

Statistical Connectomics: Homework 2

Statistical Decision Theoretic for Stochastic Block Model

1. Sample Space

Our sample space consists of all vertices, edges, and vector labels.

- a. $G = (V, E, Y)$
 - i. V = number of vertices – $\{V_1, \dots, V_n\}$
 - ii. E = number of edges- $\{E_1, \dots, E_n\}$
 - iii. Y = vector labels $\{0,1\}^n$ - each vertex as a label 0 or 1

2. Model

$J = \text{SBM}_n^k(p, B, \dots)$ for $k = 2$

- a. $\rho \in \delta_2$
- b. $\beta \in (0,1)^{2 \times 2}$

3. Action Space

$A = \{y \in \{0,1\}^n\}$ – This is the assignments of the cluster

4. Decision Rule Class

- a. The decision rule class is determined by the method of clustering

5. Loss Function

- a. Using the adjusted rand index, we get the cost of the associated action

$$l: G_n \times A \rightarrow \mathbb{R}_+$$

$$l = \sum_{i=1}^n (\theta) \hat{y}_i = y_i$$

6. Risk Function

- a. The risk function is as followed

$$i. R = J * l$$

1. This can be interpreted as the expectation of the loss function

$$a. R = E[l]$$