MS INFORMATION SYSTEMS 2402 – 1: MATH FOR FINANCE WITH R

PROJECT REPORT FOR CASE

DRIVING INNOVATIONS: OPTIMIZATION OF SUPPORT STAFFING AT TESLA

CLASS: MS FINANCE AND ANALYTICS – FALL 2023 (M/W – 5:45 P.M.)

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MSIS 2402-1: Group 5 - Project report
Driving Innovations: Optimization of Support Staffing at Tesla

<u>Global Assumptions & data-Points for all 3 scenarios:</u>

1. *Defining variables for the model:*
   Let:
   a. Full-time agents be **X (**Can work 8 hours but will rotate every 2 hours between technical and call shifts. This means they will service calls for 4 hours and they can start working from 7 A.M – 9 A.M until 3 P.M - 5 P.M)
   b. Part-time agents be: **Y (**Can work 4 hours continuously without rotating shifts but will start to shift either during the 3 P.M -5 P.M shift or 5 P.M-7 P.M shift)

2. *Each agent can efficiently handle around 6 calls/hour across all scenarios.*

3. *Cost of Full-time and Part-time agents are the same across all scenarios, costs per hour and per shift are as below: (each shift is 2 hours)*
   a. Cost per hour for calls during shifts between 7 A.M and 5 P.M:
      i. Cost per Agent: $30/hour
      ii. Hence, Cost per shift per Agent will be → $30*2 hours → $60/shift

   b. Cost per hour for calls after 5 P.M. shift:
      iii. Cost per Agent: $45/hour
      iv. Hence, the Cost per shift per Agent will be → $45*2 hours → $90/shift

4. *Work shifts and calls per hour: (Calculating the average number of agents needed per shift)*

Work shift	**Average number of Calls (A)**	**Average number of agents needed per shift (A/6 calls per hour)**
7 A.M. – 9 A.M.	40 calls per hour	~6.67
9 A.M. – 11 A.M.	85 calls per hour	~14.17
11 A.M. – 1 P.M.	70 calls per hour	~11.67
1 P.M. – 3 P.M.	95 calls per hour	~15.83
3 P.M. – 5 P.M.	80 calls per hour	~13.33
5 P.M. – 7 P.M.	35 calls per hour	~5.83
7 P.M. – 9 P.M.	10 calls per hour	~1.67

5. *Calls from English speakers – 80% and Non-English speakers (Spanish) – 20%*

Scenario 1:

Full-time agents can either service calls from English speakers or Spanish speakers. And Part-time agents can only speak English, hence can service calls from only English speakers.

A. *Variable to be used for Scenario 1:*

Let :

a. Full-time English-speaking agents be: **X**
b. Full-time Spanish-speaking agents be: **Z (**Since Full-time agents can either take Spanish or English language calls**)**
c. Part-time English speaking agents be: **Y (**Part-time agents can only take English language calls**)**

B. *Work shifts and calls per hour: (Calculating average number of agents needed per shift)*

Work shift	**Average number of Calls (A)**	**The average number of English-speaking agents needed per shift (A*80%)/6 calls per hour)**	**The average number of Spanish-speaking agents needed per shift (A*20%)/6 calls per hour)**
7 A.M. – 9 A.M.	40 calls per hour	~5.33	~1.33
9 A.M. – 11 A.M.	85 calls per hour	~11.33	~2.83
11 A.M. – 1 P.M.	70 calls per hour	~9.33	~2.33
1 P.M. – 3 P.M.	95 calls per hour	~12.67	~3.17
3 P.M. – 5 P.M.	80 calls per hour	~10.67	~2.67
5 P.M. – 7 P.M.	35 calls per hour	~4.67	~1.17
7 P.M. – 9 P.M.	10 calls per hour	~1.33	~0.33

C. *Considering the constraints regarding shifts and working hours, we assume :*

i. X1 (Full-time English agent) will start shift 7 A.M. – 9 A.M.
ii. In the next shift of 9 A.M to 11 A.M, X1 will rotate and start their technical work. So, X2 needs to be hired.
iii. X1 will rotate again and take calls from 11 A.M. – 1 P.M. X3 can be hired for any shortfalls in this shift and so on.
iv. From 3 P.M. onwards Part-time agents can be hired. So, Y1 can be hired to cover any shortfalls in the availability of Full-time agents. They can work continuously for 4 hours
v. Similarly, Z1 will start the first shift for 20% Spanish calls and continue rotating for every 2 hours. Z2 and Z3 et cetera can be hired for every alternative shift.

The above considerations will translate into the following table:

Work shift	The average number of English-speaking agents needed per shift	The average number of Spanish-speaking agents needed per shift	Variables for English speaking agents in each shift	Variables for Spanish speaking agents in each shift
7 A.M. – 9 A.M.	~5.33	~1.33	X1	Z1
9 A.M. – 11 A.M.	~11.33	~2.83	X2	Z2
11 A.M. – 1 P.M.	~9.33	~2.33	X1 + X3	Z1 + Z3
1 P.M. – 3 P.M.	~12.67	~3.17	X2 + X4	Z2 + Z4
3 P.M. – 5 P.M.	~10.67	~2.67	X3 + X5 + Y1	Z3 + Z5
5 P.M. – 7 P.M.	~4.67	~1.17	X4 + Y1 + Y2	Z4
7 P.M. – 9 P.M.	~1.33	~0.33	X5 + Y2	Z5

Let us build the Cost function to be minimized:

i. Cost @ 60/hour (from Global assumption Point 3a ii) for:

a. 7 A.M. – 9 A.M. → X1 * 60 + Z1 * 60 → **60 X1 + 60 Z1**

b. 9 A.M. – 11 A.M. → X2 * 60 + Z2 * 60 → **60 X2 + 60 Z2**

c. 11 A.M. – 1 P.M. → X1 * 60 + X3 * 60 + Z1 *60 + Z3 * 60 → **60 X1 + 60 X3 + 60 Z1 + 60 Z3**

d. 1 P.M. – 3 P.M. → X2 * 60 + X4 * 60 + Z2 *60 + Z4 * 60 → **60 X2 + 60 X4 + 60 Z2 + 60 Z4**

e. 3 P.M. –5 P.M. → X3 * 60 + X5 * 60 + Y1 * 60 + Z3 * 60 + Z5 *60 → **60 X3 + 60 X5 + 60 Y1 + 60 Z3 + 60 Z5**

ii. Cost @ 90/hour (from Global assumption Point 3b ii) for:

f. 5 P.M. – 7 P.M. → X4 * 90 + Y1 * 90 + Y2 *90 + Z4 *90→ **90 X4 + 90 Y1 + 90 Y2 + 90 Z4**

g. 7 P.M. – 9 P.M. → X5 * 90 + Y2 *90 + Z5 *90→ **90 X5 + 90 Y2 + 90 Z5**

Hence, the Total cost equation that needs to be minimized to optimize cost will be as follows:

Scenario 1: Mathematic model (Minimization function)

120X1+ 120Z1 + 120X2 + 120Z2 + 120X3 + 120Z3 + 150X4 + 150Z4 + 150X5 + 150Z5 + 150Y1 + 180Y2

Inequality constraints for Scenario 1:

1. Equation 1: Constraint 1 → X1 >= 5.33
2. Equation 2: Constraint 2 → Z1 >= 1.33
3. Equation 3: Constraint 3 → X2 >= 11.33
4. Equation 4: Constraint 4 → Z2 >= 2.83
5. Equation 5: Constraint 5 → X1 + X3 >= 9.33
6. Equation 6: Constraint 6 → Z1 + Z3 >= 2.33
7. Equation 7: Constraint 7 → X2 + X4 >= 12.67
8. Equation 8: Constraint 8 → Z2 + Z4 >= 3.17
9. Equation 9: Constraint 9 → X3 + X5 + Y1 >= 10.67
10. Equation 10: Constraint 10 → Z3 + Z5 >= 2.67
11. Equation 11: Constraint 11 → X4 + Y1 + Y2 >= 4.67
12. Equation 12: Constraint 12 → Z4 >= 1.17
13. Equation 13: Constraint 13 → X5 + Y2 >= 1.33
14. Equation 14: Constraint 14 → Z5 >= 0.33

Since we cannot have the number of agents as a negative figure, we have to define non-negative constraints as given below :

1. X3 >= 0
2. Z3 >= 0
3. X4 >= 0
4. X5 >= 0
5. Y1 >= 0
6. Y2 >= 0

Initial points for R code, that strictly satisfy all the above constraints for Scenario 1:

a. X1 → 5.4
b. Z1 → 1.4
c. X2 → 11.4
d. Z2 → 2.9
e. X3 → 4.1
f. Z3 → 1.1
g. X4 → 1.4
h. Z4 → 1.2 (higher between equation 12 and equation 8)
i. X5 → 4.4
j. Z5 → 1.8 (higher between equation 14 and equation 10)
k. Y1 → 2.3
l. Y2 → 1.3

Scenario 1: Answers (refer to file 'R Scenario 1_answer a_b.R')

a. *How many Full-time English-speaking agents, Full-time Spanish-speaking agents, and Part-time agents should Alex hire for each 2-hour shift to minimize operating costs while attending to all calls? (Please round each number to the nearest integer.)*

Work shift	Output from R for English agents	Output from R for Spanish agents	Total number of English speaking agents in each shift	Total number of Spanish speaking agents in each shift
7 A.M. – 9 A.M.	X1 = 5.33	Z1 = 1.33	5.33 or 5	1.33 or 1
9 A.M. – 11 A.M.	X2 = 11.33	Z2 = 2.83	11.33 or 11	2.83 or 3
11 A.M. – 1 P.M.	X1 = 5.33 + X3 = 4.00	Z1 = 1.33 + Z3 = 2.34	9.34 or 9	3.67 or 4
1 P.M. – 3 P.M.	X2 = 11.33 + X4 = 1.34	Z2 = 2.83 + Z4 = 1.17	12.67 or 13	4.00 or 4
3 P.M. – 5 P.M.	X3 = 4.01 + X5 = 3.34 + Y1 = 3.33	Z3 = 2.34 + Z5 = 0.33	10.67 or 11	2.67 or 3
5 P.M. – 7 P.M.	X4 = 1.34 + Y1 = 3.33 + Y2 = 0	Z4 = 1.17	4.67 or 5	1.17 or 1
7 P.M. – 9 P.M.	X5 = 3.33 + Y2 = 0	Z5 = 0.33	3.33 or 3	0.33 or 1

From our R Optimization code, the values of X, Y and Z are as follows:

X1 = 5.33	Z1 = 1.33	Y1 = 3.33
X2 = 11.33	Z2 = 2.83	Y2 = 0.00
X3 = 4.00	Z3 = 2.34	
X4 = 1.34	Z4 = 1.17	
X5 = 3.34	Z5 = 0.33	

b. *What is the minimum cost for the optimization model to assist Alex's decision in hiring all agents that she needs? (Please round to two decimal places, e.g., 123.45.)*

Based on our R Code, the minimum cost for the optimization model under Scenario 1 is **$ 4685.67**

Scenario 2:

Due to a preference among Full-time agents to avoid late evening shifts, Alex can find only one qualified English-speaking agent willing to start work at 1 P.M. and 3 P.M.

Since there is no change in the deployment of the agents, the total cost equation that needs to be minimized to optimize cost will be the same as Scenario 1:

Scenario 2: Mathematic model (Minimization function)

120X1+ 120Z1 + 120X2 + 120Z2 + 120X3 + 120Z3 + 150X4 + 150Z4 + 150X5 + 150Z5 + 150Y1 + 180Y2

Since Alex can find only one Agent who is willing to start work at 1 P.M and 3 P.M, the constraints need to be revised to reflect this.

Inequality Constraints for Scenario 2:

1. Equation 1: Constraint 1 → X1 >= 5.33
2. Equation 2: Constraint 2 → Z1 >= 1.33
3. Equation 3: Constraint 3 → X2 >= 11.33
4. Equation 4: Constraint 4 → Z2 >= 2.83
5. Equation 5: Constraint 5 → X1 + X3 >= 9.33
6. Equation 6: Constraint 6 → Z1 + Z3 >= 2.33
7. Equation 7: Constraint 7 → X2 + X4 >= 12.67
8. Equation 8: Constraint 8 → Z2 + Z4 >= 3.17
9. Equation 9: Constraint 9 → X3 + X5 + Y1 >= 10.67
10. Equation 10: Constraint 10 → Z3 + Z5 >= 2.67
11. Equation 11: Constraint 11 → X4 + Y1 + Y2 >= 4.67
12. Equation 12: Constraint 12 → Z4 >= 1.17
13. Equation 13: Constraint 13 → X5+ Y2 >= 1.33
14. Equation 14: Constraint 14 → Z5 >= 0.33

Now, we have to define equality constraints for Agent starting to shift from 1 P.M and 3 P.M. Also, we cannot have the number of agents as a negative figure, we have to define non-negative constraints as given below:

1. X3 >= 0
2. Z3 >= 0
3. X4 = 1 (subject to new constraint)
4. X5 = 1 (subject to new constraint)
5. Y1 >= 0
6. Y2 >= 0

Initial points for R code, that strictly satisfy all the above constraints for Scenario 2:

a. X1 → 5.4
b. Z1 → 1.4
c. X2 → 11.7
d. Z2 → 2.9 (to satisfy equations 12 and 14)
e. X3 → 4
f. Z3 → 1.1
g. X4 → 1.0
h. Z4 → 1.2 (to satisfy equation 12 and 14)
i. X5 →1.0
j. Z5 → 1.6 (changed to satisfy equality constraint X5 = 1)
k. Y1 → 5.7 (changed to satisfy equality constraint X5 = 1)
l. Y2 → 0.4 (changed to satisfy equality constraint X4 = 1)

Scenario 2: Answers (refer to file 'R Scenario 2 answer c d.R')

c. *How many Full-time English-speaking agents, Full-time Spanish-speaking agents, and Part-time agents should Alex hire for each 2-hour shift to minimize operating costs while attending to all calls? (Please round each number to the nearest integer.)*

Work shift	Output from R for English agents	Output from R for Spanish agents	Total number English speaking agents in each shift	Total number Spanish speaking agents in each shift
7 A.M. – 9 A.M.	X1 = 5.33	Z1 = 1.33	5.33 or 5	1.33 or 1
9 A.M. – 11 A.M.	X2 = 11.67	Z2 = 2.83	11.67 or 12	2.83 or 3
11 A.M. – 1 P.M.	X1 = 5.33 + X3 = 5.76	Z1 = 1.33 + Z3 = 2.34	11.09 or 11	3.67 or 4
1 P.M. – 3 P.M.	X2 = 11.67 + X4 = 1.00	Z2 = 2.83 + Z4 = 1.17	12.67 or 13	4.00 or 4
3 P.M. – 5 P.M.	X3 = 5.76 + X5 = 1.00 + Y1 = 3.91	Z3 = 2.34 + Z5 = 0.33	10.67 or 11	2.67 or 3
5 P.M. – 7 P.M.	X4 = 1.00 + Y1 = 3.91 + Y2 = 0.33	Z4 = 1.17	5.24 or 5	1.17 or 1
7 P.M. – 9 P.M.	X5 = 1.00 + Y2 = 0.33	Z5 = 0.33	1.33 or 1	0.33 or 1

From our R Optimization code, the values of X, Y, and Z are as follows:

X1 = 5.33	Z1 = 1.33	Y1 = 3.91
X2 = 11.67	Z2 = 2.83	Y2 = 0.33
X3 = 5.76	Z3 = 2.34	
X4 = 1.00	Z4 = 1.17	
X5 = 1.00	Z5 = 0.33	

d. *What is the minimum cost for the optimization model to assist Alex's decision in hiring all agents that she needs? (Please round to two decimal places.)*

Based on our R Code, the minimum cost for the optimization model under Scenario 2 is **$ 4682.25**

Scenario 3:
Alex can hire bilingual agents and all agents hired are bilingual

A. *Variables to be used for Scenario 3:*
Let Full-time agents be **X** and Part-time agents be **Y**

B. *Work shifts and calls per hour: (Calculating average number of agents needed per shift)*

Work shift	The average number of calls (A)	The average number of bilingual agents needed per shift (A/6 calls per hour)
7 A.M. – 9 A.M.	40 calls per hour	~6.67
9 A.M. – 11 A.M.	85 calls per hour	~14.17
11 A.M. – 1 P.M.	70 calls per hour	~11.67
1 P.M. – 3 P.M.	95 calls per hour	~15.83
3 P.M. – 5 P.M.	80 calls per hour	~13.33
5 P.M. – 7 P.M.	35 calls per hour	~5.83
7 P.M. – 9 P.M.	10 calls per hour	~1.67

C. *Considering the constraints regarding shifts and working hours, we assume :*

i. X1 (Full-time English agent) will start shift 7 A.M. – 9 A.M.
ii. In the next shift of 9 A.M to 11 A.M, X1 will rotate and start their technical work. So, X2 needs to be hired.
iii. X1 will rotate again and take calls from 11 A.M. – 1 P.M. X3 can be hired for any shortfalls and so on.
iv. From 3 P.M. onwards, Y1 can be hired to cover for any shortfalls in the availability of Full-time agents. They can work continuously for 4 hours.

The above considerations will translate into the following table:

Work shift	Average number of bilingual agents needed per shift (A/6 calls per hour)	Variable for Bilingual agents in each shift
7 A.M. – 9 A.M.	~6.67	X1
9 A.M. – 11 A.M.	~14.17	X2
11 A.M. – 1 P.M.	~11.67	X1 + X3
1 P.M. – 3 P.M.	~15.83	X2 + X4
3 P.M. – 5 P.M.	~13.33	X3+ X5 + Y1
5 P.M. – 7 P.M.	~5.83	X4 + Y1 + Y2
7 P.M. – 9 P.M.	~1.67	X5 + Y2

Let us build the Cost function to be minimized:

i. Cost @ 60/hour (from Global assumption Point 3a ii) for:

a. 7 A.M. – 9 A.M. → X1 * 60 → **60 X1**
b. 9 A.M. – 11 A.M. → X2 * 60 → **60 X2**
c. 11 A.M. – 1 P.M. → X1 * 60 + X3 * 60 → **60 X1 + 60 X3**
d. 1 P.M. – 3 P.M. → X2 * 60 + X4 * 60 → **60 X2 + 60 X4**
e. 3 P.M. –5 P.M. → X3 * 60 + X5 * 60 + Y1 * 60 → **60X3 + 60X5 + 60Y1**

ii. Cost @ 90/hour (from Global assumption Point 3b ii) for:

f. 5 P.M. – 7 P.M. → X4 * 90 + Y1 * 90 + Y2 *90 → **90X4 + 90Y1 + 90Y2**
g. 7 P.M. – 9 P.M. → X5 * 90 + Y2 *90 → **90X5 + 90Y2**

Hence, the Total cost equation that needs to be minimized to optimize cost will be as follows:

Scenario 3: Mathematic model (Minimization Function)

120X1 + 120X2 + 120X3 + 150X4 + 150X5 + 150Y1 + 180Y2

Inequality Constraints for Scenario 3:

1. Equation 1: Constraint 1 → X1 >= 6.67
2. Equation 2: Constraint 2 → X2 >= 14.17
3. Equation 3: Constraint 3 → X1 + X3 >= 11.67
4. Equation 4: Constraint 4 → X2 + X4 >= 15.83
5. Equation 5: Constraint 5 → X3 + X5 + Y1 >= 13.33
6. Equation 6: Constraint 6 → X4 + Y1 + Y2 >= 5.83
7. Equation 7: Constraint 7 → X5+ Y2 >= 1.67

Since we cannot have the number of agents as a negative figure, we must define non-negative constraints as given below:

1. X3 >= 0

2. X4 >= 0
3. X5 >= 0
4. Y1 >= 0
5. Y2 >= 0

Initial points for R code, that strictly satisfy all the above constraints for Scenario 3:

a. X1 → 6.7
b. X2 → 14.2
c. X3 → 5.1
d. X4 → 1.7
e. X5 → 5.2
f. Y1 → 3.2
g. Y2 → 1.0

Scenario 3: Answers (refer to file 'R Scenario 3 answer e f.R')

e. *How many Full-time and Part-time agents should Alex hire for each 2-hour shift to minimize operating costs while attending to all calls? (Please round each number to the nearest integer.)*

Work shift	Output from R for Bilingual agents	Total number of Bilingual agents in each shift
7 A.M. – 9 A.M.	X1 = 6.67	6.67 or 7
9 A.M. – 11 A.M.	X2 = 14.17	14.17 or 14
11 A.M. – 1 P.M.	X1 = 6.67 + X3 = 5.00	11.67 or 12
1 P.M. – 3 P.M.	X2 = 14.17 + X4 = 1.66	15.83 or 16
3 P.M. – 5 P.M.	X3 = 5.00 + X5 = 4.16 + Y1 = 4.17	13.33 or 13
5 P.M. – 7 P.M.	X4 = 1.66 + Y1 = 4.17 + Y2 = 0.00	5.83 or 6
7 P.M. – 9 P.M.	X5 = 4.16 + Y2 = 0.00	4.16 or 4

From our R Optimization code, the values of X and Y are as follows:

X1 = 6.67	Y1 = 4.17
X2 = 14.17	Y2 = 0.00
X3 = 5.00	
X4 = 1.66	
X5 = 4.16	

f. *What is the minimum cost for the optimization model to assist Alex's decision in hiring all agents that she needs? (Please round to two decimal places.)*

Based on our R Code, the minimum cost for the optimization model under Scenario 3 is **$ 4599.28**

g. *What is the maximum percentage increase in the hourly wage rate that Alex can offer to bilingual agents over monolingual agents without increasing the total operating costs? (Please round to one decimal place, e.g., 8.7%.)*

Answer : ($ 4685.67 - $ 4599.28)/$ 4685.67 = 1.84%