

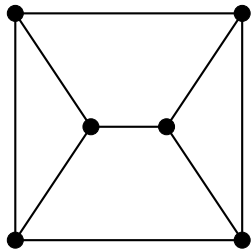
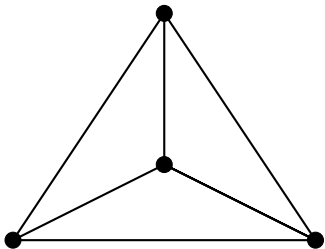
The Graph Minor Theorem

Aaron Tharsius

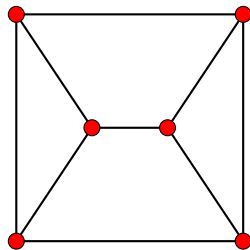
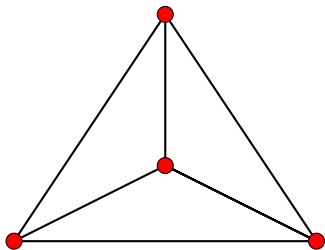
Advisor: Dr. Bogdan Oporowski
Department of Mathematics
Louisiana State University

April 9, 2019

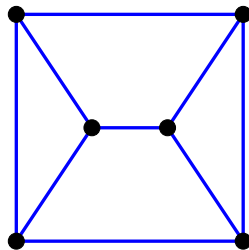
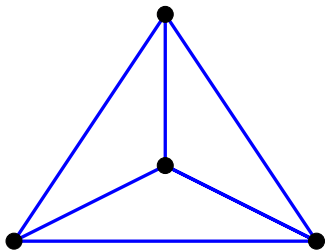
Graphs



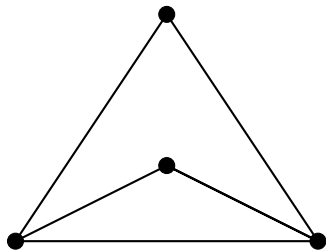
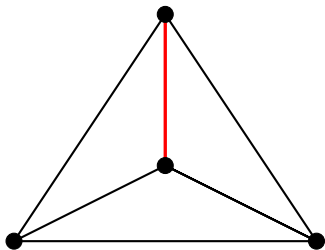
Graphs (Vertices)



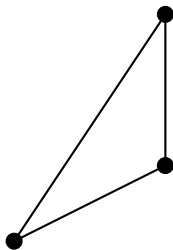
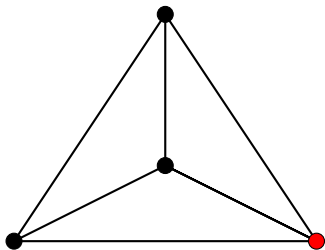
Graphs (Edges)



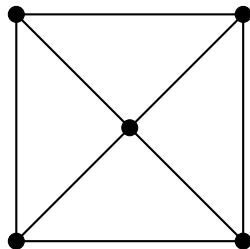
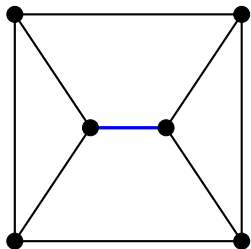
Edge Deletion



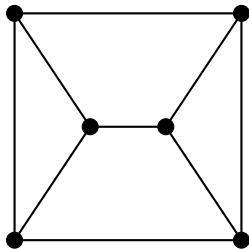
Vertex Deletion



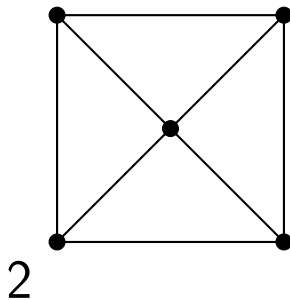
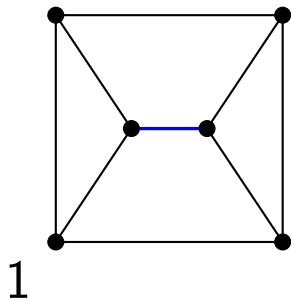
Edge Contraction



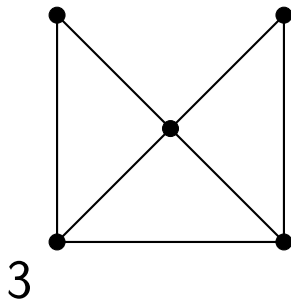
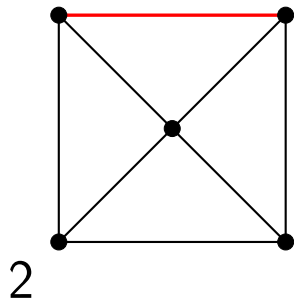
Minor Relation



Minor Relation

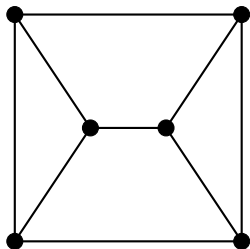


2 is a minor of 1

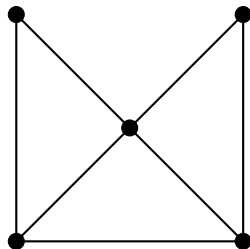


3 is a minor of 1 and 2

Planar Graphs

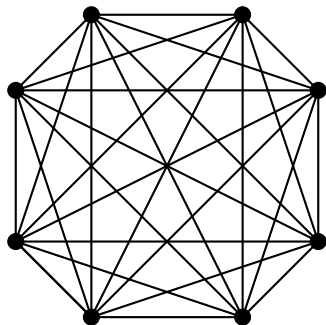


1

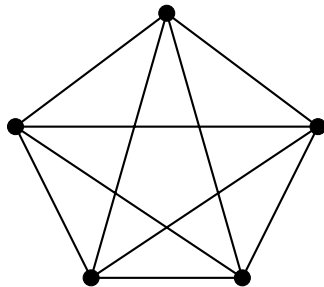
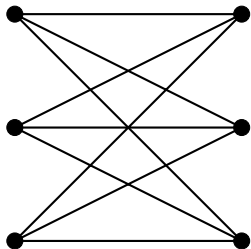


3

Planar Graphs



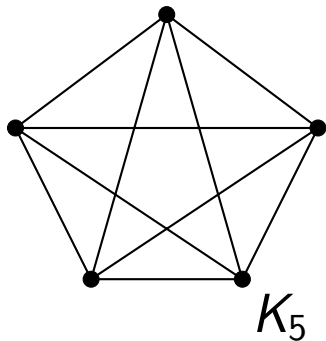
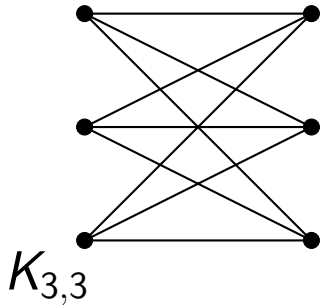
Planar Graphs



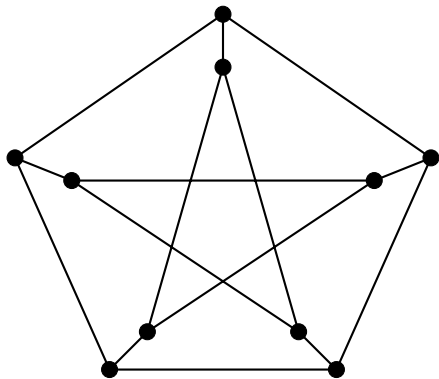
Kuratowski's Theorem

Theorem: Kuratowski (Wagner's Variation)

A graph is embeddable in the plane if it does not have K_5 or $K_{3,3}$ as minors.

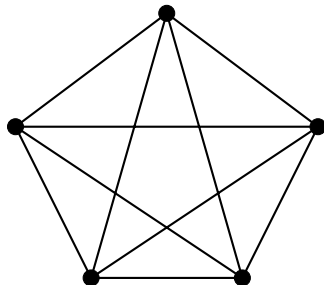
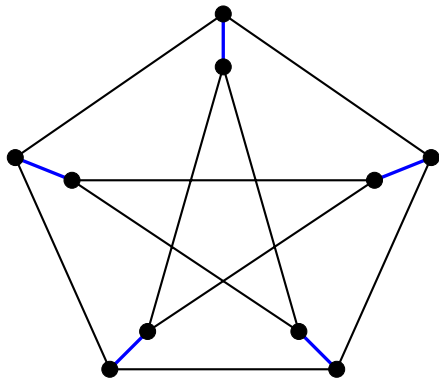


Petersen Graph

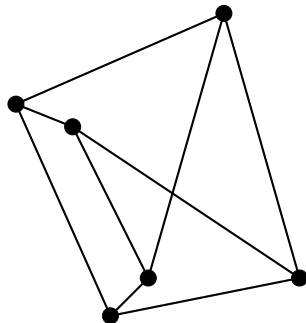
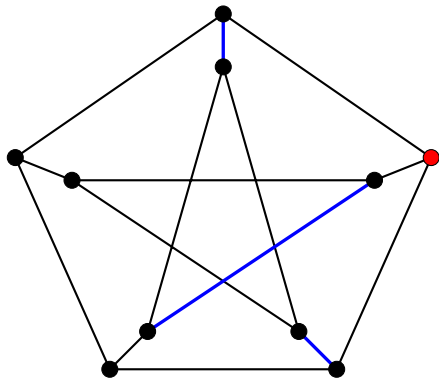


$GP(5, 2)$

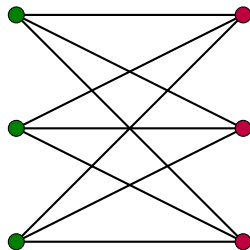
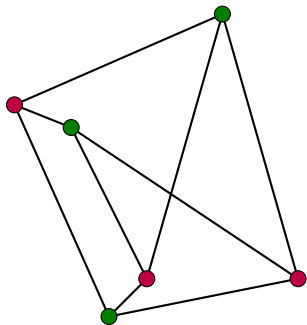
Petersen Graph



Petersen Graph

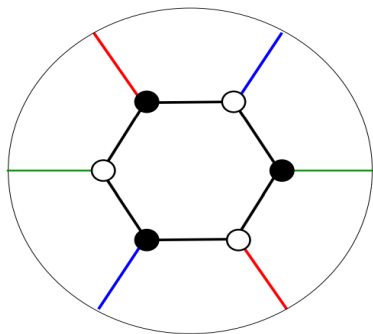


Petersen Graph

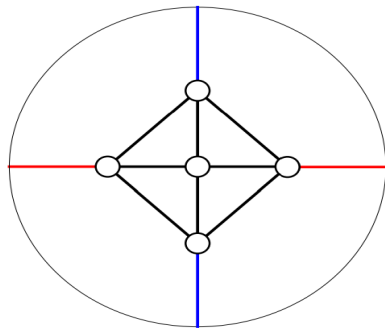


Question: Does a Kuratowski-like theorem exist for other surfaces?

Projective Plane (Archdeacon)

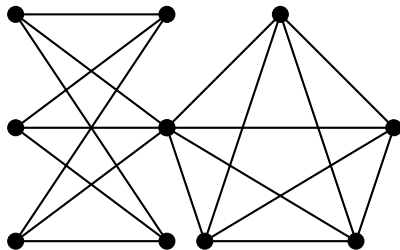
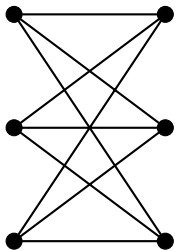
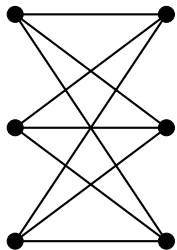


$K_{3,3}$

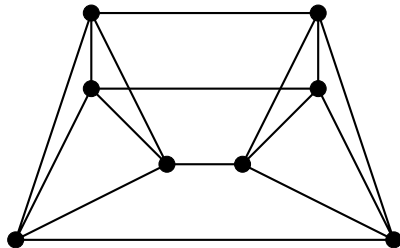
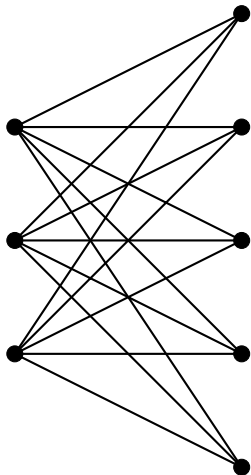


K_5

Projective Plane (Archdeacon)



Projective Plane (Archdeacon)



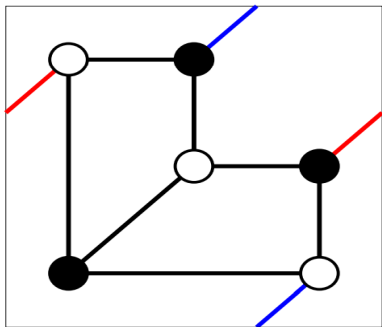
Projective Plane (Archdeacon)

Theorem (Archdeacon)

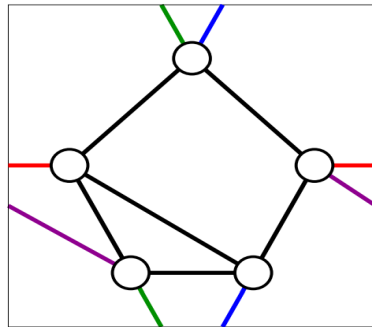
The class of graphs embeddable on the projective plane has 35 forbidden minors.

Corollary to Theorem

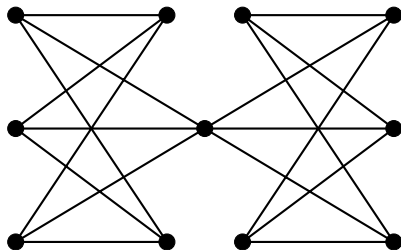
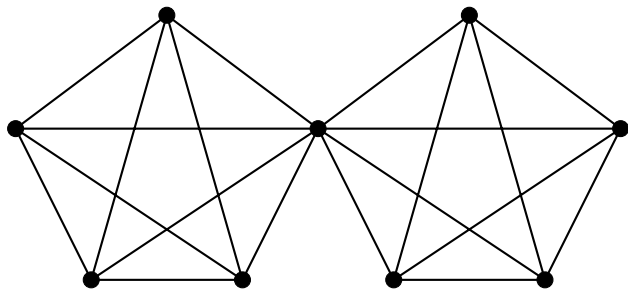
It is possible to check if a graph is embeddable on the projective plane by testing for a finite list of forbidden minors.



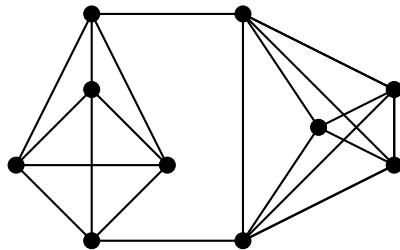
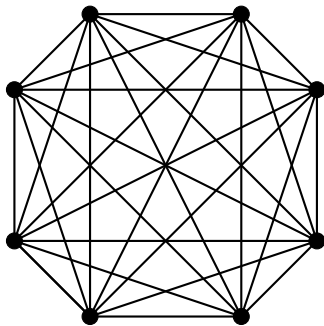
$K_{3,3}$



K_5



Torus



Torus (Chambers and Myrvold)

Statement

The class of graphs embeddable on the projective plane has **at least 16,629** forbidden minors.

Problem

Is the list of forbidden minors finite?

The Graph Minor Theorem

The Graph Minor Theorem (Robertson, Seymour)

A class of graphs closed under minors has a finite list of forbidden minors.

Classes of Graphs Closed Under Minors

- Graphs embeddable on orientable surfaces.
- Graphs embeddable on non-orientable surfaces.
- Graphs that are outerplanar.
- Graphs linklessly and knotlessly embeddable in Euclidean space.
- Graphs that are apex outerplanar. (Ding, Dziobiak)
- And many more...

Classes of Graphs Closed Under Minors

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Thank You!