Q1: Given $V_1 = \{2, -3, 5\}$
$V_2 = \{6, 2, 1\}$

Then;
Euclidean distance formula:

$$d(i,j) = \sqrt{(x_{i1} - x_{j1})^2 + (x_{i2} - x_{j2})^2 \cdots (x_{in} - x_{jn})^2}$$

$$V_1 = \{\underset{x_{i1}}{2}, \underset{x_{i2}}{-3}, \underset{x_{i3}}{5}\} \qquad V_2 = \{\underset{x_{j1}}{6}, \underset{x_{j2}}{2}, \underset{x_{j3}}{1}\}$$

$$= \sqrt{(2-6)^2 + (-3-2)^2 + (5-1)^2}$$

$$= \sqrt{(-4)^2 + (-5)^2 + (4)^2}$$

$$= \sqrt{16 + 25 + 16}$$

$$= \sqrt{57} = \cancel{7}.549$$

Q2: $x = \{6, -8, 0\}$ $\quad x = 6i + -8j + 0k$

$$|x| = \sqrt{(6)^2 + (-8)^2 + 0}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100} = 10$$

$$x = \frac{6i}{10} + \frac{-8j}{10} + \frac{0k}{10} = \left(\frac{6}{10}, \frac{-8}{10}, 0\right)$$

$$x = \left(3/5, -4/5, 0\right)$$

Q3.

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 1 & 5 \\ 6 & 0 & -1 \end{bmatrix}$$

$$\det(A) = 3(-1-0) - 4(-2-30) + 2(0-6)$$

$$= 3(-1) - 4(-32) + 2(-6)$$

$$= -3 + 128 + (-12)$$

$$= 128 - 15$$

$$= \underline{\underline{113}}$$

Q.4.)

$$B = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \frac{1}{\det(B)} \times \operatorname{adj}(B)$$

$$= \frac{1}{6-(-1)} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \frac{1}{7} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3/7 & 1/7 \\ -1/7 & 2/7 \end{bmatrix}$$

Q.5

$$V_1 = \{1, 2, -1\}$$

$$V_2 = \{3, -6, 2\}$$

$$V_1 \cdot V_2 = (1 \times 3) + (2 \times -6) + (-1 \times 2)$$

$$= 3 + (-12) + (-2)$$

$$= -11$$

Q.6)

$V_1 = \{1, 2\}$ $V_2 = \{3, 4\}$

$V_1 \cdot V_2 = (1 \times 3) + (2 \times 4)$

$= 3 + 8$

$= 11$

$\|V_1\| = \sqrt{(1)^2 + (2)^2}$

$= \sqrt{5}$

$= 2.236$

$\|V_2\| = \sqrt{(3)^2 + (4)^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$

angle $\cos\theta = \dfrac{a \cdot b}{\|a\| \cdot \|b\|}$

$\cos\theta = \dfrac{11}{2.236 \times 5} = \dfrac{11}{11.18} =$

$\theta = \cos^{-1}(0.983)$

$= 10.579$

Q.7)

$C = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Characteristic equation $|A - \lambda I| = 0$

$$\left[\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] = 0$$

$$\left[\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right] = 0$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)(2-\lambda) - 1$$

$$(4 - 2\lambda - 2\lambda + \lambda^2) - 1$$

$$4 - 4\lambda + \lambda^2 - 1 = 0$$

$$\underline{\lambda^2 - 4\lambda + 3} = 0$$

$$\cancel{(\lambda+4)(\lambda+1) = 0}$$

$$(\lambda-3)(\lambda-1) = 0$$

$$\lambda = 3, 1$$

~~product = -4~~
~~sum = 3~~

$(-4, -1)$

product $= 3$
sum $= -4$

Eigen values = 3 and 1

Q.8.) $P(\text{red ball}) = \frac{4}{9}$

$P(\text{blue ball}) = \frac{3}{9}$

$P(\text{green ball}) = \frac{2}{9}$

$P(\text{drawing a red or blue ball}) = \frac{\text{favourable outcome}}{\text{Total outcome}}$

$= \frac{4+3}{9}$

$= \frac{7}{9} \,// = \underline{\underline{0.777}}$

Q.9.) $P(\text{disease}) = 0.01$

$P(\text{no disease}) = 0.99$

$P(\text{Test. Positive/disease}) = 0.95$

$P(\text{Test. Positive/no disease}) = 1 - 0.95$
$= 0.05$

Using Baye's Theorem

$$P(D/TP) = \frac{P(\text{Test } P/D) \cdot P(D)}{P(\text{Test } P)}$$

Law of total Probability

$$P(\text{Test } P) = P(\text{Test } P / D) \cdot P(D) + P(\text{Test } P / \neg D) \cdot P(\neg D)$$

$$= (0.95 \times 0.01) + (0.05 \times 0.99)$$

$$= \underline{\underline{0.059}}$$

$$P(D / \text{Test } P) = \frac{P(\text{Test } P / D) \cdot P(D)}{0.059}$$

$$= \frac{0.95 \times 0.01}{0.059} = \frac{0.0095}{0.059}$$

$$= \underline{\underline{0.161}} \text{ or } \underline{\underline{16.1\%}}$$

Q.10.) $P(\text{Student pass math}) = 0.7$

$P(\text{Student pass physics}) = 0.5$

$P(\text{Math \& physics}) = 0.3$

$$\text{Conditional Probability} = \frac{P(\text{Math} \cap \text{physics})}{P(\text{Math})}$$

$$= \frac{0.3}{0.7}$$

$$= \underline{\underline{0.428}}$$

Q.11.)

$$H(X) = -\sum P(x) \log P(x)$$

$P(H) = 0.8$

$P(T) = 0.2$

Then; $H(X) = -[P(H) \log_2 P(H) + P(T) \log_2 P(T)]$

$$= -[0.8 \log_2 (0.8) + 0.2 \log_2 (0.2)]$$

$$= -[0.8 \times (-0.3[illegible]) + 0.2 \times (-2.3221)]$$

$$= -[-0.2568 + -0.4642]$$

$$= \underline{\underline{0.721}}$$

Q.12.) $P(1) = 0.1$

$P(2) = 0.2$

$P(3) = 0.3$

$P(4) = 0.4$

" $E(X) = \sum_{i=1}^{4} (x_i) P(x_i)$ "

Then expected value of $X = (1 \times 0.1) + (2 \times 0.2) + (3 \times 0.3) + (4 \times 0.4)$

$$= 0.1 + 0.4 + 0.9 + 1.6$$

$$E(X) = 3 \,//$$

Q.13.)

P(total outcomes) = 36

Two dice sum > 8

possibilities

(3, 6)
(4, 5)
(4, 6)
(5, 4)
(5, 5)
(5, 6)
(6, 3)
(6, 4)
(6, 5)
(6, 6) = 10 favourable outcomes

$$P(\text{sum} > 8) = \frac{\text{favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{10}{36} = \frac{5}{18} = 0.277$$

Q.14) Inorder to calculat probability of getting 7 heads, we need to use binomial co-efficient.

Binomial co-efficient $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

n = no. of trials (10)

k = no. of success in trials (7)

$$\binom{10}{7} = \frac{10!}{7!(3)!}$$

$$= \frac{10 \times 9 \times 8 \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1} (3)!}$$

$$= \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = \frac{720}{6} = \underline{\underline{120}}$$

Then; $P(x=7)$ for head $\quad P(\text{Heads}) = 0.6$

$P(x=4)$ for tail $\quad P(\text{Tail}) = 0.4$

$$P(x=7) = 120 \times (0.6)^7 \times (0.4)^3$$

$$= 120 \times 0.027 \times 0.064$$

$$= \underline{\underline{0.20736}}$$

Q. 15.

vectors = $2i + 4j$

$= i + 2j$

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$$

$\det(A) = (2 \times 2) - (1 \times 4)$

$= 4 - 4 = 0$

Since, through det(A) we got 0 as solution.
Then we can finalise that the given vectors are linearly dependent.

Q. 16. $a = (1, -2, 3)$

$b = (4, 0, -1)$

$a \cdot b = (1 \times 4) + (-2 \times 0) + (3 \times -1)$

$= 4 + 0 - 3$

$= 1$

Q.17.) Total card = 52

$P(\text{ace}) = \frac{4}{52}$

$P(\text{heart}) = \frac{13}{52}$

$P(\text{ace or heart}) = \frac{1}{52}$

$$\text{Conditional Probability} = A \cap B$$

$$= \frac{4 + 13 - 1}{52}$$

$$= \frac{16}{52} = \frac{4}{13}$$

$$= \underline{\underline{0.307}}$$

18.) Two vectors are said to be orthogonal if dot product is equal to zero.

i.e $v = (2, -3, z)$
$u = (1, 4, 5)$

$$v \cdot u = (2 \times 1) + (-3 \times 4) + (z \times 5) = 0$$

$$2 - 12 + 5z = 0$$

$$-10 + 5z = 0$$

$$5z = 10$$

$$z = 10/5 = 2 /\!/$$

Q. 19.) $P(\text{Rain}) = 0.3$

$P(\text{Umbrella}) = 0.6$

$P(\text{Rain} \& \text{umbrella}) = 0.2$

$$P(\text{Rain} / \text{umbrella}) = \frac{P(\text{Rain} \& \text{umbrella})}{P(\text{umbrella})}$$

$$= \frac{0.2}{0.6}$$

$$= 0.333$$

Q. 20.) $C = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

$[A - \lambda I] = 0$

$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$

$\begin{bmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{bmatrix} = 0$

$(2-\lambda)(3-\lambda)$

$6 - 2\lambda - 3\lambda + \lambda^2$

$6 - 5\lambda + \lambda^2$

prod = 6
Sum = -5

$$(\lambda - 2)(\lambda - 3) = 0$$

$$\lambda = 2, 3$$