Adam Jump
MATH 385
HW #1

- Prove De Morgan's Law

  Proof. $(A \cup B)^{\complement} = A^{\complement} \cap B^{\complement}$,

  1. $(A \cup B)^{\complement} \subseteq A^{\complement} \cap B^{\complement}$ Let $x \in (A \cup B)^{\complement}$,
     so $x \notin (A \cup B)$, definition complement,
     and $x \notin A$ or $x \notin B$, definition $\cup$,

     (a) $x \notin A$
         then $x \in A^{\complement}$, definition complement,
         and $x \in A^{\complement} \cap B^{\complement}$ by composition
     (b) Follows from **1a**.
     $\therefore (A \cup B)^{\complement} \subseteq A^{\complement} \cap B^{\complement}$
  2. $A^{\complement} \cap B^{\complement} \subseteq (A \cup B)^{\complement}$
     Let $x \in A^{\complement} \cup B^{\complement}$,
     so $x \in A^{\complement}$ and $x \in B^{\complement}$ by definition $\cap$,
     which implies $x \notin A$ or $x \notin B$ by definition complement,
     and $x \notin (A \cup B)$ definition $\cup$,
     so $x \in (A \cup B)^{\complement}$ definition complement,
     $\therefore (A^{\complement} \cap B^{\complement}) \subseteq (A \cup B)^{\complement}$
     which means that $(A \cup B)^{\complement} = A^{\complement} \cap B^{\complement}$

  Q.E.D.

Prove Triangle Inequality using cases
Show $|x + y| \le |x| + |y|$

Proof.
This means that we have four cases to consider,

1. $x \ge 0, y \ge 0$
2. $x \ge 0, y < 0$
3. $x < 0, y \ge 0$
4. $x < 0, y < 0$

1. $|x + y| = x + y$
and
$|x| + |y| = x + y$
$\implies |x + y| = |x| + |y|$
and $|x + y| \le |x| + |y|$
2.

1. $|y| \ge x$
   $|x + y| = -(x + y)$
   and
   $|x| + |y| = x - y$

$\implies 0 \leq x$

and $0 \leq 2x \implies 0 \leq x + x$

$\implies -y \leq x + x - y$

$\implies -x - y \leq x - y$

$\implies -(x + y) \leq x - y$

$\implies |x + y| = -(x + y) \leq x - y = |x| + |y|$

$\implies |x + y| \leq |x| + |y|$

$\implies |x + y| \leq |x| + |y|$

2. $|y| < x$ which implies $|x + y| = x + y$ and $|x| + |y| = x - y$,

   $y < 0$ by assumption,

   $\implies 2y < 0$,

   $x + y \leq x - y$,

3. Follows from 2

4. $|x + y| = -(x + y) = -x - y$

and so $|x| + |y| = -x - y$

$\implies |x + y| = |x| + |y|$

$\implies |x + y| \leq |x| + |y|$

Q.E.D.

Example 1.3.5 Determine the set $a = \{x \mid |x - 17| = |2x + 8|$

Example 1.3.6 For $c \geq 0$, define the set $A_c = \{x \mid |x - 2| + |x - 10| = c\}$