

# Comparison of two theorem provers: Isabelle & Coq

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Comparison of two  
theorem provers:  
Isabelle & Coq

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S. Tripakis

Foundations of  
Formal Approach

The Formal System

Properties of a Formal  
System

Classical and Intuitionistic  
Logics

Two Theorem  
Provers

Isabelle

Coq

Comparison of the  
Theorem Provers

Common Features

Major Differences

Summary

# Outline

## Foundations of Formal Approach

- The Formal System

- Properties of a Formal System

- Classical and Intuitionistic Logics

## Two Theorem Provers

- Isabelle

- Coq

## Comparison of the Theorem Provers

- Common Features

- Major Differences

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# Definition of the Formal System

- ▶ // TODO
- ▶ My second point.

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# Properties of a Formal System

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A formal system  $\Gamma = \langle A, V, \Omega, R \rangle$  is called:

- ▶ *consistent*, if  $\nexists \phi \in \Gamma : \Gamma \vdash \phi \wedge \Gamma \vdash \neg \phi \Leftrightarrow \Gamma \not\vdash \perp$ ;
- ▶ *complete*, if  $\forall \phi \in U : A \vdash \phi \vee A \vdash \neg \phi$ ;
- ▶ *independent*, if  $\nexists a \in A : A \vdash a$ .

# Classical and Intuitionistic Logics

// Classical: set of axioms

// Intuitionistic: same, but without EM

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# Isabelle: First Acquaintance

- ▶ a generic proof assistant
- ▶ a successor of HOL theorem prover //TODO: cite
- ▶ created in 1986 by
  - ▶ Larry Paulson @ University of Cambridge, and
  - ▶ Tobias Nipkow @ Technische Universität München
- ▶ based on classical higher-order logic
- ▶ uses powerful functional language HOL
- ▶ has large collection of formalised theories //TODO: HOL, ZF, CCL, ...

## Example 1: Definition of basic datatypes

```
datatype bool =  
  True | False
```

```
datatype nat =  
  zero ("0") | Suc nat
```

## Example 2: ???

???

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# Coq: First Acquaintance

- ▶ a formal proof management system
- ▶ created at INRIA (Paris, France) in 1984
- ▶ based on Calculus of Inductive Constructions theory (an implementation of intuitionistic logic)
- ▶ uses powerful functional language Gallina
- ▶ has large collection of formalised theories //TODO
- ▶ widely used in software verification (proof code extraction)

## Example 3: Definition of basic datatypes

```
Inductive False : Prop := .
```

```
Inductive True : Prop := I : True.
```

```
Inductive nat : Type :=  
  | O : nat  
  | S : nat -> nat.
```

## Example 4: ???

???

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# Blocks

## Block Title

You can also highlight sections of your presentation in a block, with it's own title

## Theorem

*There are separate environments for theorems, examples, definitions and proofs.*

## Example

Here is an example of an example block.

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# Summary

- ▶ The **first main message** of your talk in one or two lines.
- ▶ The **second main message** of your talk in one or two lines.
- ▶ Perhaps a **third message**, but not more than that.
- ▶ Outlook
  - ▶ Something you haven't solved.
  - ▶ Something else you haven't solved.

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# For Further Reading I

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Appendix

For Further Reading



A. Author.

*Handbook of Everything.*

Some Press, 1990.



S. Someone.

On this and that.

*Journal of This and That*, 2(1):50–100, 2000.