfrac{1}{24}\right) + \left(3^2 * \tfrac{1}{12}\right) + \left(4^2 * \tfrac{3}{24}\right) + \left(5^2 * \tfrac{1}{6}\right) + \left(6^2 * \tfrac{1}{6}\right) + \left(7^2 * \tfrac{1}{6}\right) + \left(8^2 * \tfrac{3}{24}\right) + \left(9^2 * \tfrac{1}{12}\right) + \left(10^2 * \tfrac{1}{24}\right)$$

$$0.1666666667 + 0.75 + 2 + 4.1666666667 + 6 + 8.1666666667 + 8 + 6.75 + 4.1666666667$$

$$= 40.1666666667$$

$$Var(X) = E[X^2] - \mu^2$$

$$\mu^2 = 6^2 = 36$$

$$E[X^2] = 40.1666666667$$

$$40.1666666667 - 36$$

$$= 4.1666666667$$

$1/24 = \sim 0.0416$

$1/12 = \sim 0.08\overline{333}$

$3/24 = 0.125$

$1/6 = \sim 0.1666\ldots7$