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Dynamic modeling of the airship with Matlab using geometrical aerodynamic parameters

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ABSTRACT

Airships provide an attractive solution to many challenges in the aviation industry. The buoyancy force offered by the airship enables to be used for long endurance aero applications. Key objective is to operate the vehicle with a sufficient level of autonomy under extreme range of atmospheric and wind conditions. In order to achieve these goals, a guidance and control system is needed which controls the airship in air at the specified height. The accurate control system first requires an accurate model of the airship dynamics. Based on six degrees of freedom, an airship dynamic model is presented in this paper. The aerodynamic part of the model is taken from the geometrical configuration of the vehicle instead of expensive wind tunnel test setup. The computer simulation algorithm is then derived and the easily available Matlab environment is used to simulate it and its initial validation for YEZ-2A airship is demonstrated.

1. Introduction

Vehicles with lighter-than-air (LTA) concept represent a unique and promising platform for many applications that involve a long-duration airborne presence. Having their lift by the buoyancy force, these vehicles require much less power than traditional aircraft. Major applications include roving or hovering, surveillance and communication utilities for both military and commercial use, plus a variety of remote-sensing instruments for scientific applications. In order to achieve these goals, a robust guidance and control system capable of auto-piloting and controlling the airship flight is required. Successful design of such a system necessitates an accurate model of airship dynamics. As a basis, a six degrees-of-freedom dynamic model of a non-rigid airship is required.

Many efforts have been directed to the development of semi-autonomous vehicles with navigation and control systems capable of executing trajectories on the basis of high level human-planned missions [4,6]. Despite significant developments in this field, a lot need to be explored so that lighter-than-air (LTA) vehicles could be used as unmanned robotic platforms [18].

For the development of meaningful control and navigation strategies, an accurate dynamic model of the airship is required. A small perturbation modeling technique is described in [9,3]. System identification approach for determining flight dynamic characteristics of an airship from flight data is discussed in [10]. Dynamics of flexible and non-flexible airship is illustrated in [12]. Hull shape optimization is described in [13]. A detailed work for investigation of flight dynamics using wind tunnel database is given in [5].

In this paper, we suggest a comprehensive six degrees-of-freedom dynamic model of a non-rigid airship modified for its aerodynamic part. This model is based on that of a remotely operated underwater vehicle (ROV), upgraded to model airship dynamics [11]. The model has all inertial, dynamic, aerodynamic, buoyant, gravitational, and propulsion forces. Most of these terms are relatively easy to obtain, the aerodynamic term is usually very hard to evaluate. In this model, parameters for aerodynamic and control behavior are taken from the 600 hours wind tunnel experimental data on the scaled model of the YEZ-2A airship. Direct measurements of aerodynamic coefficients require wind tunnel test setup. A scaled model is required to make necessary analysis. In order to address these issues, six degrees-of-freedom aerodynamic forces and moments model based upon analytical formulation is used instead of wind tunnel database only. A remarkable work relating airship aerodynamics was reported in literature [19–21,16,7,15,1]. Airship aerodynamics has been explained in terms of geometrical formulation in recent work of Jones and Mueller [8,14].

The paper is divided into five sections. Next section includes a six degrees-of-freedom based complete dynamic model of the non-rigid airship. In Section 3 we discuss the simulation strategy with some results by applying different control inputs and validating it with YEZ-2A wind tunnel results. And finally some conclusions

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Nomenclature

A 6×1 aerodynamic vector

Acc Acceleration matrix containing state variable derivatives with respect to time

A_X, A_Y, A_Z Aerodynamic force components in Newton along $0X$, $0Y$, $0Z$ direction in airship body axes system

A_L, A_M, A_N Aerodynamic moment components in Newton about $0X$, $0Y$, $0Z$ direction in airship body axes system

C_x, C_y, C_z Non-dimensional aerodynamic force coefficients along $0X$, $0Y$, $0Z$ direction in airship body axes system.

C_l, C_m, C_n Non-dimensional moment coefficients about $0X$, $0Y$, $0Z$ direction in airship body axes system

a_x, a_y, a_z Coordinates of center of gravity (C.G) with respect to $0X$, $0Y$ and $0Z$ direction in airship body axes system .... m

B Buoyancy force in Newton acting at the center of volume point (C.V) in airship body axes system

$C_{Dh_0}, C_{Df_0}, C_{Dg_0}$ Hull, fin and gondola zero-incidence drag coefficients, respectively

$C_{Dch}, C_{Dcf}, C_{Dcg}$ Hull, fin and gondola cross-flow drag coefficients, respectively

C_{tf} Leading edge fin suction coefficient

$(\partial C_L/\partial\alpha)_f$ Derivative of fin-lift coefficient with respect to angle-of-attack at zero incidence

$(\partial C_L/\partial\delta)_f$ Derivative of fin-lift coefficient with respect to the flap deflection angle

DCM Direction Cosine Matrix

d Airship envelope max diameter .... m

d_x Horizontal distance between $0Z$ axis of airship hull and the line connecting the two diesel engine thrust lines .... m

d_y Horizontal distance between $0XZ$ plane of airship hull and the diesel engine thrust line .... m

d_z Vertical distance between $0XZ$ plane of airship hull and the line connecting the two diesel engine thrust line .... m

$\mathbf{F_d}$ 6×1 dynamic vector

G 6×1 gravitational vector

g Gravitational acceleration .... m/s^2

H Airship heaviness .... kg

I_x, I_y, I_z Moments of inertia of hull about $0X$, $0Y$ and $0Z$ axes of airship body, respectively .... kg m^2

I_{xz} Product of inertia of hull about $0Y$ axis of airship body .... kg m^2

k_1, k_2, k' Lamb's inertia ratio for movements along $0X$, $0Y$ and rotation about $0Y$, respectively in body axis system

L Airship envelope length .... m

l_{f1} X-distance from hull nose to aerodynamic center of fin .... m

l_{f2} X-distance from hull nose to geometric center of fin .... m

l_{f3} Y, Z-distance from origin to aerodynamic center of fin .... m

l_h Length of the hull up to the leading edge of the fin m

l_{gx}, l_{gz} X and Z distances from origin to center of gondola, respectively .... m

$L_{\dot p}, M_{\dot q}, N_{\dot r}, N_{\dot p}, L_{\dot r}, L_{\dot v}, M_{\dot u}, M_{\dot w}, N_{\dot v}$ Virtual inertia terms (derivatives of moment components with respect to linear and axial acceleration perturbation)

M 6×6 mass matrix

m Airship total mass .... kg

P 6×1 propulsion vector

Position 3×1 position vector (distance traveled by vehicle with respect to Earth axes system) .... m

p, q, r Airship angular velocities in roll, pitch and yaw with respect to body axes system, respectively .... rad/s

S Surface area of the airship hull .... m^2

S_h, S_f, S_g Airship hull reference area ($V^{2/3}$), fin and gondola reference area, respectively .... m^2

T_{d_s}, T_{d_p} Thrust of the starboard and port side diesel engine N

T Simulation time variable .... s

U_0 Airship flying with air speed velocity .... m/s

u, v, w Airship linear velocities along $0X$, $0Y$ and $0Z$ axes of airship body, respectively .... m/s

V Airship envelope volume .... m^3

Vel Velocity vector containing the value of three linear and three angular velocities in the body axes

$V_{North}, V_{East}, V_{Up}$ Velocity components with respect to Earth axes system .... m/s

W Airship weight, acting at the center of gravity (C.G) N

$X_{\dot u}, Y_{\dot v}, Z_{\dot w}, X_{\dot q}, Y_{\dot p}, Y_{\dot r}, Z_{\dot q}$ Virtual mass terms (derivatives of force component with respect to linear and axial acceleration perturbation)

x_{cv} Distance from the nose to center of volume (C.V) of the airship .... m

α, β Longitudinal and lateral incidences of airship .... rad

ρ Air density .... kg/m^3

μ Angle of the rotation through which the diesel engine thrust vector direction can be varied synchronously in a plane parallel to $0XZ$ .... rad

λ_{ij} Elements of the DCM matrix

ϕ, θ, φ Vehicle's Euler angles in Earth inertial system .... rad

η_f, η_k Fin and hull efficiency factor accounting for effects of hull on the fin and fin on hull, respectively

$\delta_{RUDT}, \delta_{RUDB}$ Deflections of top and bottom trailing edge flap of the rudder .... rad

$\delta_{ELEVL}, \delta_{ELEVR}$ Deflections of left and right trailing edge flap of the elevator .... rad and future recommendations are described in Sections 4 and 5, respectively.

2. Dynamic modeling of the airship

The concept of mathematical model about this flight simulation work is derived from a mathematical model used for Remotely Operated Underwater Vehicle (ROV). Its small perturbation model is used as a basis for study of the airship flight dynamics. This concept was adopted by S.B.V. Gomes to draw a full six-degrees-of-freedom (6-DOF) non-linear mathematical model of the airship flight for computer simulation. The model describes the dynamic, gravitation, propulsion, aerodynamic and control behavior of the vehicle. The model can explain the stability and control performance as well as flight modes of the airship.

Traditional methods for the aerodynamic model make use of extensive wind tunnel data. An alternative approach is to use an analytical model and to determine the constants in this model from limited wind tunnel data sets. This has a more general modeling approach that could be easily adapted to the other geometries.

2.1. Assumptions and axes systems

In this simulation, two orthogonal axes sets are assumed. First axes set is known as body axes which is pointing towards the three orthogonal $0X$, $0Y$ and $0Z$ directions and is centered at airship center of volume (C.V). Main concept behind the mathematical model of the simulation is the calculation of the three linear velocities, u, v and w, and the three angular velocities, p, q and r, with reference to $0X$, $0Y$ and $0Z$, respectively. The second set of orthogonal axes used in the simulation is inertial or Earth axes. Here, at the start of simulation, $0X$ axis is aligned with the North direction, $0Y$ with the East and $0Z$ axis points downwards to the center of earth. Earth axes set stays stationary throughout the simulation run, whereas the body axes set moves along with the vehicle. The effects of the earth curvature are not taken into account in this simulation (flat earth is assumed).

Aerodynamic control surfaces (rudders and elevators) are attached to the empennage surfaces (Fig. 1), which controls left/right and up/down movements.

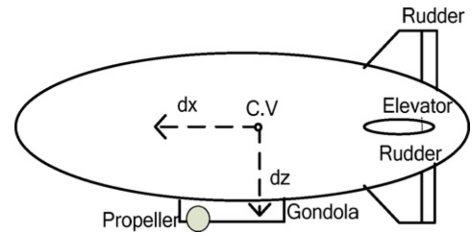

Fig. 1. Vertical and horizontal tail fins (Rudder and Elevator), propeller and gondola of airship.

Fig. 2 shows the set of body axes system employed. C.V is assumed to lie on the geometrical longitudinal axis of the envelope (its center line), and X-axis is aligned with it. XZ plane defines longitudinal plane of symmetry, because the body is assumed to be symmetric, therefore the center of gravity (C.G) lies on this plane. Important consequences from this assumption are:

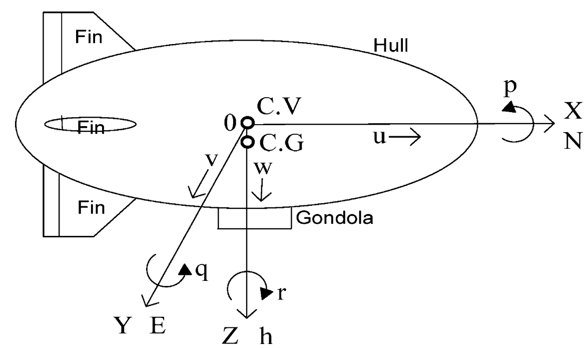

Fig. 2. Body axes system of airship showing C.V, C.G and linear (u, v, w) as well as angular (p, q, r) velocities around the $0X$, $0Y$, and $0Z$ axes respectively.

Firstly, if C.G coordinates (with respect to the stated axes system) are a_x, a_y and a_z then it means $a_y = 0$. Secondly, inertia terms I_{xy} and I_{yz} become zero. It is assumed that the airship is rigid, i.e. aero elastic effects are ignored.

For simplicity, it is assumed that the maximum flight altitude is limited to small height where temperature and pressure conditions remain almost constant and so the C.G location is the same.

2.2. Equations of motions

LTA vehicle dynamics can be modeled by a mass with forces and moments applied to it. After some manipulations, the final format of the equation is stated in [19] as:

$$\mathbf{M}\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\\ \dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \mathbf{F_d}(u, v, w, p, q, r) + \mathbf{A}(u, v, w, p, q, r) + \mathbf{G}(\lambda_{31}, \lambda_{32}, \lambda_{33}) + \mathbf{P} \tag{1}$$

where, u, v, w are linear velocities along $0X$, $0Y$, $0Z$ axes of airship and p, q, r are angular velocities about these axes defined in airship body axes system. $\mathbf{M}$ is a 6×6 matrix that contains mass and inertia terms associated with the airship. $\mathbf{F_d}$ is the 6×1 column matrix that contains the dynamic terms associated with inertial velocities u, v, w, p, q and r. $\mathbf{A}$ is a 6×1 column matrix that contains the aerodynamic forces and moments acting on the vehicle. $\mathbf{G}$ is 6×1 column matrix that contains the terms associated with gravitational and buoyancy forces and moments of airship. $\mathbf{P}$ is a 6×1 column matrix containing the terms associated with the propulsive forces and moments of airship.

2.2.1. The mass matrix **M**

In contrast to aircraft simulation, additional terms specific to buoyant vehicles like airships need to be incorporated in the mass matrix. These are called 'added' or 'virtual' masses and inertias which arise due to the vehicle mass being of the same order of magnitude as the mass of the displaced air. Conventional formulation of using aerodynamic stability derivative notation to denote these terms is employed here.

Incorporating simplifications due to vehicle's symmetry, the mass matrix is written as:

$$\mathbf{M} = \begin{pmatrix} m_x & 0 & 0 & 0 & ma_z - X_{\dot{q}} & 0 \\ 0 & m_y & 0 & -ma_z - Y_{\dot{p}} & 0 & ma_x - Y_{\dot{r}} \\ 0 & 0 & m_z & 0 & -ma_x - Z_{\dot{q}} & 0 \\ 0 & -ma_z - L_{\dot{v}} & 0 & J_x & 0 & -J_{xz} \\ ma_z - M_{\dot{u}} & 0 & -ma_x - M_{\dot{w}} & 0 & J_y & 0 \\ 0 & ma_x - N_{\dot{v}} & 0 & -J_{xz} & 0 & J_z \end{pmatrix} \tag{2}$$

where,

$$m_x = m - X_{\dot{u}}, \qquad m_y = m - Y_{\dot{v}}, \qquad m_z = m - Z_{\dot{w}}$$

$$J_x = I_x - L_{\dot{p}}, \qquad J_y = I_y - M_{\dot{q}}, \qquad J_z = I_z - N_{\dot{r}}$$

$$J_{xz} = I_{xz} + N_{\dot{p}} = I_{xz} + L_{\dot{r}}$$

In practice, some virtual masses and inertia terms have to be disregarded, whereas other terms can be estimated using the following standard expressions:

$$X_{\dot{u}} = -k_1\bar{m} \tag{3}$$

$$Y_{\dot{v}} = -k_2\bar{m} \tag{4}$$

$$M_{\dot{q}} = -k'\bar{I}_y \tag{5}$$

$$Z_{\dot{w}} = Y_{\dot{v}} \tag{6}$$

$$N_{\dot{r}} = M_{\dot{q}} \tag{7}$$

where, $\bar{m} = B/g$ and $\bar{I}_y = \bar{m}[l^2 + d^2]/20$.

k_1, k_2 are Lamb's inertia ratios for movement along longitudinal $(0X)$ and lateral $(0Y)$ axes and k' is Lamb's inertia ratio for rotation about lateral $(0Y)$ axis. These factors can be estimated by the given procedure [20].

The remaining acceleration derivative terms of the mass matrix are neglected.

2.2.2. The dynamics vector $\mathbf{F_d}$

It is the 6×1 column matrix that contains the dynamic terms associated with inertial linear and angular velocities u, v, w, p, q and r and can be expressed as:

$$\mathbf{F_d} = [\, f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5 \quad f_6 \,]^{\mathrm{T}} \tag{8}$$

where the simplified dynamic terms can be expressed as:

$$f_1 = -m_z wq + m_y rv + m\{a_x[q^2 + r^2] - a_z rp\} \tag{9}$$

$$f_2 = -m_x ur + m_z pw + m[-a_x pq - a_z rq] \tag{10}$$

$$f_3 = -m_y vp + m_x qu + m\{-a_x rp + a_z[q^2 + p^2]\} \tag{11}$$

$$f_4 = -[J_z - J_y]rq + J_{xz}pq + ma_z[ur - pw] \tag{12}$$

$$f_5 = -[J_x - J_z]pr + J_{xz}[r^2 - p^2] + m\{a_x[vp - qu] - a_z[wq - rv]\} \tag{13}$$

$$f_6 = -[J_y - J_x]qp - J_{xz}qr + m\{-a_x[ur - pw]\} \tag{14}$$

2.2.3. The aerodynamic vector $\mathbf{A}$

The 6×1 column matrix that contains the aerodynamic forces and moments has computations based on the geometrical configuration of the vehicle. In practice, the aerodynamic vector is expressed as:

$$\mathbf{A} = [\, A_X \quad A_Y \quad A_Z \quad A_L \quad A_M \quad A_N \,]^{\mathrm{T}} \tag{15}$$

where,
A_X: Aerodynamic total force on $0X$ axis (body axis)
A_Y: Aerodynamic total force on $0Y$ axis (body axis)
A_Z: Aerodynamic total force on $0Z$ axis (body axis)
A_L: Aerodynamic total moment about $0X$ axis (body axis)
A_M: Aerodynamic total moment about $0Y$ axis (body axis)
A_N: Aerodynamic total moment about $0Z$ axis (body axis)

The equations for the aerodynamic forces and moments are derived using the procedure outlined in [8] and explained in [14] as given:

$$A_X = 0.5\rho U_0^2 S\big[C_{X1}\cos^2\alpha\cos^2\beta + C_{X2}\big(\sin(2\alpha)\sin(\alpha/2) + \sin(2\beta)\sin(\beta/2) + C_{X3}\big)\big] \tag{16}$$

$$A_Y = 0.5\rho U_0^2 S\big[C_{Y1}\cos(\beta/2)\sin(2\beta) + C_{Y2}\sin(2\beta) + C_{Y3}\sin(\beta)\sin\big(|\beta|\big) + C_{Y4}(\delta_{RUDT} + \delta_{RUDB})\big] \tag{17}$$

$$A_Z = 0.5\rho U_0^2 S\big[C_{Z1}\cos(\alpha/2)\sin(2\alpha) + C_{Z2}\sin(2\alpha) + C_{Z3}\sin(\alpha)\sin\big(|\alpha|\big) + C_{Z4}(\delta_{ELEVL} + \delta_{ELEVR})\big] \tag{18}$$

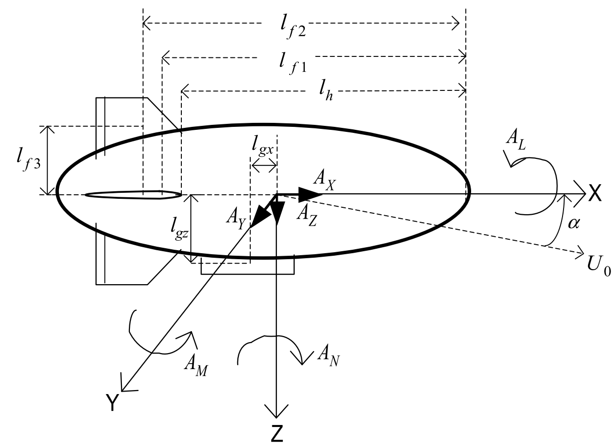

Fig. 3. Steady-state aerodynamic model.

$$A_L = 0.5\rho U_0^2 lS\big[C_{L1}(\delta_{ELEVL} - \delta_{ELEVR} + \delta_{RUDB} - \delta_{RUDT}) + C_{L2}\big(\sin(\beta)\sin\big(|\beta|\big)\big) + 0.5\rho S\big(C_{L3}r|r| + C_{L4}p|p|\big)\big] \tag{19}$$

$$A_M = 0.5\rho U_0^2 lS\big[C_{M1}\cos(\alpha/2)\sin(2\alpha) + C_{M2}\sin(2\alpha) + C_{M3}\sin(\alpha)\sin\big(|\alpha|\big) + C_{M4}(\delta_{ELEVL} + \delta_{ELEVR}) + 0.5\rho SC_{M5}q|q|\big] \tag{20}$$

$$A_N = 0.5\rho U_0^2 lS\big[C_{N1}\cos(\beta/2)\sin(2\beta) + C_{N2}\sin(2\beta) + C_{N3}\sin(\beta)\sin\big(|\beta|\big) + C_{N4}(\delta_{RUDT} + \delta_{RUDB}) + 0.5\rho SC_{M5}r|r|\big] \tag{21}$$

where,

$$\alpha = \arctan(w/u) \tag{22}$$

$$\beta = \text{arcsine}(v/V_{Total}) \tag{23}$$

and

$$V_{Total} = \sqrt{u^2 \quad v^2 \quad w^2} \tag{24}$$

Surface area of the hull with axes a and b, can be defined for prolate ellipsoid, if $a > b$ as:

$$S = \pi b\big(b + a^2\sin(e)/e\big) \tag{25}$$

where,

$$e = \sqrt{\big(a^2 - b^2\big)/a} \tag{26}$$

Surface area of the hull for oblate ellipsoid, if $a < b$ as:

$$S = \pi b\big(b + a^2\sinh(f)/f\big) \tag{27}$$

where,

$$e = \sqrt{\big(b^2 - a^2\big)/b} \tag{28}$$

$$f = be/a \tag{29}$$

The geometric configuration of complete airship with fins and gondola is shown in Fig. 3.

The coefficients used in Eqs. (16)–(21) are direct functions of airship geometry. These are defined in Appendix A.

It is to note that we can get the non-dimensional static coefficients of forces and moments $(C_x, C_y, C_z, C_l, C_m, C_n)$ from Eqs. (16)–(21) by dividing with $0.5\rho U_0^2 S$.

2.2.4. The gravity and buoyancy vector **G**

It is 6×1 column matrix that contains the terms associated with gravitational and buoyancy forces and moments and is given by:

$$\mathbf{G} = \begin{bmatrix} \lambda_{31}[W - B] \\ \lambda_{32}[W - B] \\ \lambda_{33}[W - B] \\ -\lambda_{32} a_z W \\ [\lambda_{31} a_z - \lambda_{33} a_x] W \\ \lambda_{32} a_x W \end{bmatrix} \tag{30}$$

λ_{ij} are the elements of the Direction Cosine Matrix (DCM).

The value of total airship weight and buoyancy can be determined as:

$$W = B + Hg \tag{31}$$

$$B = V \rho g \tag{32}$$

where, H is airship heaviness value set by the user in kg (positive for heaviness force greater than airship weight and negative for the opposite).

2.2.5. The propulsion vector **P**

The 6×1 column matrix containing the terms associated with the propulsive forces and moments is given by:

$$\mathbf{P} = [X_{prop}, Y_{prop}, Z_{prop}, L_{prop}, M_{prop}, N_{prop}]^{\mathrm{T}} \tag{33}$$

X_{prop}: Total thrust along the $0X$ axis (body axis)
Y_{prop}: Total thrust along the $0Y$ axis (body axis)
Z_{prop}: Total thrust along the $0Z$ axis (body axis)
L_{prop}: Total thrust moment about the $0X$ axis (body axis)
M_{prop}: Total thrust moment about the $0Y$ axis (body axis)
N_{prop}: Total thrust moment about the $0Z$ axis (body axis)

The derivation of the mathematical expressions making each element of **P** is:

$$X_{prop} = (T_{d_s} + T_{d_p}) \cos \mu \tag{34}$$

$$Y_{prop} = 0 \tag{35}$$

$$Z_{prop} = -(T_{d_s} + T_{d_p}) \sin \mu \tag{36}$$

$$L_{prop} = (T_{d_p} - T_{d_s}) \sin \mu . d_y \tag{37}$$

$$M_{prop} = (T_{d_s} + T_{d_p})(d_z \cos \mu - d_x \sin \mu) \tag{38}$$

$$N_{prop} = (T_{d_p} - T_{d_s}) \cos \mu . d_y \tag{39}$$

The following expression is used to estimate thrust generated by each engine.

$$T_{d_s} = T_{d_p} = \frac{P_a \eta_p}{V_{Total}} \tag{40}$$

where P_a is available power and η_p is propeller efficiency.

2.2.6. The quaternion method

This method is used to find Direction Cosine Matrix (DCM). Quaternion equation relates the four parameters, e_0, e_1, e_2 and e_3 to the airship rates of roll, pitch and yaw, i.e., p, q and r respectively, in the following:

$$\begin{bmatrix} \dot{e}_0 \\ \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = 0.5 \begin{bmatrix} -e_1 & -e_2 & -e_3 \\ e_0 & -e_3 & e_2 \\ e_3 & e_0 & -e_1 \\ -e_2 & e_1 & e_0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{41}$$

The above equation is integrated to obtain the values of the four e-parameters. Once the new e-parameter values are obtained, the DCM could be determined from the relation:

$$\mathbf{DCM} = \begin{bmatrix} e_0^2 + e_1^2 - e_2^2 - e_3^2 & 2[e_1 e_2 - e_0 e_3] & 2[e_0 e_2 - e_1 e_3] \\ 2[e_0 e_3 + e_1 e_2] & e_0^2 - e_1^2 + e_2^2 - e_3^2 & 2[e_2 e_3 - e_0 e_1] \\ 2[e_1 e_3 - e_0 e_2] & 2[e_0 e_1 + e_2 e_3] & e_0^2 - e_1^2 - e_2^2 + e_3^2 \end{bmatrix} \tag{42}$$

$$\mathbf{DCM} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix} = [\lambda_{ij}] \tag{43}$$

2.2.7. End solution

Solution to this simulation work is found by inverting the mass matrix **M** and multiplying both sides of Eq. (1) by $\mathbf{M}^{-1}$.

Eq. (1) will then have the form:

$$[\mathbf{Acc}]^{\mathrm{T}} = [\, \dot{u} \quad \dot{v} \quad \dot{w} \quad \dot{p} \quad \dot{q} \quad \dot{r} \,] \tag{44}$$

Once the acceleration matrix **Acc** is found, it is integrated over a small time interval Δt, the values of three linear and three angular velocities in the body axes are automatically found in the form of velocity matrix as:

$$[\mathbf{Vel}]^{\mathrm{T}} = [\, u \quad v \quad w \quad p \quad q \quad r \,] \tag{45}$$

The three linear velocities are transformed into Earth (inertial) axes by using the following relation as:

$$\begin{bmatrix} V_{North} \\ V_{East} \\ V_{Up} \end{bmatrix} = \begin{bmatrix} & \mathbf{DCM} & \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{46}$$

The Direction Cosine Matrix (DCM) is to be calculated at each time interval Δt so that necessary transformation can take place during the simulation.

Eq. (46) is integrated over the small time interval Δt, the displacements in the North, East and vertical directions are found. i.e., we get **Position** vector as:

$$[\mathbf{Position}]^{\mathrm{T}} = [\, North \quad East \quad H \,] \tag{47}$$

3. Simulation and results

3.1. Program flow diagram and airship parameters

A complete program flow chart is described in Fig. 4. Algorithm in this simulation follows the procedures given in [17,2]. Initial values of the $u, v, w, p, q, r, \phi, \theta, \varphi$, *North*, *East* and H are given at the start of simulation. Body axes system is made to coincide with Earth axes system at $t = 0$. From the given values of ϕ, θ and φ, values of e-parameters are calculated. The control surface deflection input (step type in our case) is given. Vehicle geometry is also given before start of simulation. Before the start of simulation, number of required iterations is given. After the start of simulation, mass matrix and its inverse are calculated first, afterwards dynamic, propulsion, aerodynamic, gravitational vectors are calculated and from these values, acceleration matrix is found. The desired vehicle velocities and their positions at each time step are calculated after performing integration operation on acceleration matrix. It is important to mention that in this work, actual geometric configuration is used to simulate the results instead of scaled configurations used in the wind tunnel test setup.

3.1.1. Example airship

In this airship, ellipsoid geometry with ratio of 1.3 is used for rear to front ellipsoid length. The length and maximum diameter of hull is 129.5 m and 32 m, respectively. Four control surfaces (2-Rudders and 2-Elevators) in "+" configuration are employed. Two

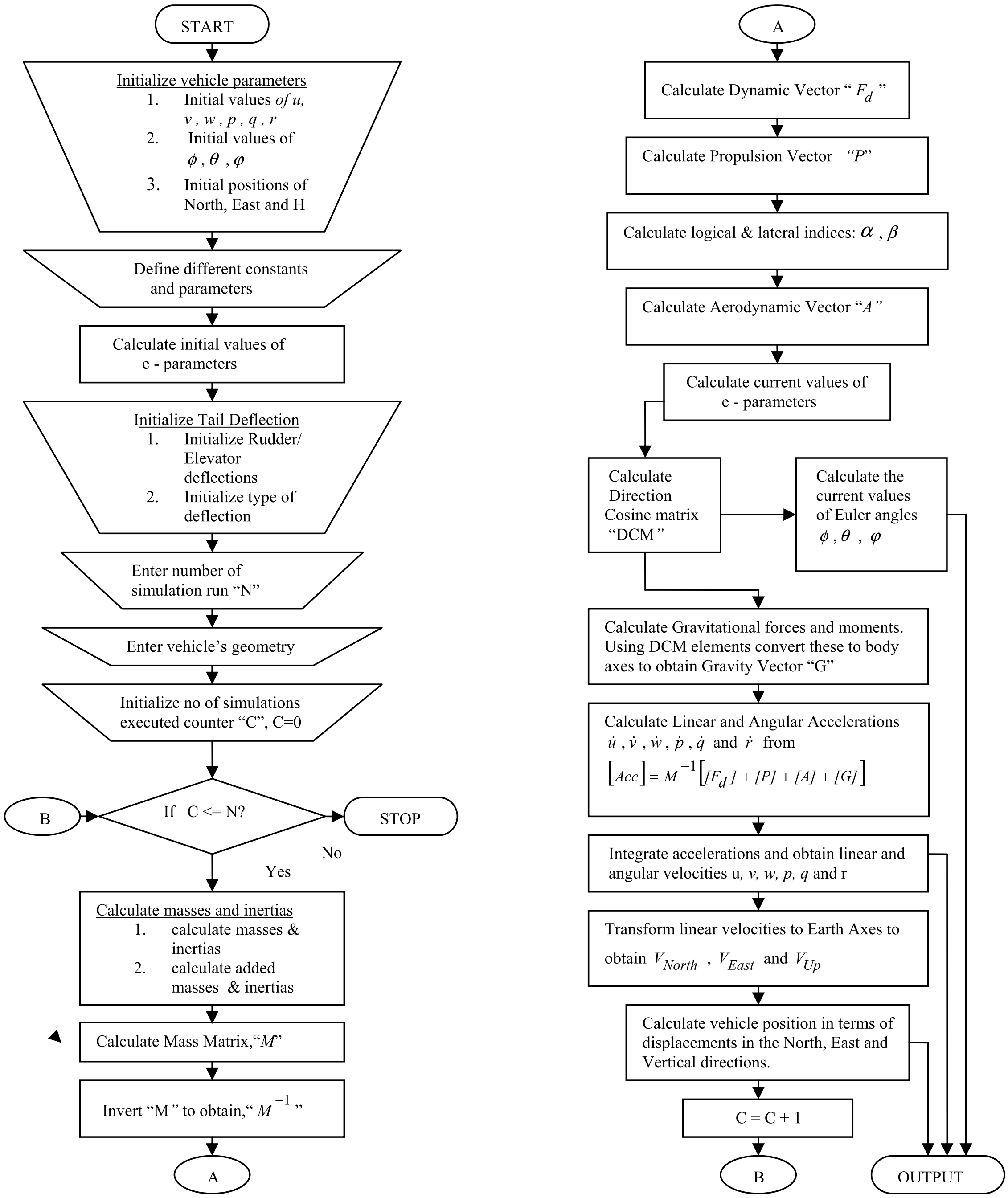

Fig. 4. Flow chart for Matlab simulation of airship flight dynamics.

propellers (one in starboard and other in port side) are installed. Integration step size equal to 0.1 s is used and the simulation was carried out for 100 s.

3.2. Simulation results

Simulation results for non-dimensional coefficients of force and moments for YEZ-2A airship and Example airship are presented in Figs. 5–7. These coefficients are taken at a lateral incidence "beta" equal to −5 deg. Results shows that there is a close approximation between analytical and wind tunnel methods.

At sea level, neutral buoyancy condition with initial flying speed of 25 m/s, when we apply input of 5 deg rudders to starboard side, high variations in velocity along Y, roll and yaw rates take place due to application of control input. It is to note that when we ini-

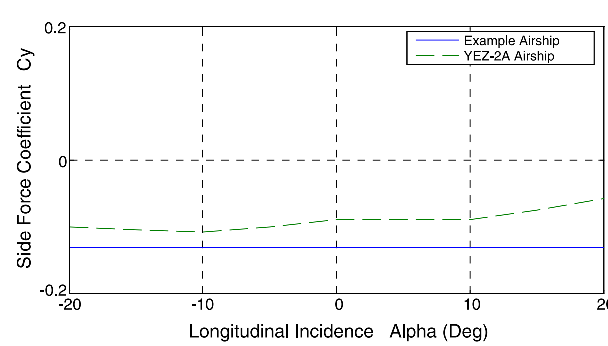

Fig. 5. Non-dimensional static side force coefficient taken at −5 deg lateral incidence for Example and YEZ-2A airships.

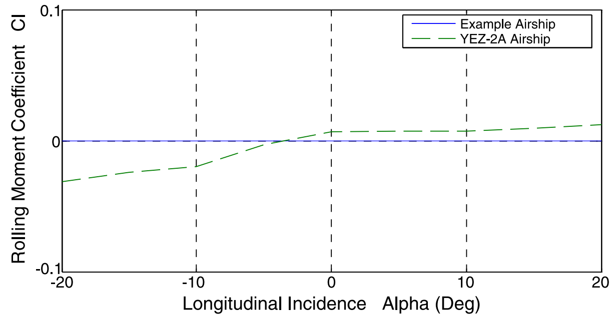

Fig. 6. Non-dimensional static rolling moment coefficient taken at −5 deg lateral incidence for Example and YEZ-2A airships.

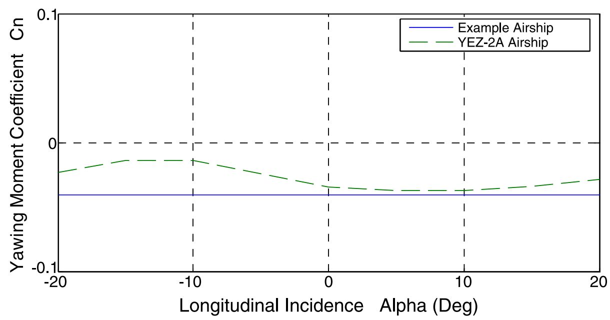

Fig. 7. Non-dimensional static yawing moment coefficient taken at −5 deg lateral incidence for Example and YEZ-2A airships.

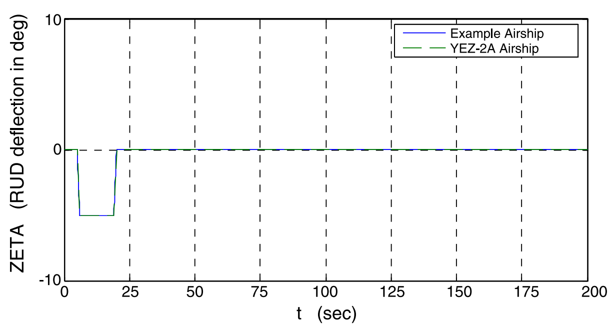

Fig. 8. Rudder deflection (ZETA) as a function of time of Example and YEZ-2A airships.

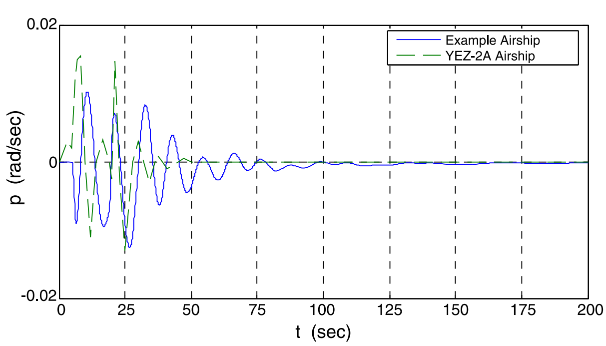

Fig. 9. Roll rate about x-axis (p) as a function of time of Example and YEZ-2A airships.

tiate the input, it excites the roll oscillations in the vehicle. When we remove the input, Example airship tries to attain the same attitude more accurately as before application of the input which can be viewed in Figs. 8–13. Simulation results show that there is a good agreement between the Example airship and YEZ-2A airship with the only exception of magnitude values. This deviation is due

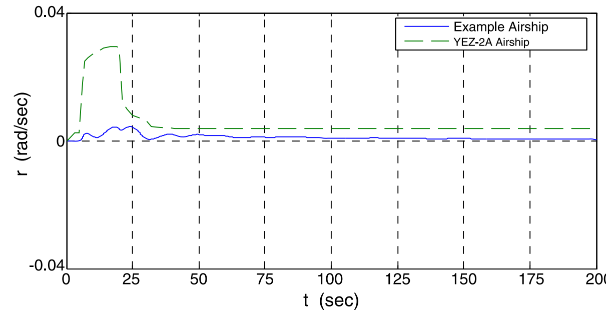

Fig. 10. Yaw rate about z-axis (r) as a function of time of Example and YEZ-2A airships.

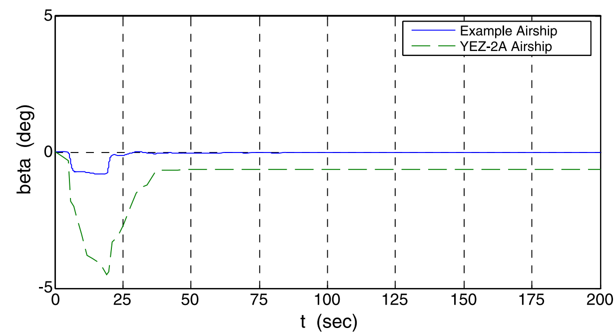

Fig. 11. Lateral incidence (beta "β") as a function of time of Example and YEZ-2A airships.

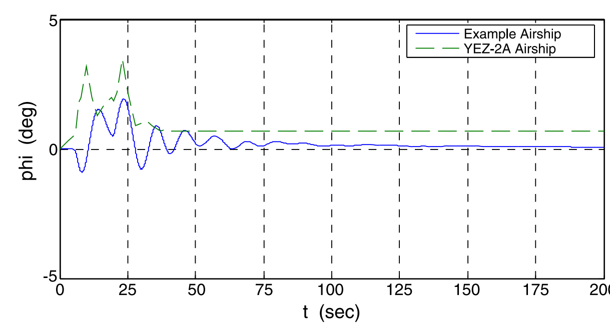

Fig. 12. Roll angle (phi "ϕ") as a function of time of Example and YEZ-2A airships.

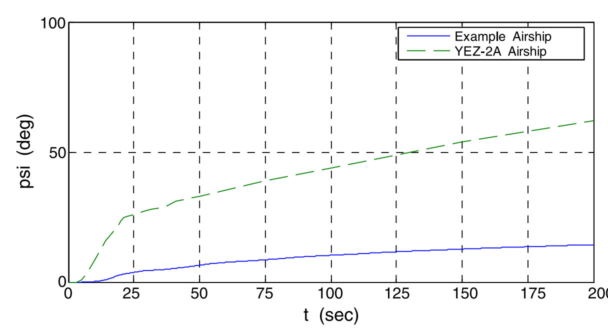

Fig. 13. Yaw angle (psi "φ") as a function of time of Example and YEZ-2A airships.

to assumptions of geometrical parameters in the Example airship and taking its full scale model as compared to down scaled model used in wind tunnel testing.

4. Conclusions

A general simulation methodology for airship has been developed. The aerodynamic model is analytic and can be tuned to any kind of airship with minimum effort. A limited experimental data is required to carry out the simulation for analysis of different airship designs.

Design and control of airship is simple if mathematical model of the airship is available. The dynamic model is similar to that of aircraft with additional buoyancy effect. The 6-DOF modeling approach used in this paper illustrates the airship response to control inputs. In this model aerodynamic part is analytically formulated using airship geometric parameters which are otherwise found using wind tunnel setup in conventional approaches.

A complete simulation strategy is described in easily understandable Matlab environment. This may be further used to design controller for different applications. The significance of this approach is described by simulating a typical airship design and validating it with YEZ-2A airship wind tunnel test results. This approach can be used to find longitudinal and lateral aerodynamic stability modes. With this type of computer simulation, the performance of the vehicle is tested before actually manufacturing the airship. This work provides a first step for developing the navigation and control system of autonomous airship.

5. Future recommendations

The prospective researchers interested to pursue their research in the area of airship dynamic modeling may investigate longitudinal behavior of airship. Investigation of stability modes could be another promising area for the researchers. Implementation of control algorithm for autonomous operations and high altitude missions will be the future dimensions of research in this area.

Appendix A

Aerodynamic force and moment equations defined in Section 2 contain coefficients. These coefficients are defined below as:

$$C_{X1} = -[C_{Dh_0} S_h + C_{Df_0} S_f + C_{Dg_0} S_g] \tag{A.1}$$

$$C_{X2} = (k_2 - k_1)\eta_k I_1 S_h \tag{A.2}$$

$$C_{X3} = C_{tf} S_f \tag{A.3}$$

$$C_{Y1} = -C_{X2} \tag{A.4}$$

$$C_{Y2} = -0.5(\partial C_L/\partial\alpha)_f S_f \eta_f \tag{A.5}$$

$$C_{Y3} = -[C_{Dch} J_1 S_h + C_{Dcf} S_f + C_{Dcg} S_g] \tag{A.6}$$

$$C_{Y4} = -0.5(\partial C_L/\partial\delta)_f S_f \eta_f \tag{A.7}$$

$$C_{Z1} = -C_{X2} \tag{A.8}$$

$$C_{Z2} = C_{Y2} \tag{A.9}$$

$$C_{Z3} = -[C_{Dch} J_1 S_h + C_{Dcf} S_f] \tag{A.10}$$

$$C_{Z4} = C_{Y4} \tag{A.11}$$

$$C_{L1} = (\partial C_L/\partial\delta)_f S_f \eta_f l_{f3} \tag{A.12}$$

$$C_{L2} = C_{Dcg} S_g l_{gz} \tag{A.13}$$

$$C_{L3} = C_{Dcg} S_g l_{gz} l^2 \tag{A.14}$$

$$C_{L4} = -(C_{Dcg} S_g l_{gz} + C_{Dcf} S_f l_{f3}) d^2 \tag{A.15}$$

$$C_{M1} = -(k_2 - k_1)\eta_k I_3 S_h l \tag{A.16}$$

$$C_{M2} = -0.5(\partial C_L/\partial\alpha)_f S_f \eta_f l_{f1} \tag{A.17}$$

$$C_{M3} = -(C_{Dch} J_2 S_h l + C_{Dcf} S_f l_{f2}) \tag{A.18}$$

$$C_{M4} = -0.5(\partial C_L/\partial\delta)_f S_f \eta_f l_{f1} \tag{A.19}$$

$$C_{M5} = -C_{Dcf} S_f l_{f2} l^2 \tag{A.20}$$

Table A.1
Parameter values selected for calculations of aerodynamic coefficients.

Parameter	Value
L	129.5 m
d	32 m
l_{f1}	109.1 m
l_{f2}	113.7 m
l_{f3}	13.8 m
l_h	103.6 m
l_{gx}	45.3 m
l_{gz}	17.6 m
x_{cv}	63.47 m
S_h	1689.3 m^2
S_f	709.7 m^2
S_g	24.7 m^2
η_f	0.3789
η_k	1.0518
C_{Dh_0}	0.0250
C_{Df_0}	0.006
C_{Dg_0}	0.01
C_{Dch}	0.50
C_{Dcf}	1
C_{Dcg}	1
C_{tf}	0
$(\partial C_L/\partial\alpha)_f$	5.73
$(\partial C_L/\partial\delta)_f$	1.24

$$C_{N1} = -C_{M1} \tag{A.21}$$

$$C_{N2} = -C_{M2} \tag{A.22}$$

$$C_{N3} = C_{Dch} J_2 S_h l + C_{Dcf} S_f l_{f2} + C_{Dcg} S_g l_{gx} \tag{A.23}$$

$$C_{N4} = -C_{M4} \tag{A.24}$$

$$C_{N5} = -(C_{Dcf} S_f l_{f2} + C_{Dcg} S_g l_{gx}) l^2 \tag{A.25}$$

where,

$$I_1 = (1/S_h) \int_{-a_1}^{a_2} (dA_c/dx)\, dx \tag{A.26}$$

$$I_3 = (1/S_h l_h) \int_{-a_1}^{a_2} x\big(dA_c(x)/dx\big)\, dx \tag{A.27}$$

$$J_1 = (1/S_h) \int_{-a_1}^{a_2} 2r_h(x)\, dx \tag{A.28}$$

$$J_2 = (1/S_h l_h) \int_{-a_1}^{a_2} 2r_h(x) x\, dx \tag{A.29}$$

Here A_c is the hull cross-sectional area, r_h is the hull radius; x is the axial distance along the hull from the nose; a_1 is axial distance, from hull nose to C.V point and a_2 is axial distance from C.V point to the hull tail.

Solution to the above integrals in (A.26)–(A.29) is:

$$I_1 = \pi\big(b^2/V^{2/3}\big)\big(1 - f^2\big) \tag{A.30}$$

$$I_3 = \pi\big(b^2/3lV^{2/3}\big)\big(a_1 - 2a_2 f^3 - 3a_1 f^2\big) - x_{cv} I_1/l \tag{A.31}$$

$$J_1 = \big(b/2V^{2/3}\big)\big(\pi a_1 + a_2\sqrt{1 - f^2}\, f + 2a_2 \sin^{-1}(f)\big) \tag{A.32}$$

$$J_2 = (J_1/l)(a_1 - x_{cv}) + \big(2b/3lV^{2/3}\big)\big(a_2^2 - a_1^2 - a_2^2(1 - f^2)^{3/2}\big) \tag{A.33}$$

where,

$$f = (l_h - a_1)/a_2 \tag{A.34}$$

$$x_{cv} = a_1 - (4\pi/3)(a_1 - a_2) \tag{A.35}$$

Here,

$$a_1 + a_2 = l = L \quad \text{(length of airship)}$$

$$2b = 2r_h = d \quad \text{(diameter of airship)}$$

The parameters used in the above equations can be estimated according to the definitions presented in Refs. [9,8,14].

Parameter values selected for calculations of aerodynamic coefficients are listed in Table A.1.

References

[1] H.J. Allen, E.W. Perkins, A study of viscosity on flow over slender inclined bodies of revolution, NACA report 1048, 1951.

[2] S.A. Bhatti, Numerical Analysis with C++, 4th ed., A-One Publishers, 2002.

[3] M.V. Cook, The linearised small perturbation equation of motion for an airship, Working paper No. WP 8, Aerodynamics Department, College of Aeronautics, Cranfield, 1990.

[4] A. Elfes, S.S. Bueno, M. Bergerman, J.G. Ramos, A semi-autonomous robotics airship for environmental monitoring missions, in: Proc. IEEE International Conference on Robotics and Automation, Leuven, Belgium, 1998, pp. 3449–3455.

[5] S.B.V. Gomes, An investigation of the flight dynamics of airships with application to the YEZ-2A, PhD thesis, College of Aeronautics, Cranfield University, 1990.

[6] S.B.V. Gomes, J.J.G. Ramos, Airship dynamic modeling for autonomous operation, in: Proc. IEEE International Conference on Robotics & Automation, Leuven, Belgium, 1998, pp. 3462–3467.

[7] R. Jones, Acceleration derivatives in the case of a body moving in ideal fluid, ARC R&M 748, 1921.

[8] S.P. Jones, J.D. DeLaurier, Aerodynamic estimation techniques for aerostats and airships, in: AIAA Lighter-than-Air Systems Conference, AIAA, Annapolis, MD, 1981.

[9] G.A. Khoury, J.D. Gillett, Airship Technology, Cambridge Aerospace Series, vol. 10, 2004.

[10] A. Kornienko, System identification approach for determining flight dynamical characteristics of an airship from flight data, PhD thesis, University of Stuttgart, 2006.

[11] D.J. Lewis, J.M. Lipscombe, P.G. Thomasson, The simulation of remotely operated underwater vehicle, in: Proc. ROV '84 Conference and Exposition, The Marine Technology Society, San Diego, CA, 1984.

[12] Y. Li, Dynamics modeling and simulation of flexible airships, PhD thesis, McGill University, Montreal, 2007.

[13] T. Lutz, S. Wagner, Drag reduction and shape optimization of airship bodies, Journal of Aircraft 35 (3) (1998) 345–351.

[14] J.B. Mueller, M.A. Paluszek, Y. Zhao, Development of an aerodynamic model and control law design for a high altitude airship, in: AIAA 3rd Unmanned Unlimited Technical Conference, Workshop and Exhibit, AIAA, Chicago, IL, 2004.

[15] M.M. Munk, Aerodynamics of airships, in: W.F. Durand (Ed.), Aerodynamics Theory, vol. 6, Springer, Berlin, 1936, pp. 32–48.

[16] M.M. Munk, The aerodynamic forces on airship hulls, NACA TR-184, 1924.

[17] S. Nakamura, Numerical Analysis and Graphic Visualization with MATLAB, Prentice Hall, Inc., 2002.

[18] D.K. Schmidt, Modeling and near-space station keeping control of a large high-altitude airship, Journal of Guidance, Control and Dynamics 30 (2) (2007) 540–547.

[19] L.B. Tuckerman, Notes on aerodynamic forces on airship hulls, NACA report No. 129, 1923.

[20] L.B. Tuckerman, Inertia factors of ellipsoids for use in airship design, NACA report No. 210, 1926.

[21] R. Upson, W. Klikoff, Application of practical hydrodynamics to airship design, British Aeronautical Research Committee Report and Memoranda, No. 405, 1933.