Centrality Measures in Weighted Bimodal Graphs

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Overview

Mission Santa Catalina de Guale is a 17th century Spanish mission off the coast of Georgia. During excavations of the mission, numerous beads were found buried with those found in the cemetery. When groups of people would enter into the mission, they would often bring beads with them, and they would be divided among people based off of the social relationships there. The connections between people buried there and the bead types found in the cemetery are best structured as a weighted bimodal graph. However, this is not a commonly used network type in archaeology, resulting in difficulties in analyzing the social relationships.

These difficulties in quantitative analysis for this network structure include measuring network centrality, which measures how central a given object is to the rest of the social network. Multiple definitions of centrality have been designed, such as degree, closeness, betweenness, and eigenvector centralities. However, using these measures for weighted bimodal graphs has been limited in use up to now.

This work looks at designing centrality algorithms to apply to weighted bimodal graphs in archaeology, specifically in application to understanding the social relationships found in Mission Santa Catalina de Guale, as well as implementing the algorithms using the network analysis package NetworkX in Python. Using a custom Python script to parse the bimodal data set and run the algorithms on the parsed weighted data, the results detail centrality values mapped to all values within the network.

Figure 1: Sample of beads found in Santa Catalina de Guale

Weighted Bimodal Graphs

A weighted bimodal graph is a network consisting of two different types of values, and undirected relationships mapping from a value of one type to a value of the other type with a numerical weight. These relationships also have a numerical positive nonzero weight associated with them, adding an additional dimension to the graph.

Centrality Measures

Degree Centrality

Degree centrality measures the sum of all relationships for a given value in a weighted bimodal graph.

$$deg(v, D) = \sum_{u \in D} e(v, u)$$

Closeness Centrality

Closeness centrality relates the distance traveled for all shortest paths between a given vertex and a subset of the vertexes in a graph to how central a given value is.

$$c_c(v, D, T) = (\sum_{u \in D} d(u, v, T))^{-1}$$

Betweenness Centrality

Betweeness centrality uses the ratio of shortest paths between vertexes in the graph compared to the shortest paths between vertexes containing a given vertex in order to determine its significance to the graph.

$$c_b(v, D, T) = \sum_{s,t \in D - \{v\}} \frac{\sigma(s, t, v)}{\sigma(s, t, \emptyset)}, \ \sigma : S(T, D) \to \mathbb{N}$$

Eigenvector Centrality

Eigenvector centrality measures how important a value in a weighted bimodal graph is based on how important its neighbors are; the more central a value's neighbors are, the more central that value is.

$$c_e(v, D) = \frac{1}{\lambda} \sum_{t \in D} a_{v,t} x_t$$

Centrality Visualizations

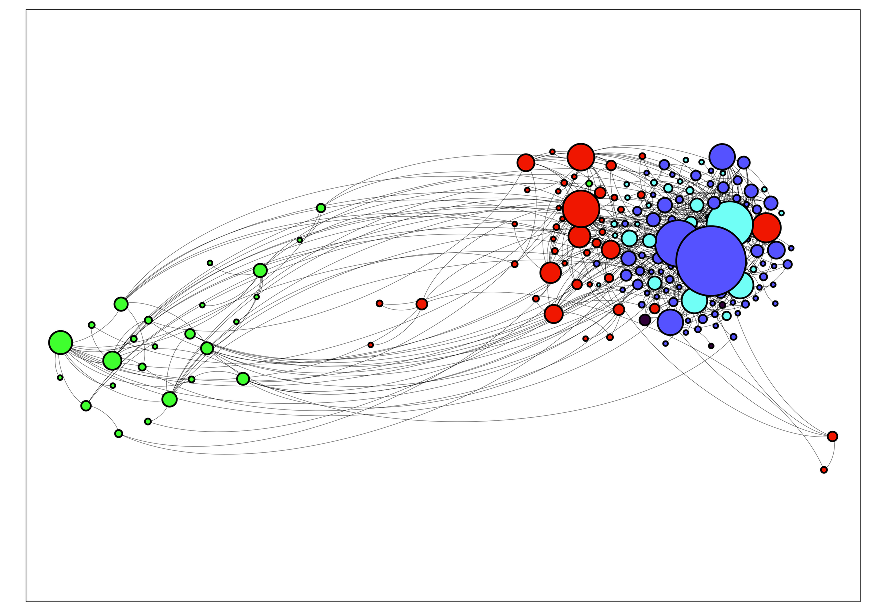

Figure 2: Sizing based on degree centrality

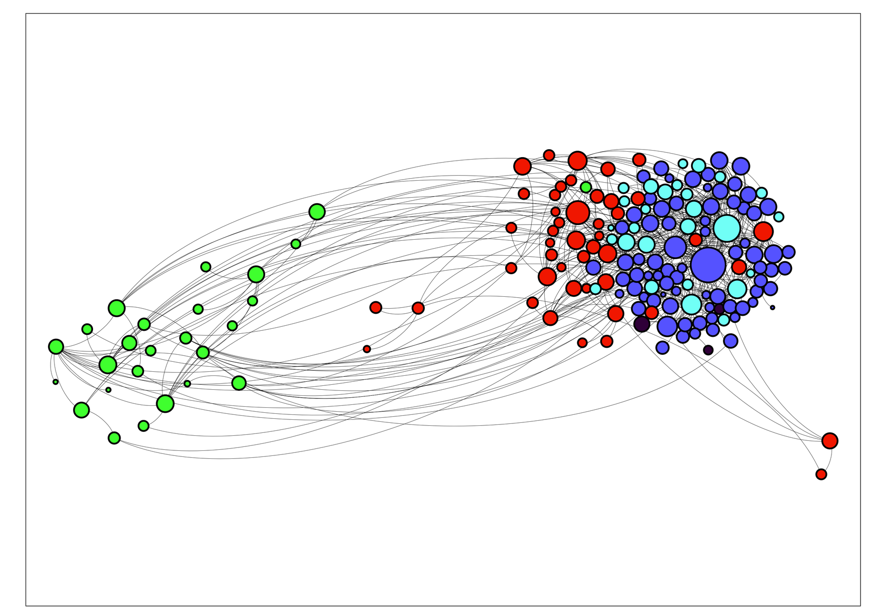

Figure 3: Sizing based on closeness centrality

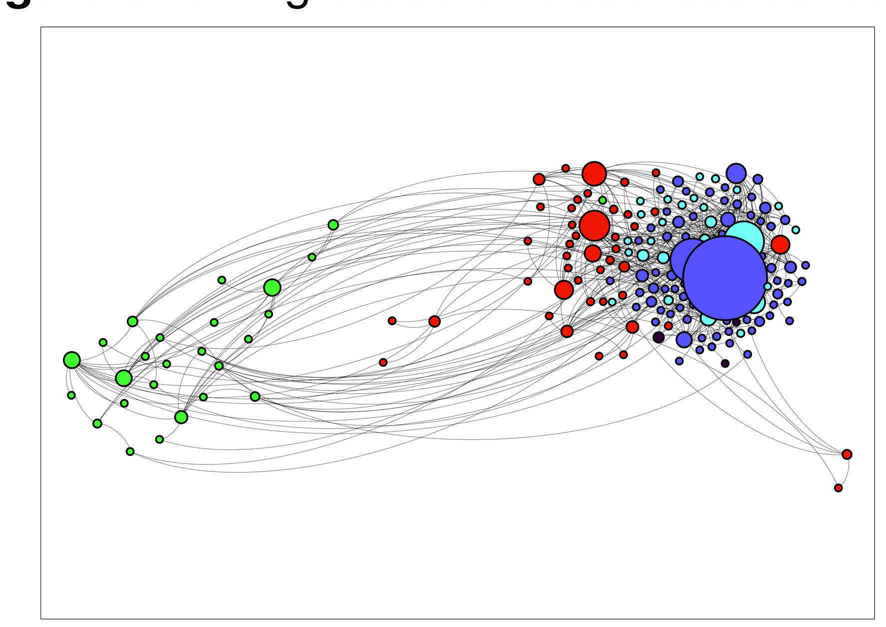

Figure 4: Sizing based on betweenness centrality

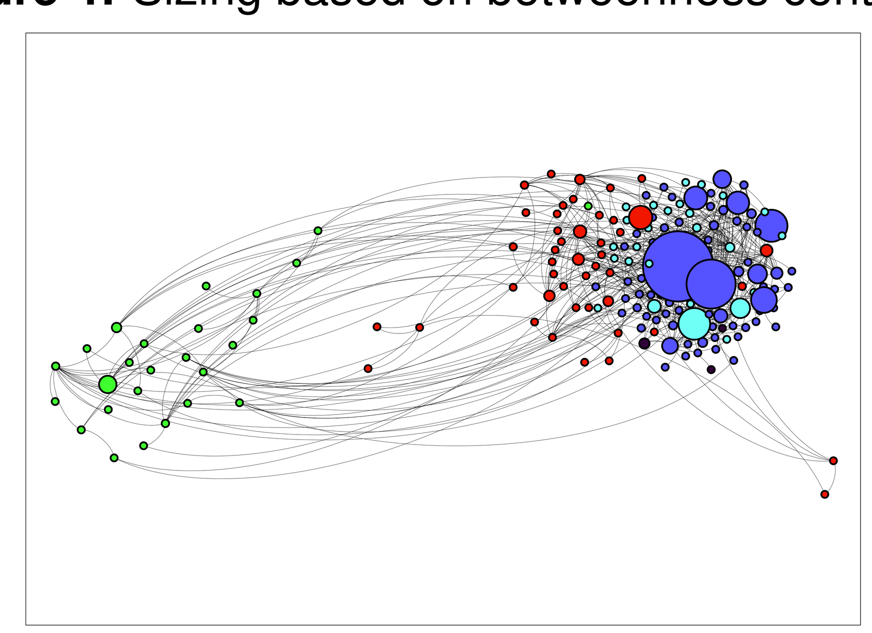

Figure 5: Sizing based on eigenvector centrality

Future Work

- Modifying other centrality measures to work with weighted bimodal graphs.
- Developing an efficient way to find centrality values for sub-modules of weighted bimodal graphs.
- Generalizing the code used for calculations to openly share with others.

References

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Figure 6: Overhead map of the cemetery at Mission Santa Catalina de Guale