

Complexity of Computing Categorified Invariants

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ABSTRACT

A popular subfield of topology, knot theory, revolves around the question of how circles can be embedded in 3-dimensional space. These embedded circles, called knots, can be manipulated up to isotopy, allowing for two knots to be considered equivalent if there is an orientation-preserving homeomorphism of \mathbb{R}^3 which takes one knot into the other. To distinguish different types of knots we use knot invariants, which are quantities computed for each knot that are equal for all equivalent knots. Recently, homological invariants that categorify other popular knot invariants have developed; however, determining the computational complexity of these new homological invariants has not been actively studied. This project centered on this question, focusing on the relationship between the complexity of computing the Khovanov Homology invariant and the complexity of computing the signed Bollobas-Riordan-Tutte Homology invariant.

THEOREMS

THEOREM I (Loebel & Moffat, 2007): The Khovanov Homology of any knot can be recovered from the signed Bollobas-Riordan-Tutte Homology of the related fatgraph.

THEOREM II (Bar-Natan, 2002): The graded dimensions of the homology groups are link invariants, and hence $\text{Kh}(L)$, a [graded Poincaré] polynomial in the variables t and q , is a link invariant that specializes to the unnormalized Jones polynomial at $t = -1$.

BIG QUESTION

What can the Bollobas-Riordan-Tutte Homology tell us about computing the homology groups of Khovanov Homology? In particular, what is the complexity of computing said homology groups?

REFERENCES

M. Loebel and I. Moffat, "The chromatic polynomial of fatgraphs and its categorification," arXiv:math/0511557 [math.CO], Nov. 2007

D. Bar-Natan, "On Khovanov's categorification of the Jones Polynomial," arXiv:math/0201043 [math.QA], Jun. 2002

O. Dasbach et. al., "The Jones polynomial and graphs on surfaces", arXiv:math/0605571 [math.GT], July 2007

D. Bar-Natan, "Fast Khovanov Homology Computations", arXiv:math/0606318 [math.GT], Jun 2006

EXAMPLE: LEFT-HANDED TREFOIL KNOT

Figure #1 : Jones Polynomial

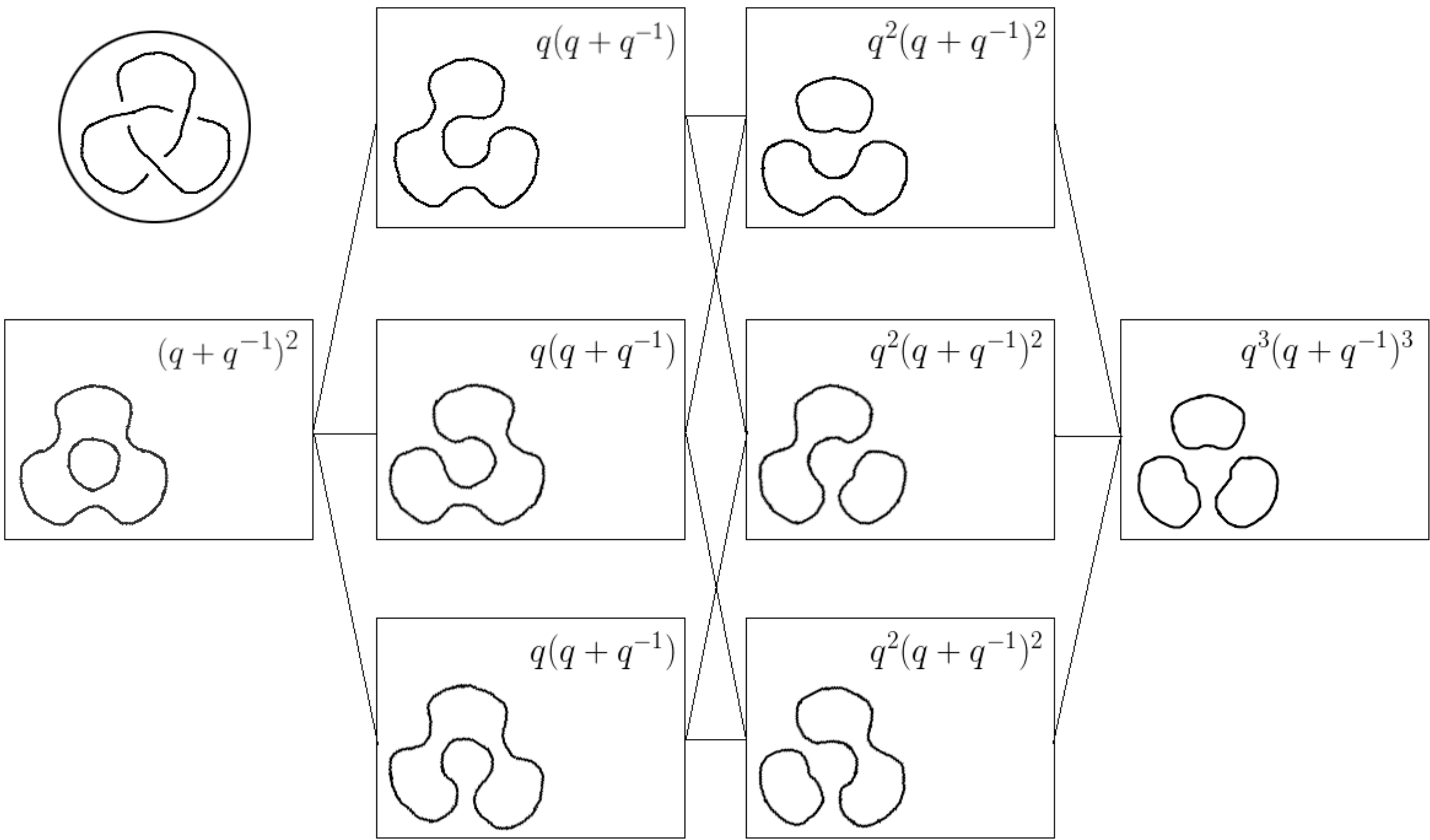


Figure #2 : Khovanov Homology

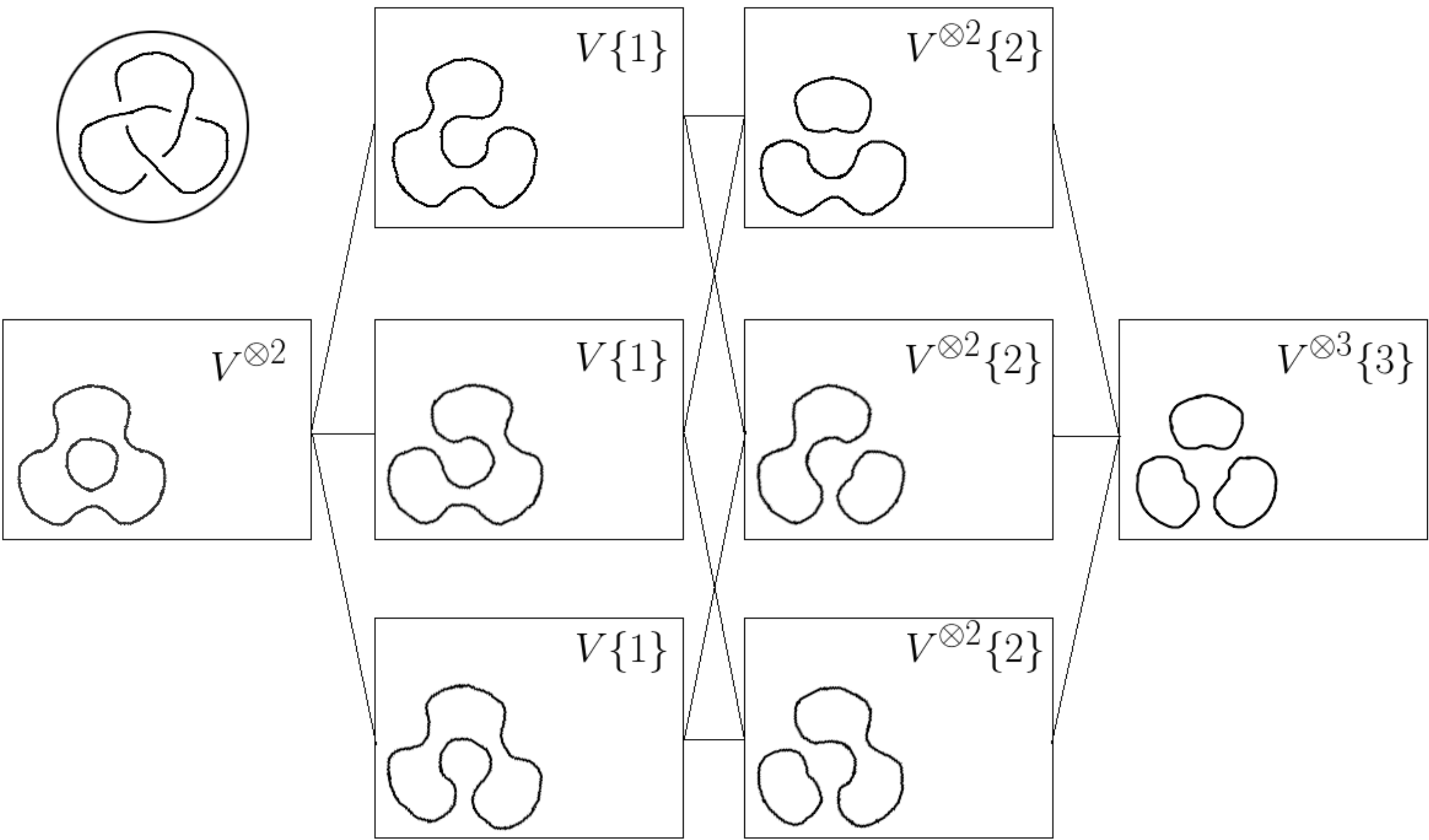


Figure #3: Bollobas-Riordan-Tutte Polynomial

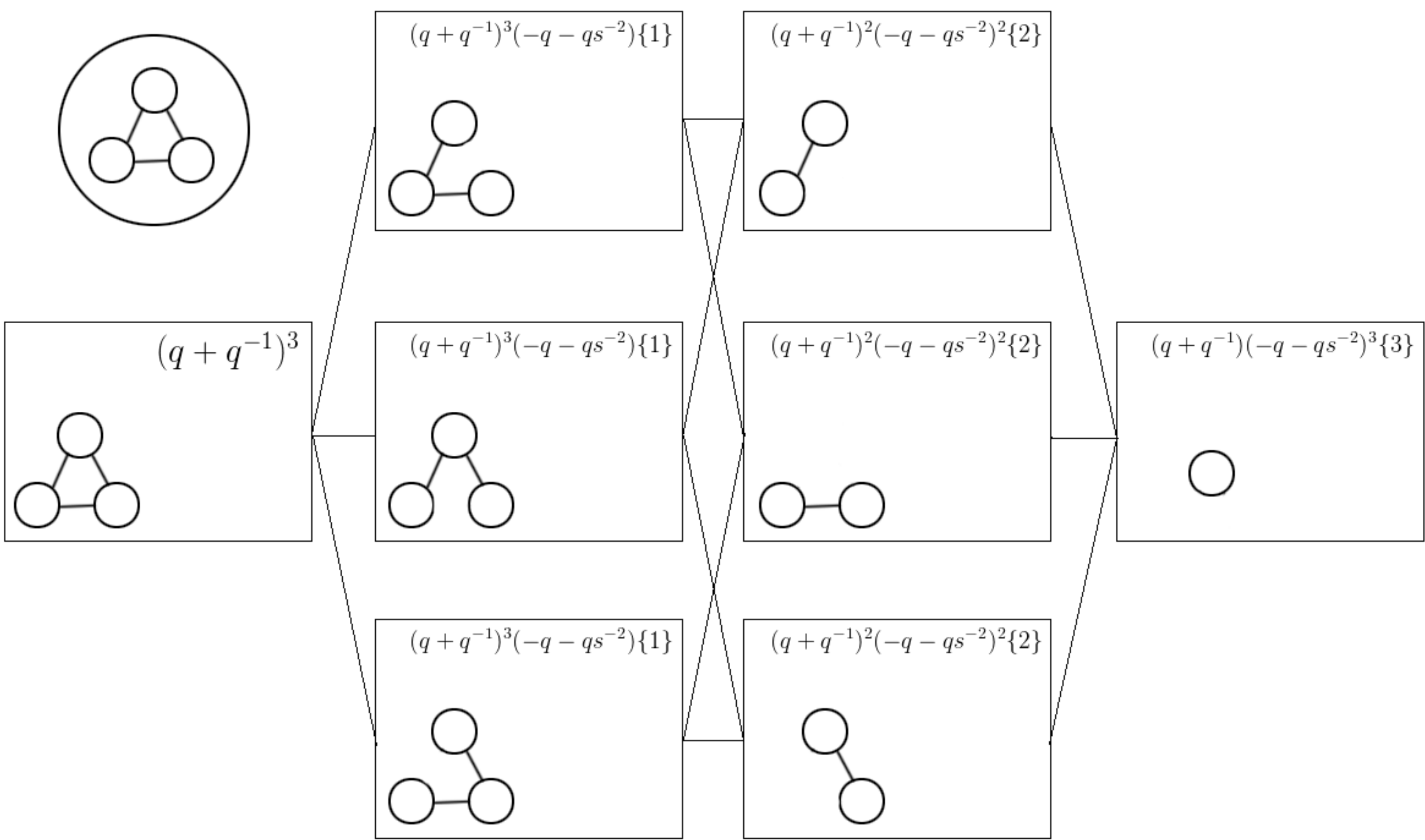


Figure #4: Bollobas-Riordan-Tutte Homology

