Definition 1. *Zero-Divisors*
A zero-divisor is a nonzero element a of a commutative ring R such that there is a nonzero element $b \in R$ with $ab = 0$.

Definition 2. *Integral Domain*
An integral domain is a commutative ring with unity and no zero-divisors.

EXAMPLE 1
The ring of integers is an integral domain.
EXAMPLE 2
The ring of Gaussian integers $\mathbb{Z}[i] = (a + bi \,|\, a, b \in \mathbb{Z})$ is an integral domain.
EXAMPLE 3
The ring $\mathbb{Z}[x]$ of polynomials with integer coefficients is an integral domain.
EXAMPLE 4
The ring $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} \,|\, a, b \in \mathbb{Z}\}$ is an integral domain.
EXAMPLE 5
The ring, Z_p, of integers modulo a prime p is an integral domain.
EXAMPLE 6
The ring $\mathbb{Z}_n$ of integers modulo n is not an integral domain when n is not prime.
EXAMPLE 7
The ring $M_2(\mathbb{Z})$ of 2×2 matrices over the integers is not an integral domain.
EXAMPLE 8
$Z \bigoplus Z$ is not an integral domain.

Theorem 1. *Let $a, b,$ and c belong to an integral domain. If $n \neq 0$ and $ab = ac$, Then $b = c$.*

Definition 3. *Field*
A field is a commutative ring with unity in which every nonzero element is a unit.

Theorem 2. *A finite integral domain is a field.*

Theorem 3. *For every prime p, $\mathbb{Z}_p$, The ring of integers modulo p is a field.*

Definition 4. *Characteristic of a Ring*
The characteristic of a ring R is the least positive integer n such that nnx I 0 for all x in R. If no such integer exists, we say that R has characteristic 0. The characteristic of R is denoted by char(R).

Theorem 4. *Let R be a ring with unity 1. If 1 has infinite order under addition, then the characteristic of R is 0. If 1 has order n under addition, then the characteristic of R is n.*

Theorem 5. *The characteristic of an integral domain is 0 or prime.*