Utilizations of Queuing Theory and Digraphs for more Efficient Merging in Tollways

Adam Jump, Richard Quackenbush, Amy Vennos

Mathematics and Computer Science
Salisbury University

Abstract

We propose a model that optimizes toll plaza design for maximum throughput, showing improvement by more than a standard deviation in mean system time. Furthermore, we describe how this system will function using driverless vehicles and electronic toll collection. A cost-benefit analysis of installation shows that the implementation cost for refitting existing toll plazas with electronic toll collection equipment is completely offset by the staffing and maintenance cost savings in the scenario of demolition of a manual toll collection booth and replacement with a completely new electronic toll collection booth.

Key Terms

MTC Manual Toll Collection. A human operated toll booth in which the driver pays the toll before continuing.

ETC Electronic Toll Collection. A device transmits data to the ETC booth which automatically raises the booth gate upon electronic payment.

FOZ Fan Out Zone. The merge lanes from the input lane headed into the toll collection area.

FIZ Fan In Zone. The merge lanes after the toll collection area headed into the egress lane.

LOE Lanes of Egress / Egress Lane. The travel lanes in the original highway.

Assumptions

- All booths are open.
- Drivers will choose shortest queue.
- Human factor always costs time.
- All cars drive the speed limit.
- Peripheral lanes require more time to merge than center lanes.
- All booths have an electronic barrier.
- All self-driving cars have an ETC transponder.

The Current Model

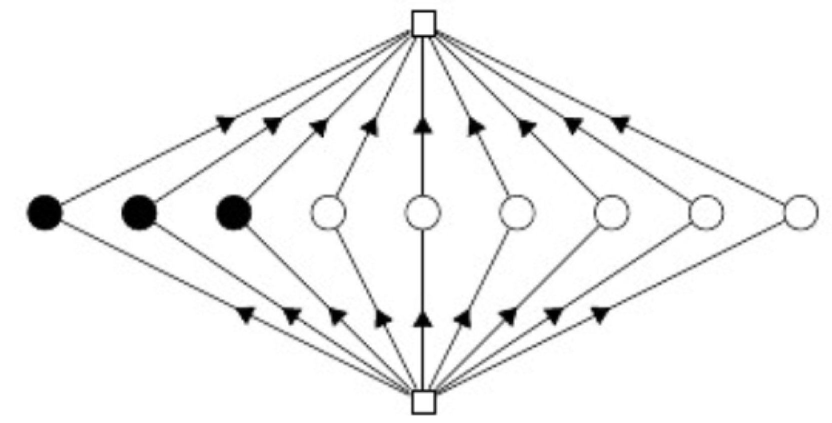

Figure 2: Current Model

The current model illustrates a standard toll plaza. ETC booths are placed on the left, and MTC booths are on the right.

The Proposed Model

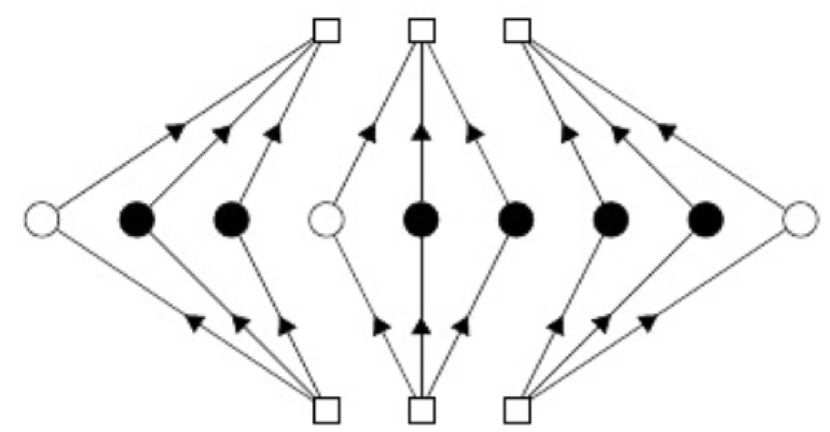

Figure 3: Current Model

This graph is automorphic to any combination of nodes within a partition, as long as the ETC booths are placed together.

Main Result

We propose a model that optimizes toll plaza design for maximum throughput, showing improvement by more than a standard deviation in mean system time.

Methods

We must take into account human error to model the flow of traffic in the FOZ- randomness is taken into account using the Erlang function, shown below:

$$\text{Erlang } c = \frac{1}{\mu} + \frac{c\mu\Gamma(c)\left(\frac{\lambda}{\mu}\right)^c}{(c\mu-\lambda)\left(c\mu\Gamma(c)\left(\frac{\lambda}{\mu}\right)^c - c!e^{\lambda/\mu}(\lambda-c\mu)\Gamma\left(c,\frac{\lambda}{\mu}\right)\right)}$$

s_i	the number of input lanes
s	the number of booths
P	the number of partitions
l_T	the total number of lanes within a partition
l_e	the number of ETC lanes within a partition
μ	the mean service rate of the queue
λ	the mean arrival rate of the queue

We use the Erlang function to calculate the mean system time of each queue. The equation for mean system time of each queue in the proposed model is shown below:

$$\frac{\sum_{n=1}^{s_i}\left(\left(\sum_{n=1}^{l_e}\text{Erlang}_n 1 + \sum_{n=1}^{l_m}\text{Erlang}_n 1\right)/\,l\right)}{s_i} = \text{average mean system time}$$

Queuing Theory and Partitioning

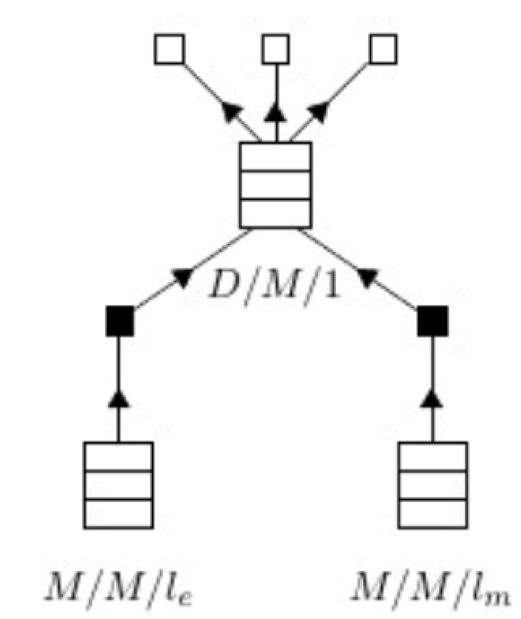

Figure 4: Queuing diagram

$$s = (s_i)(l_T) \tag{1}$$

$$l_e \geq \lceil \frac{l_t}{2} \rceil \tag{2}$$

$$|P| = \frac{s}{s_i} \in \mathbb{N} \tag{3}$$

Conclusion

The use of concrete barriers to partition lanes is more time efficient in both the FIZ and FOZ. Though we are mainly concerned with the efficiency in the FIZ, we must consider the toll plaza as a whole to effectively model the FIZ. Our proposed model decreases travel time through the toll plaza by more than one standard deviation, allowing toll plazas to service more customers as well as improve customer satisfaction.

Additional Information

Simulations, cost benefit analysis, and sensitivity analysis were computed in the WL. The paper on which this project was based was a Meritorious Winner in the Comap MCM.

Acknowledgements

Salisbury University
Department of Mathematics and Computer Science

Contact Information

- Email:
  ajump2@gulls.salisbury.edu,
  rquackenbush1@gulls.salisbury.edu,
  avennos3@gulls.salisbury.edu