Multiple Traveling Salesman Problem (mTSP)

The single-depot multiple TSP (SD-MTSP) is a simple extension of the standard TSP, in which more than one salesman is allowed to visit the set of interconnected cities, such that each city is visited exactly once (by a single salesman) and the total cost of the traveled subtours is minimized.

MinMax SD-MTSP, the objective is to minimize the longest subtour. If the lengths of the m subtours (corresponding to the m salesmen) are encoded as a vector v in R^m, the standard objective (minimizing the total cost) may be formulated as minimizing the Euclidean norm of v.

SD-MTSP can be formalised using a directed graph G = (V, A), where V is the set of nodes and A the set of arcs. The graph has associated a cost matrix C = (cij) for each arc (i, j) ∈ A. Let xij be a binary variable that is equal to 1 if arc (i, j) is selected in the candidate solution and 0 otherwise. ui denotes the number of nodes visited on a salesman's path from the origin/depot to node i, for any salesman, i.e. the position of node i in a subtour.

The formulation of the standard SD-MTSP aims at minimizing the total cost/length of the subtours and can be formulated as an integer linear program as follows (node 1 is considered to be the origin/depot):

$$\min \quad \sum_{(i,j)\in A} c_{ij}x_{ij} \qquad (1)$$

$$\text{s.t.} \quad \sum_{j=2}^{n} x_{1j} = m \qquad (2)$$

$$\sum_{j=2}^{n} x_{j1} = m \qquad (3)$$

$$\sum_{i=1}^{n} x_{ij} = 1,\ j = 2, .., n \qquad (4)$$

$$\sum_{j=1}^{n} x_{ij} = 1,\ i = 2, .., n \qquad (5)$$

$$x_{1i} + x_{i1} \leq 1,\ i = 2, .., n \qquad (6)$$

$$u_i - u_j + (n-m) \cdot x_{ij} \leq n - m - 1, \qquad (7)$$
$$2 \leq i \neq j \leq n$$

The MinMax formulation for SD-MTSP requires a third dimension (k) in order to clearly distinguish among the arcs assigned to each salesman:

$$\min \quad T \tag{9}$$

$$\text{s.t.} \quad \sum_{j=2}^{n} x_{1jk} = 1, \; k = 1, .., m \tag{10}$$

$$\sum_{j=2}^{n} x_{j1k} = 1, \; k = 1, .., m \tag{11}$$

$$\sum_{i=1}^{n} \sum_{k=1}^{m} x_{ijk} = 1, \; j = 2, .., n, \; i \neq j \tag{12}$$

$$\sum_{j=1}^{n} \sum_{k=1}^{m} x_{ijk} = 1, \; i = 2, .., n, \; i \neq j \tag{13}$$

$$\sum_{i=1}^{n} x_{ijk} = \sum_{i=1}^{n} x_{jik}, \; j = 2, .., n, \; k = 1, .., m, \; i \neq j \tag{14}$$

$$u_i - u_j + (n - m) \cdot \sum_{k=1}^{m} x_{ijk} \leq n - m - 1, \quad 2 \leq i \neq j \leq n \tag{15}$$

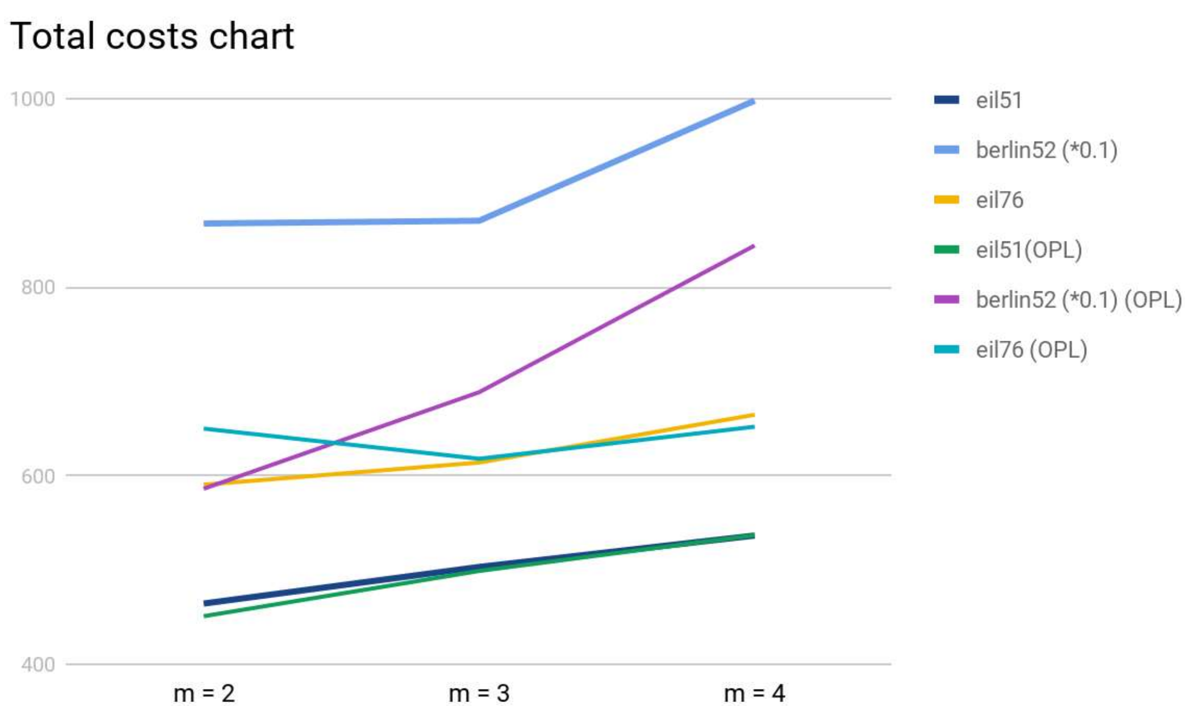
$$\sum_{(i,j) \in A} c_{ij} x_{ijk} \leq T, \; k = 1, .., m \tag{16}$$

$$x_{ijk} \in \{0, 1\}, \; \forall (i, j) \in A, \; k = 1, .., m. \tag{17}$$

The objective here is to minimize the longest subtour, and is clearly expressed by introducing inequality (16) in conjunction with the new variable T.

- **SOM - Results**

TSP instance	n (# cities)	m (#salesmen)	Total cost	Time (s)	OPL cost
eil51	**51**	**2**	464.418	1085.077	451
		3	503.245	490.592	499
		5	536.565	193.401	537
berlin52	**52**	**2**	8675.680	1001.593	5862
		3	8704.536	573.025	6886
		5	9978.843	209.845	8442
eil76	**76**	**2**	590.514	3105.371	650
		3	614.003	1885.580	618
		5	664.670	723.326	652



Experiments were performed on:

- Core i5 CPU 2.4GHz
- 8GB RAM
- 900GB hard disk.
- running Windows 10 OS

Algorithms:

- **SOM**
  - implemented in python 3.6 and using:
    - matplotlib 2.1.0
    - pandas 0.20.3
    - numpy 1.14.0
  - parameters:
    - nr_iterations = 200
    - alfa = 0.03
    - learning_rate = 0.6
    - weights = 0.3
- **CPLEX**
  - implemented in OPL
  - settings:
    - time limit = 10800.0
    - solution limit = 100

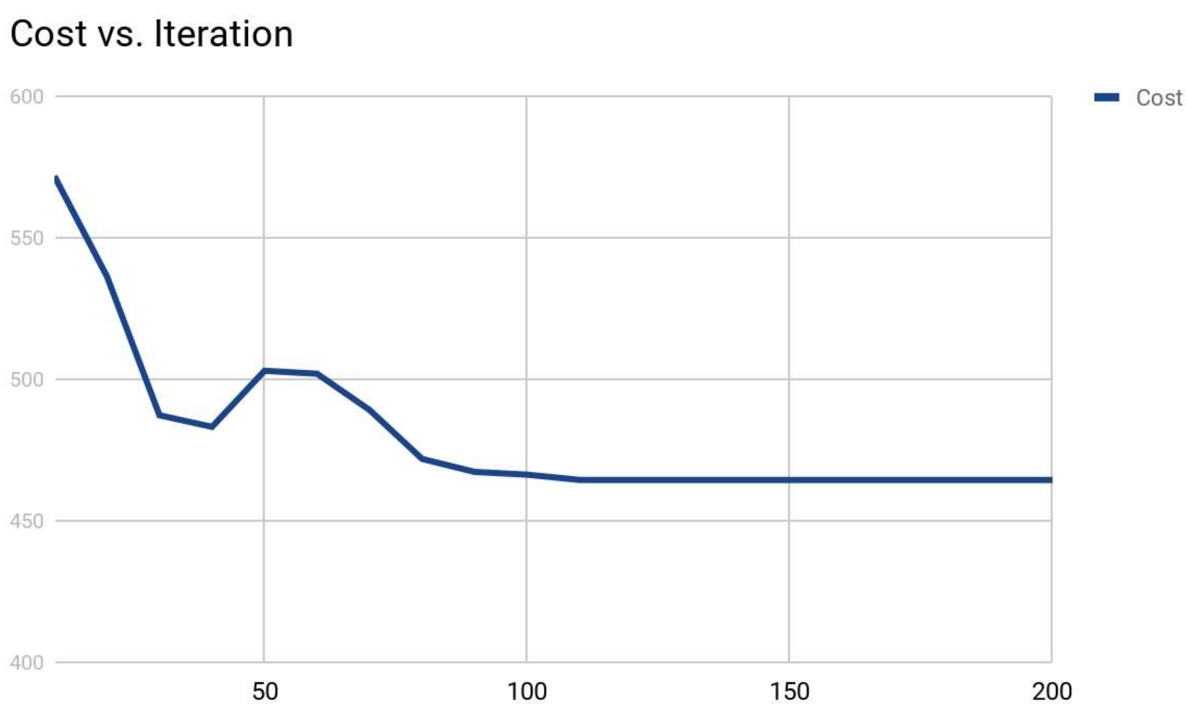
- ## SOM - Detailed results

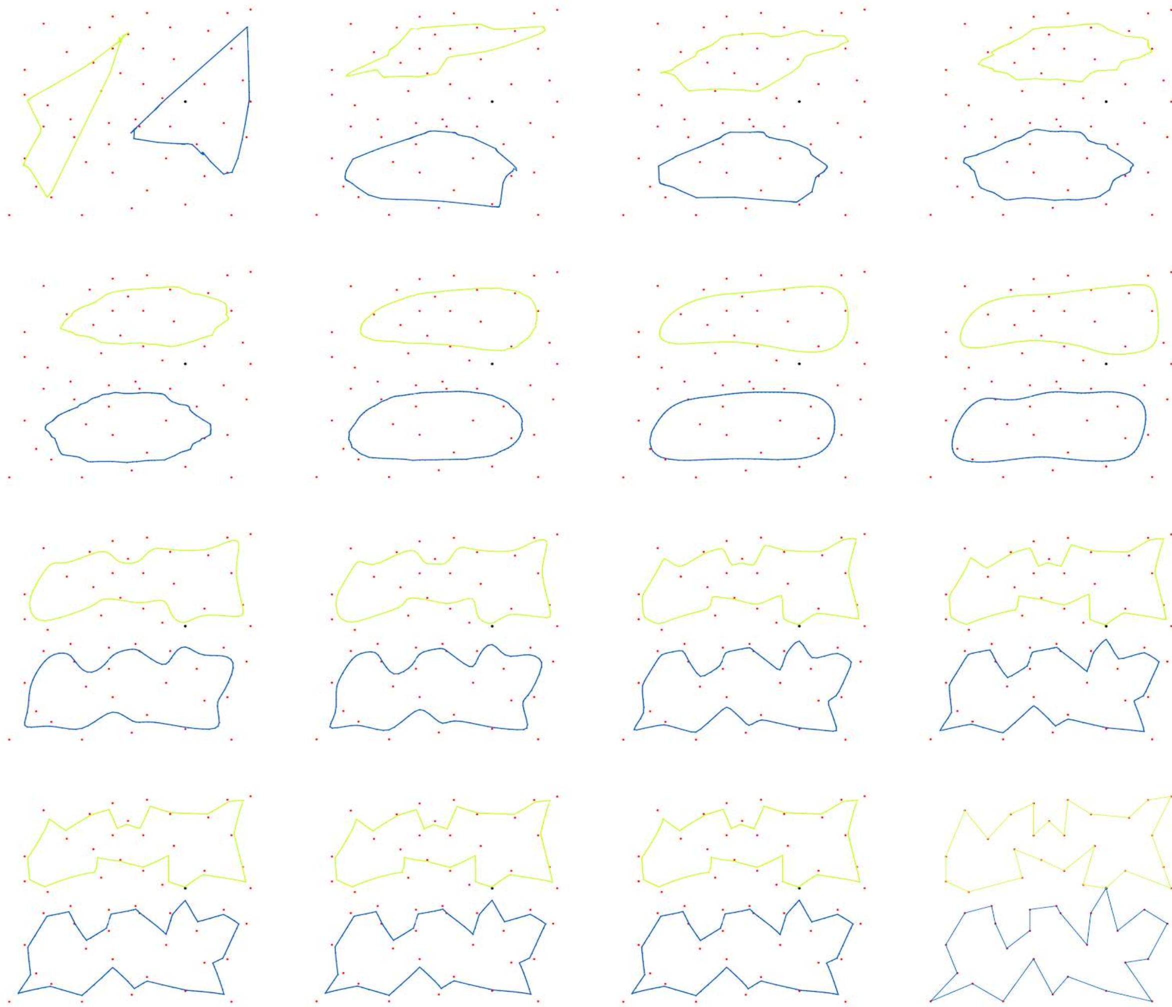
- **Eil51, m=2**

S1: 1 27 6 51 46 12 47 4 17 37 44 42 19 40 41 13 18 25 14 24 43 23 7 26 8 48 1
Total cost: 236.86527874731289

S2: 1 22 31 28 3 36 35 20 29 21 16 50 9 34 30 10 39 33 45 15 5 49 38 11 2 32 1
Total cost: 227.5533059822798

Total sum of costs is: 464.4185847295927
Elapsed seconds: 1085.077





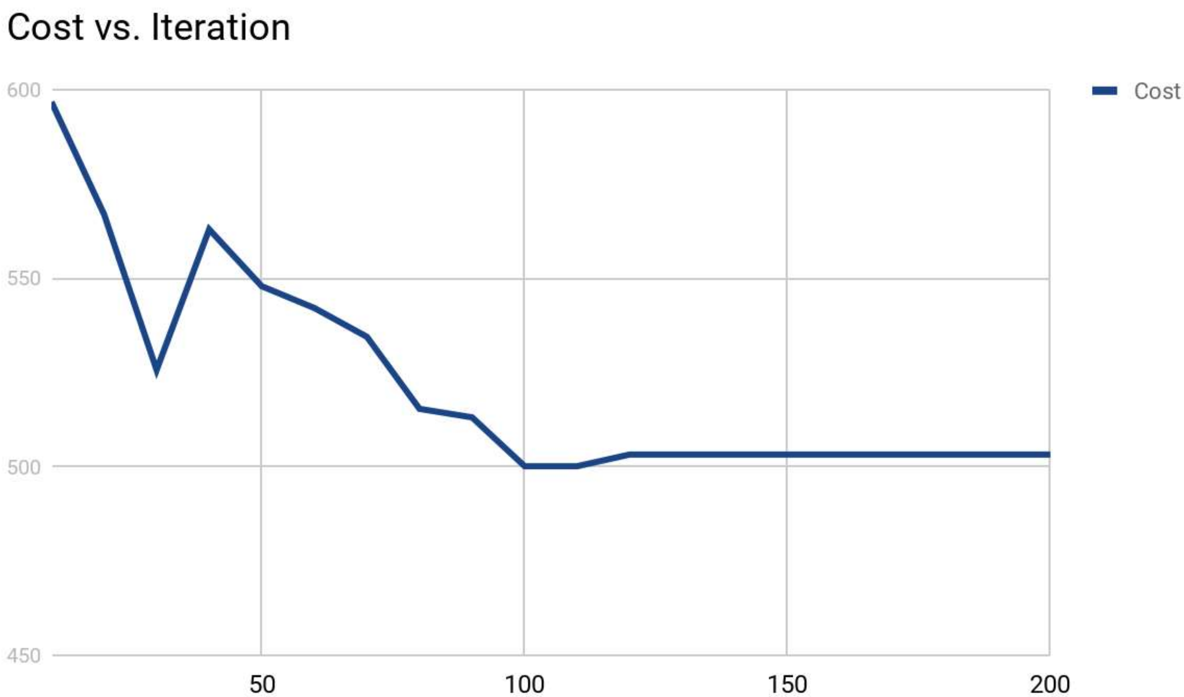
- **Eil51, m=3**

S1: 1 32 27 6 14 24 43 7 23 48 26 8 31 28 22 1
Total cost: 137.19262931767327

S2: 1 11 16 38 9 49 10 39 30 34 50 21 29 20 35 36 3 2 1
Total cost: 165.7289999935344

S3: 1 46 12 5 17 37 15 33 45 44 42 19 40 41 13 25 18 4 47 51 1
Total cost: 200.3240206352064

Total sum of costs is: 503.24564994641406
Elapsed seconds: 490.592


Cost vs. Iteration
Cost
600
550
500
450
50
100
150
200

- ❑ **Eil51, m=5**

S1: 1 6 14 25 24 43 7 23 48 1
Total cost: 99.58001316036761

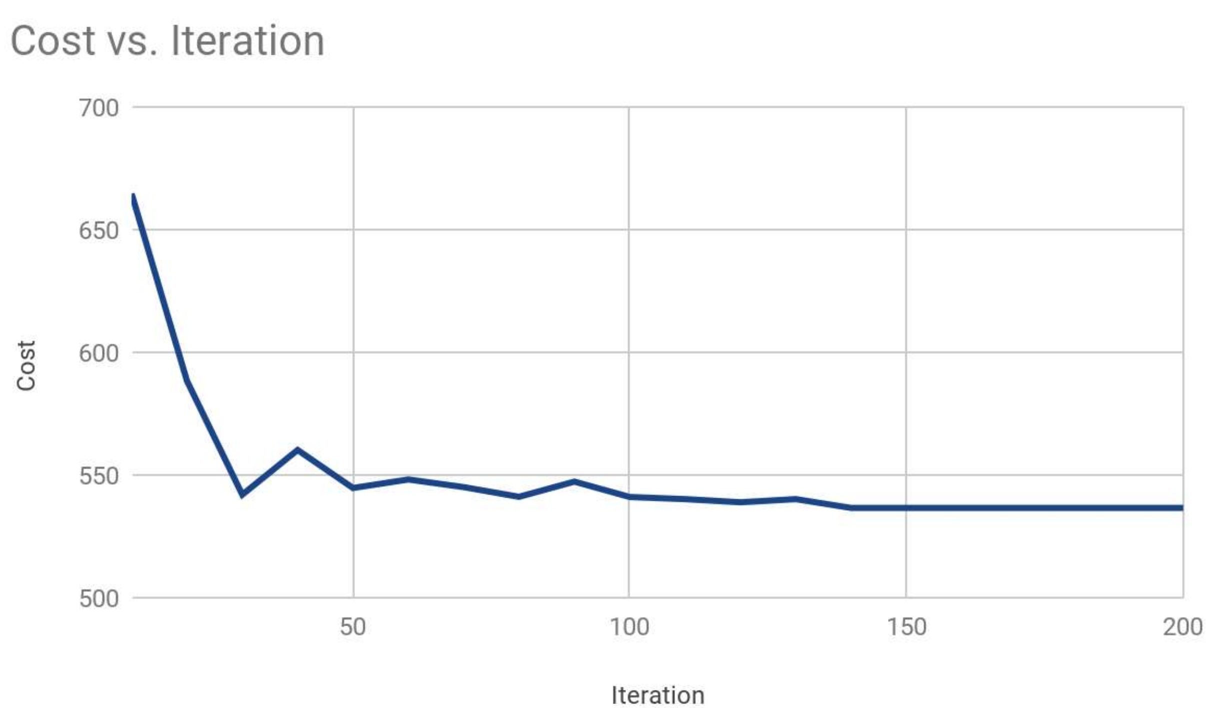
S2: 1 8 26 31 28 3 36 35 20 22 1
Total cost: 92.07551076857803

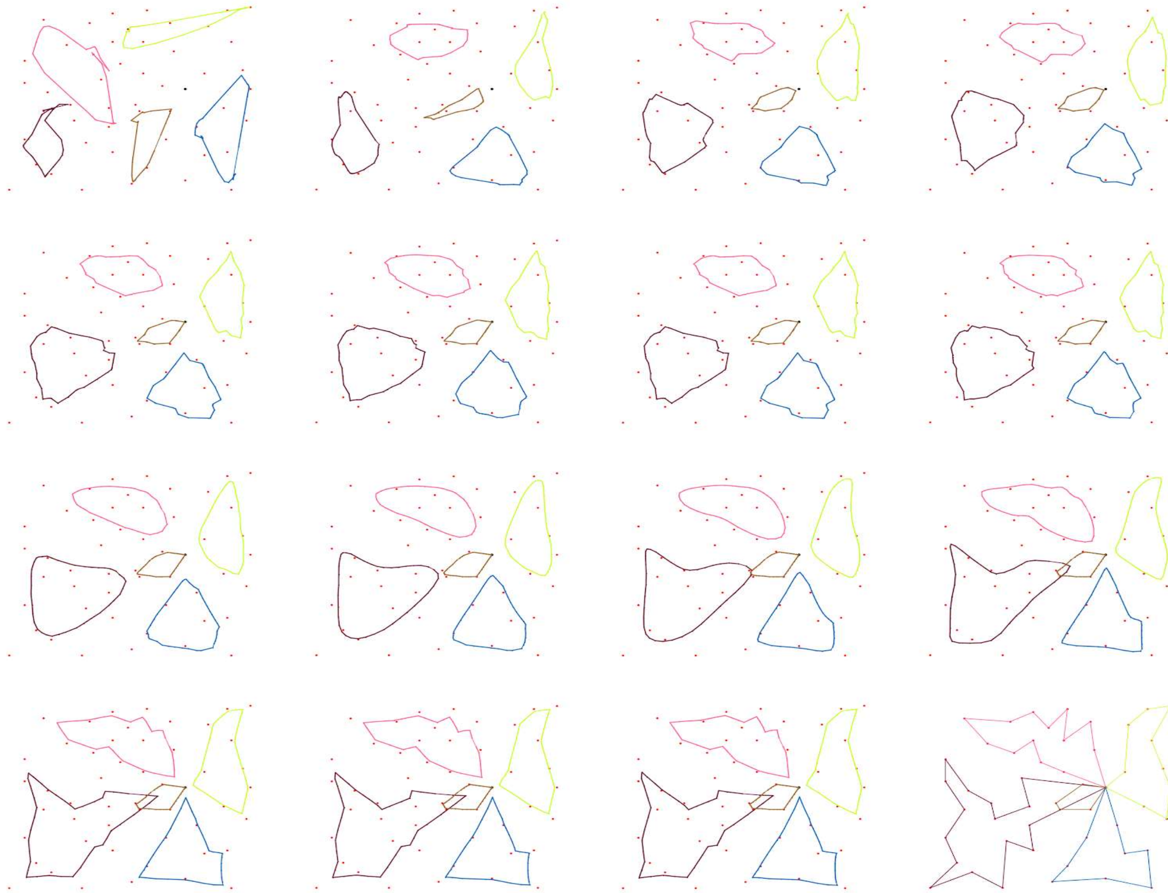
S3: 1 2 29 16 21 50 34 30 39 10 49 9 38 1
Total cost: 123.74755165670656

S4: 1 '47' 18 4 13 41 40 19 42 44 45 33 15 37 17 12 5 1
Total cost: 187.5616031820688

S5: 1 27 51 46 32 1
Total cost: 33.600632713389444

Total sum of costs is: 536.5653114811105
Elapsed seconds: 193.401





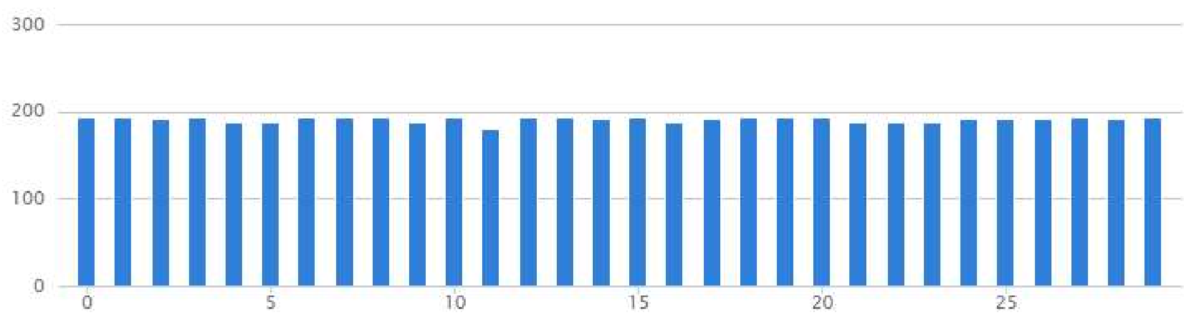
- **Eil10, m=2**

Sample Standard Deviation, s	3.137196194538
Variance (Sample Standard), s^2	9.8419999630237
Population Standard Deviation, σ	3.0844664526608
Variance (Population Standard), σ^2	9.5139332975896
Total Numbers, N	30
Sum:	5737.2984588602
Mean (Average):	191.24328196201
Standard Error of the Mean ($SE_{\bar{x}}$):	0.57277104102261

Confidence Interval Approximations, If sampling distribution of the mean follows normal distribution

Confidence Level	Range
68.3%, $SE_{\bar{x}}$	190.67051092098 - 191.81605300303
90%, $1.645SE_{\bar{x}}$	190.30107359952 - 192.18549032449
95%, $1.960SE_{\bar{x}}$	190.1206507216 - 192.36591320241
99%, $2.576SE_{\bar{x}}$	189.76782376033 - 192.71874016368
99.9%, $3.291SE_{\bar{x}}$	189.358292466 - 193.12827145801
99.99%, $3.891SE_{\bar{x}}$	189.01462984139 - 193.47193408263
99.999%, $4.417SE_{\bar{x}}$	188.71335227381 - 193.7732116502
99.9999%, $4.892SE_{\bar{x}}$	188.44128602932 - 194.04527789469

Column Chart of the Values



- **Eil10, m=5**

Sample Standard Deviation, s	9.095544904452
Variance (Sample Standard), s^2	82.728937108902
Population Standard Deviation, σ	8.942667715617
Variance (Population Standard), σ^2	79.971305871939
Total Numbers, N	30
Sum:	8209.6827952279
Mean (Average):	273.65609317426
Standard Error of the Mean ($SE_{\bar{x}}$):	1.6606117056565

Confidence Interval Approximations, If sampling distribution of the mean follows normal distribution

Confidence Level	Range
68.3%, $SE_{\bar{x}}$	271.99548146861 - 275.31670487992
90%, $1.645SE_{\bar{x}}$	270.92438691846 - 276.38779943007
95%, $1.960SE_{\bar{x}}$	270.40129423118 - 276.91089211735
99%, $2.576SE_{\bar{x}}$	269.37835742049 - 277.93382892804
99.9%, $3.291SE_{\bar{x}}$	268.19102005095 - 279.12116629758
99.99%, $3.891SE_{\bar{x}}$	267.19465302756 - 280.11753332097
99.999%, $4.417SE_{\bar{x}}$	266.32117127038 - 280.99101507815
99.9999%, $4.892SE_{\bar{x}}$	265.53238071019 - 281.77980563834

Column Chart of the Values