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MATH 385
HW #1

- Prove De Morgan's Law

*Proof.* $(A \cup B)^{\complement} = A^{\complement} \cap B^{\complement}$,

1. $(A \cup B)^{\complement} \subseteq A^{\complement} \cap B^{\complement}$ Let $x \in (A \cup B)^{\complement}$,
   so $x \notin (A \cup B)$, definition complement,
   and $x \notin A$ or $x \notin B$, definition $\cup$,

   (a) $x \notin A$
   then $x \in A^{\complement}$, definition complement,
   and $x \in A^{\complement} \cap B^{\complement}$ by composition
   (b) Follows from **1a**.

   $\therefore (A \cup B)^{\complement} \subseteq A^{\complement} \cap B^{\complement}$
2. $A^{\complement} \cap B^{\complement} \subseteq (A \cup B)^{\complement}$
   Let $x \in A^{\complement} \cup B^{\complement}$,
   so $x \in A^{\complement}$ and $x \in B^{\complement}$ by definition $\cap$,
   which implies $x \notin A$ or $x \notin B$ by definition complement,
   and $x \notin (A \cup B)$ definition $\cup$,
   so $x \in (A \cup B)^{\complement}$ definition complement,
   $\therefore (A^{\complement} \cap B^{\complement}) \subseteq (A \cup B)^{\complement}$
   which means that $(A \cup B)^{\complement} = A^{\complement} \cap B^{\complement}$

Prove Triangle Inequality
Show $|x + y| \leq |x| + |y|$

*Proof.*
This means that we have four cases to consider,

1. $x \geq 0, y \geq 0$
2. $x \geq 0, y < 0$
3. $x < 0, y \geq 0$
4. $x < 0, y < 0$

**1.** $|x + y| = x + y$
and
$|x| + |y| = x + y$
$\implies |x + y| = |x| + |y|$
and $|x + y| \leq |x| + |y|$
**2.**

1. $|y| \geq x$
   $|x + y| = -(x + y)$
   and
   $|x| + |y| = x - y$
   $\implies 0 \leq x$
   and $0 \leq 2x \implies 0 \leq x + x$
   $\implies -y \leq x + x - y$

$\implies -x-y \leq x-y$
$\implies -(x+y) \leq x-y$
$\implies |x+y| = -(x+y) \leq x-y = |x|+|y|$
$\implies |x+y| \leq |x|+|y|$
$\implies |x+y| \leq |x|+|y|$

2. $|y| < x$ which implies $|x+y| = x+y$ and $|x|+|y| = x-y$,
   $y < 0$ by assumption,
   $\implies 2y < 0$,
   $x+y \leq x-y$,

**3.** Follows from 2
**4.** $|x+y| = -(x+y) = -x-y$
and so $|x|+|y| = -x-y$
$\implies |x+y| = |x|+|y|$
$\implies |x+y| \leq |x|+|y|$

Q.E.D.

Example 1.3.5

Example 1.3.6