

Comparison of two theorem provers: Isabelle & Coq

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Comparison of two
theorem provers:
Isabelle & Coq

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PRINCIPLES OF MATHEMATICAL LOGIC

BY

D. HILBERT AND W. ACKERMANN

TRANSLATED FROM THE GERMAN BY

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Über die Bedeutung des Satzes vom ausgeschlossenen Dritten in der Mathematik, insbesondere in der Funktionentheorie ¹⁾.

Von L. E. J. Brouwer in Amsterdam.

§ 1.

Innerhalb eines bestimmten endlichen „Hauptsystems“ können Eigenschaften von Systemen, d. h. Abbildbarkeiten von Systemen auf andere Systeme mit vorgeschriebenen Elementkorrespondenzen, immer *geprüft* (d. h. entweder bewiesen oder ad absurdum geführt) werden; die durch die betreffende Eigenschaft angewiesene Abbildung besitzt nämlich auf jeden Fall nur eine endliche Anzahl von Ausführungsmöglichkeiten, von denen jede für sich unternommen und entweder bis zur Beendigung oder bis zur Hemmung fortgesetzt werden kann. (Hierbei liefert das Prinzip der mathematischen Induktion oft das Mittel, derartige Prüfungen ohne individuelle Betrachtung jedes an der Abbildung beteiligten Elementes bzw. jeder für die Abbildung bestehenden Ausführungsmöglichkeit durchzuführen; demzufolge kann die Prüfung auch für Systeme mit sehr großer Elementenzahl mitunter verhältnismäßig schnell verlaufen.)

Auf Grund der obigen Prüfbarkeit gilt für innerhalb eines bestimmten endlichen Hauptsystems konzipierte Eigenschaften der *Satz vom ausgeschlossenen Dritten*, d. h. das Prinzip, daß jede Eigenschaft für jedes System entweder richtig oder unmöglich ist, und insbesondere der *Satz von der Reziprozität der Komplementärspezies*, d. h. das Prinzip, daß für jedes System aus der Unmöglichkeit der Unmöglichkeit einer Eigenschaft die Richtigkeit dieser Eigenschaft folgt.

Wenn z. B. die Vereinigung $\mathfrak{S}(p, q)$ zweier mathematischer Spezies p und q wenigstens 11 Elemente enthält, so folgt hieraus auf Grund des (in diesem Falle als „Disjunktionsprinzip“ auftretenden) Satzes vom ausgeschlossenen Dritten, daß entweder p oder q wenigstens 6 Elemente enthält.

Ebenso: Wenn man in der elementaren Arithmetik bewiesen hat, daß, wenn

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wenigstens 11 Elemente enthält, so folgt hieraus auf Grund des (in diesem Falle
als „Disjunktionsprinzip“ auftretenden) Satzes vom ausgeschlossenen Dritten,
daß entweder p oder q wenigstens 6 Elemente enthält.

Ebenso: Wenn man in der elementaren Arithmetik bewiesen hat, daß, wenn
ein Zahlensystem S die Eigenschaften P und Q besitzt, dann

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Elements of a Formal System

- A formula (judgement, statement) $\phi \in \Phi$:

$$\begin{aligned}\phi &:= p \mid q \mid \dots \\ &\mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2 \mid \neg \phi_1 \mid \phi_1 \rightarrow \phi_2 \\ &\mid \text{true} \mid \text{false} \\ &\mid \dots\end{aligned}$$

- Propositional variables: — $p, q, \dots \in V$
- An axiom — $\phi_A \in A$
- An inference rule τ — a transition function $\tau : \Phi \rightarrow \Phi$
- A formula ϕ provable from Φ — $\Phi \vdash \phi$
- A tautology \top — $\vdash \phi$
- A contradiction \perp — $\vdash \neg \phi$

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Definition of the formal system

A *formal system* is a quadruple $\Gamma = \langle A, V, \Omega, R \rangle$, where

- A – set of axioms
- V – set of propositional variables
- Ω – set of logical operators
- R – set of inference rules

A *formal proof* of the formula ϕ is a finite sequence of judgements

$$\psi_1 \xrightarrow{\tau_1} \psi_2 \xrightarrow{\tau_2} \dots \xrightarrow{\tau_n} \psi_n$$

where each ψ_i is either an axiom ϕ_{A_i} , or a formula inferred from the set of previously derived formulas according the rules of inference.

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Properties of a formal system

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A formal system Γ is called:

- *consistent*, if $\nexists \phi \in \Gamma : \Gamma \vdash \phi \wedge \Gamma \vdash \neg \phi \Leftrightarrow \Gamma \not\vdash \perp$;
- *complete*, if $\forall \phi \in U : A \vdash \phi \vee A \vdash \neg \phi$;
- *independent*, if $\nexists a \in A : A \vdash a$.

Classical Logic

example: The Hilbert System

Set of axioms:

$$A \rightarrow (B \rightarrow A) \quad (\text{A1})$$

$$(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C)) \quad (\text{A2})$$

$$A \vee \neg A \quad (\text{EM})$$

Single inference rule (*Modus Ponens*)

$$\llbracket A, A \rightarrow B \rrbracket \longrightarrow B \quad (\text{MP})$$

Some provable tautologies:

$$\neg\neg(A \vee \neg A) \quad (\text{nnEM})$$

$$A \rightarrow \neg\neg A \quad (\text{DNi})$$

$$\neg\neg A \rightarrow A \quad (\text{DNe})$$

$$((A \rightarrow B) \rightarrow A) \rightarrow B \quad (\text{PL})$$

$$\neg(A \wedge B) \rightarrow \neg A \vee \neg B \quad (\text{DMdi})$$

$$\neg(A \vee B) \rightarrow \neg A \wedge \neg B \quad (\text{DMci})$$

$$\neg A \wedge \neg B \rightarrow \neg(A \vee B) \quad (\text{DMce})$$

$$\neg A \vee \neg B \rightarrow \neg(A \wedge B) \quad (\text{DMde})$$

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Intuitionistic Logic

a.k.a. Constructive Logic

Set of axioms:

$$A \rightarrow (B \rightarrow A) \quad (\text{A1})$$

$$(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C)) \quad (\text{A2})$$

$$\cancel{A \vee \neg A} \quad (\text{EM})$$

Single inference rule (*Modus Ponens*)

$$\llbracket A, A \rightarrow B \rrbracket \longrightarrow B \quad (\text{MP})$$

Some provable tautologies:

$$\neg\neg(A \vee \neg A) \quad (\text{nnEM})$$

$$A \rightarrow \neg\neg A \quad (\text{DNi})$$

$$\cancel{\neg\neg A \rightarrow A} \quad (\text{DNe})$$

$$\cancel{((A \rightarrow B) \rightarrow A) \rightarrow B} \quad (\text{PL})$$

$$\cancel{\neg(A \wedge B) \rightarrow \neg A \vee \neg B} \quad (\text{DMdi})$$

$$\neg(A \vee B) \rightarrow \neg A \wedge \neg B \quad (\text{DMci})$$

$$\neg A \wedge \neg B \rightarrow \neg(A \vee B) \quad (\text{DMce})$$

$$\neg A \vee \neg B \rightarrow \neg(A \wedge B) \quad (\text{DMde})$$

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Isabelle: first acquaintance

- a generic proof assistant
- based on classical higher-order logic
- created in 1986 by
 - Larry Paulson @ University of Cambridge, and
 - Tobias Nipkow @ Technische Universität München
- uses powerful functional language HOL
- the proof system core *Isabelle* is extended by various theories: Isabelle/HOL, Isabelle/ZF, Isabelle/CCL, etc.

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Example 3: Definition of basic datatypes

```
datatype bool =  
  True | False
```

```
datatype nat =  
  zero ("0") | Suc nat
```

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Example 5: Definition of basic datatypes

```
datatype bool =  
  True | False
```

```
datatype nat =  
  zero ("0") | Suc nat
```

Example 6: Definition of addition over nat

```
fun add :: "nat  $\Rightarrow$  nat  $\Rightarrow$  nat"  
  where  
    "add 0 n = n" |  
    "add (Suc m) n = Suc (add m n)"
```

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Coq: first acquaintance

- a formal proof management system
- based intuitionistic logic (Calculus of Inductive Constructions)
- created at INRIA (Paris, France) in 1984
- uses powerful functional language `Gallina`
- has large collection of formalised theories
- widely used in software verification (proof code extraction)

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Example 9: Definition of basic datatypes

```
Inductive False : Prop := .  
Inductive True : Prop :=  
  I : True.
```

```
Inductive nat : Type :=  
  | O : nat  
  | S : nat -> nat.
```

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- uses powerful functional language Gallina
- has large collection of formalised theories
- widely used in software verification (proof code extraction)

Example 11: Definition of basic datatypes

```
Inductive False : Prop := .
Inductive True : Prop :=
  I : True.

Inductive nat : Type :=
  | O : nat
  | S : nat -> nat.
```

Example 12: Definition of addition over `nat`

```
Fixpoint add (n m: nat) : nat :=
  match n with
  | O => m
  | S n' => S (n' + m)
  end
where "n + m" :=
  (add n m) : nat_scope.
```

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Major similarities:

- both work in a similar way of *verifying* the proof or *assisting* in creation of the new one
- $\text{premises} \xrightarrow{\text{tactics}} \text{goals}$ (forward proof)
- $\text{goals} \xrightarrow{\text{tactics}} \text{premises}$ (backward proof)
- both have large amount of libraries with formalised theories
- both dispose the set of highly automated tactics
- both are being actively developed these days

Major differences:

- based on different logics \Rightarrow
 - ★ unprovable statements and invalid proofs in Coq
 - ★ sometimes more complex proof in Coq
 - ★ *constructive* proof in Coq

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Isabelle: proof example in propositional logic

The screenshot shows the Isabelle2017 IDE with a file named 'Scratch.thy (modified)'. The main editor displays a theory 'Scratch' with a lemma 'DeMorganQuantified_Isabelle'. The lemma's statement is $\neg (\forall x. P x) \implies \exists x. \neg P x$. The proof is structured as follows:

```
theory Scratch
  imports Main
begin

lemma DeMorganQuantified_Isabelle:
  assumes "¬ (∀ x. P x)"
  shows "∃ x. ¬ P x"
proof (rule classical)
  assume "¬ ∃ x. ¬ P x"
  have "∀ x. P x"
  proof
    fix x show "P x"
    proof (rule classical)
      assume "¬ P x"
      then have "∃ x. ¬ P x" ..
      with <¬ ∃ x. ¬ P x> show ?thesis by contradiction
    qed
  qed
  with <¬ (∀ x. P x)> show ?thesis by contradiction
qed
```

The right-hand pane shows the current state of the proof:

```
proof (state)
this:
  ¬ x. ¬ P x

goal (1 subgoal):
  1. ¬ x. ¬ P x ⟹ ∃ x. ¬ P x
```

The bottom status bar indicates the current position in the document: 9,22 (162/411).

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Isabelle: proof example over nat

The screenshot displays the Isabelle2017 IDE with a file named 'Scratch.thy (modified)'. The interface includes a menu bar (File, Edit, Search, Markers, Folding, View, Utilities, Magros, Plugins, Help), a toolbar with various icons, and a main editing area. The left pane shows the file structure with 'Scratch.thy (~/.)' selected. The right pane shows the proof state with a goal and a list of subgoals.

```
theory Scratch
  imports Main
begin

fun range_sum :: "nat => nat"
  where "range_sum n = (∑ k::nat=0..n . k)"

theorem arith_progr_sum: "2 * (range_sum n) = n * (n + 1)"
proof (induct n)
  show "2 * range_sum 0 = 0 * (0 + 1)" by simp
next
  fix n have "2 * range_sum (n + 1) = 2 * (range_sum n) + 2 * (n + 1)" by simp
  also assume "2 * (range_sum n) = n * (n + 1)"
  also have "... + 2 * (n + 1) = (n + 1) * (n + 2)" by simp
  finally show "2 * (range_sum (Suc n)) = (Suc n) * (Suc n + 1)" by simp
qed
```

The right pane shows the proof state with a goal and a list of subgoals:

```
proof (prove)
goal (1 subgoal):
1. n * (n + 1) + 2 * (n + 1) = (n + 1) * (n + 2)
```

The bottom status bar shows the file path '14,15 (387/507)' and the session information '(isabelle,isabelle,UTF-8-isabelle)Nm r o UG 202/1242MB 2:18 AM'.

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Coq: proof example in propositional logic

The screenshot shows the CoqIDE interface with a file named `*scratch*`. The main editor contains the following Coq code:

```
Theorem DeMorganPropositional_Coq:
  forall P Q : Prop, ~(P ∨ Q) <-> ~P ∧ ~Q.
Proof.
  (* 'tauto' automatically proves the equation *)
  intros P Q. unfold iff.
  split.
  - intros H_not_or. unfold not. constructor.
    + intro H_P. apply H_not_or. left. apply H_P.
    + intro H_Q. apply H_not_or. right. apply H_Q.
  - intros H_and_not H_or.
    destruct H_and_not as [H_not_P H_not_Q].
    destruct H_or as [H_P | H_Q].
    + apply H_not_P. assumption.
    + apply H_not_Q. assumption.
Qed.
```

The right-hand pane shows the current subgoal:

```
| subgoal
P, Q : Prop
H_not_or : ~ (P ∨ Q)
H_Q : Q
----- (1/1)
P ∨ Q
```

The bottom status bar indicates: "Ready, proving DeMorganPropositional_Coq", "Line: 9 Char: 33", "Coq is ready", and "0 / 0".

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Coq: proof example over nat

The screenshot shows the CoqIDE interface with a file named `*scratch*`. The editor contains the following Coq code:

```
Require Import Coq.omega.Omega.
Require Coq.Logic.Classical.

Fixpoint range_sum (n: nat) : nat :=
  match n with
  | 0 => 0
  | S p => range_sum p + (S p)
  end.

Lemma range_sum_lemma: forall n: nat,
  range_sum (n + 1) = range_sum n + (n + 1).
Proof.
  intros. induction n.
  - simpl; reflexivity.
  - simpl; omega.
Qed.

Theorem SimpleArithProgressionSumFormula_Coq:
  forall n, 2 * range_sum n = n * (n + 1).
Proof.
  intros.
  induction n.
  (* goal: '2 * range_sum 0 = 0 * (0 + 1)' *)
  - simpl; reflexivity.
  (* goal: '2 * range_sum (S n) = S n * (S n + 1)' *)
  - rewrite -> Nat.mul_add_distr_l.
    rewrite -> Nat.mul_1_r.
    rewrite -> (Nat.mul_succ_l n).
    rewrite <- (Nat.add_1_r n).
    rewrite -> range_sum_lemma.
    omega.
Qed.
```

On the right side of the interface, there is a subgoal window showing the current goal and its proof steps:

```
1 subgoal
n : nat
IHn : 2 * range_sum n = n * (n + 1)
----- (1/1)
2 * range_sum (S n) = S n * S n + S n * 1
```

At the bottom of the interface, there are tabs for Messages, Errors, and Jobs. The status bar at the very bottom indicates: "Ready, proving SimpleArithProgressionSumFormula_Coq", "Line: 26 Char: 38", "Coq is ready", and "0 / 0".

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Coq: proof example over nat + verified code

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The screenshot shows the CoqIDE interface with a file named `*scratch*`. The main editor contains Coq code for defining a function `range_sum` and proving a theorem `SimpleAr`. A yellow box highlights the extracted code for `range_sum` in Haskell and OCaml.

Figure 3. Extracted function in Haskell

```
range_sum :: Nat -> Nat
range_sum n =
  case n of {
    0 -> 0;
    S p -> add (range_sum p) (S p)}
```

Figure 4. Extracted function in OCaml

```
(** val range_sum : nat -> nat **)
```

```
let rec range_sum = function
  | 0 -> 0
  | S p -> add (range_sum p) (S p)
```

Summary

- Two widespread theorem provers were considered: Isabelle and Coq
- The **key difference** between them lie in differences between logical theories they based on
- Nonetheless, they both may be used to solve applied problems, such as software testing and verification

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