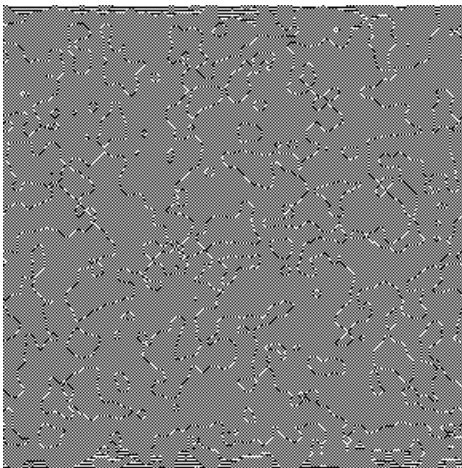


1. $K=0.0025$ ，使用 Inverse filter 結果如下圖，有使用巴特沃斯低通濾波器。



如果不使用 10 階，半徑為 30 的巴特沃斯低通濾波器結果會如下圖，可發現會無法產生圖像。



下圖為使用 Wiener Filter 的結果



下面為 Source code，包含使用的濾波器。

```
clc;clear;
%% read 5.25
fig = double(rgb2gray(imread('Fig5.25.jpg')))/255;
[H_1,W_1] = size(fig);
F_1 = fft2(center_transform(fig));
%% inverse filter
o = 5/6;
f = filter_H1(H_1,W_1,0.0025,o);
butter = butterworth_lowpass_filter(H_1,W_1,30,10);
res = (F_1./f);
res_filter = (F_1./f).*butter;
res2 = real(ifft2(res));
res_filter2 = real(ifft2(res_filter));
res_fig = center_transform(res2);
lpf_fig = center_transform(res_filter2);
lpf_fig2 = center_transform(real(ifft2(res_filter)));
lpf_fig(lpf_fig>1)=1;
lpf_fig(lpf_fig<0)=0;
res_fig(res_fig>1)=1;
res_fig(res_fig<0)=0;
r=double(lpf_fig);
g=double(lpf_fig);
b=double(lpf_fig);
rgb=cat(3,r,g,b);
figure(1)
imshow(fig, []);
figure(2)
imshow(rgb,[]);
imwrite(rgb, '5.25_lpf.png');
figure(3)
imshow(res_fig,[]);
imwrite(res_fig, '5.25_nolpf.png');

%% wiener filter
f2 = filter_H1(H_1,W_1,0.0025,o);
F_wiener = fft2(center_transform(fig));
wiener_restored = center_transform(real(ifft2(wiener_filter1(0.0025, F_1, f2) .* F_wiener)));
wiener_restored(wiener_restored>1)=1;
wiener_restored(wiener_restored<0)=0;
r_wiener=double(wiener_restored);
g_wiener=double(wiener_restored);
b_wiener=double(wiener_restored);
rgb_wiener=cat(3,r_wiener,g_wiener,b_wiener);

figure(4)
imshow(rgb_wiener, []);
imwrite(rgb_wiener, '5.25_wiener.png');
```

```

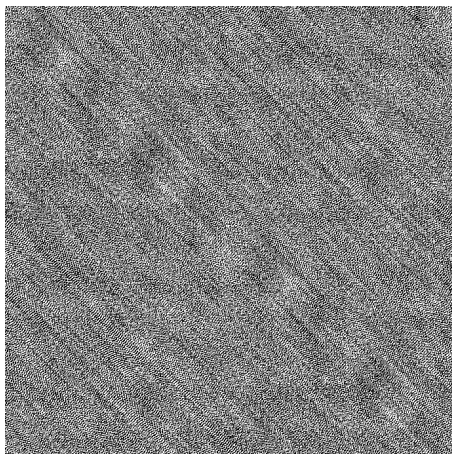
function [ output ] = filter_H1( M, N, k, o )
    u = [1:M]-M/2 ;
    v = [1:N]-N/2 ;
    [V, U] = meshgrid(v,u);
    D = (V.^2+U.^2).^o;
    output = exp(-k.*D);
end

function [ output ] = butterworth_lowpass_filter( m, n, D0, N )
% BUTTERWORTH_LOWPASS_FILTER The Butterworth lowpass filter
%   H(u, v) = 1 / (1 + (D(u, v) / D0)^2n)
    u = [0:(m-1)]-m/2;
    v = [0:(n-1)]-n/2;
    [V, U] = meshgrid(v, u);
    D = sqrt((V.^2 + U.^2));
    output = 1 ./ (1 + (D ./ D0).^(2 * N));
end

function [ output ] = wiener_filter1( k, F, H)
% WIENER_FILTER generates a typical Wiener filter by definition
    H2 = abs(H) .^ 2;
    output = (1./H).*(H2./(H2+k));
end

```

2. 下圖為使用 Inverse Filter 的結果，可發現看不出結果圖像。



下圖為使用 Wiener Filter 的結果，加入平均數 0，變異數 1 的高斯雜訊，並計算功率代入濾波器公式，可看出能一定程度的還原旋轉結果。



下面為 Source code

<pre>%% read the book cover picture fig_original = double(imread('book-cover-blurred.tif')) / 255; [H, W] = size(fig_original); F = fft2(center_transform(fig_original));</pre>	
<pre>%% inverse filter h = filter_H(H, W, 0.1, 0.1, 1); inverse_restored = center_transform(real(ifft2(F ./ h))); figure(5) imshow(inverse_restored, []); imwrite(inverse_restored, 'book_Cover_inverse.png');</pre>	
<pre>%% wiener filter noise = gaussian_noise(H, W, sqrt(1), 0) / 255; blurred_noisy_image = fig_original + noise; F_blurred_noisy = fft2(center_transform(blurred_noisy_image)); blurred_noisy_restored = center_transform(real(ifft2(wiener_filter(noise, F, h) .* F_blurred_noisy))); figure(6) imshow(blurred_noisy_restored, []); imwrite(blurred_noisy_restored, 'book_Cover_wiener.png');</pre>	
<pre>function [output] = filter_H(M, N, a, b, T) % FILTER_H generates a filter of size h * w % where $H(u, v) = T / \pi(ua+vb) \sin[\pi(ua+vb)] e^{-j\pi(ua+vb)}$ u = [1:M] - M / 2; v = [1:N] - N / 2; [V, U] = meshgrid(v, u); D = (V .* b + U .* a) .* pi; output = ones(M, N) .* T ./ D .* sin(D) .* exp(-j * D); output(D == 0) = 1; end</pre>	

```

function [ output ] = wiener_filter( N, F, H)
% WIENER_FILTER generates a typical Wiener filter by definition
    H2 = abs(H) .^ 2;
    Sn = abs(fft2(center_transform(N))) .^ 2;
    Sf = abs(F) .^ 2;
    output = (H2 ./ H ./ (H2 + Sn ./ Sf));
end

```

3.

由結果也可發現，雖然理想上知道退化轉移函數，是能用 **Inverse filter** 還原圖像的。但由於有雜訊，是無法理想的還原圖像的。並且當退化轉移函數有零點或極小值時，會放大雜訊的影響，由結果也可看出反濾波確實不能很好的還原，使用低通濾波器限制靠近原點附近的頻率值，是能改善一些的。而 **Wiener filter** 是使原影像及受污染影像之間的均方誤差最小，由結果也能看出使用 **Wiener filter** 有更好的效果。