Spares

Computer Science Society
Programming Contest
Fall 2011

In bowling, ten wooden pins stand on a floor and a bowling ball is rolled towards them. The pins still standing after the first roll are called a *leave*. If pins are left, a second ball is rolled towards them in an attempt to roll a *spare*—knocking down all of the remaining pins. Some spares are easy, and some spares are hard, depending on the leave. In this problem, you're given the positions of the pins in a leave and will compute the difficulty of rolling a spare. Here is a glossary of symbols:

$L \subseteq \{1, 2, 3, \ldots, 10\}$	is a *leave* of pins
$p \in L$	is a *pin*
(x_p, y_p)	are the *coordinates* of pin p

p	(x_p, y_p)
1	$(0, 0)$
2	$(-.5, \frac{\sqrt{3}}{2})$
3	$(.5, \frac{\sqrt{3}}{2})$
4	$(-1, \frac{2\sqrt{3}}{2})$
5	$(0, \frac{2\sqrt{3}}{2})$
6	$(1, \frac{2\sqrt{3}}{2})$
7	$(-1.5, \frac{3\sqrt{3}}{2})$
8	$(-.5, \frac{3\sqrt{3}}{2})$
9	$(.5, \frac{3\sqrt{3}}{2})$
10	$(1.5, \frac{3\sqrt{3}}{2})$

is the mapping from pins to coordinates

$i \in L$	is an *impact* pin
$j \in L$	is a *projectile* pin
$k \in L$	is a *target* pin
$\pi_{j,k}$	is the *penalty* for projectile j against target k
δ_L	is the *difficulty* of rolling a spare on leave L

Assuming the ball is rolling in the positive y-direction, min is minimum over a set of pins, and μ is mean over a set of pins,

$$\pi_{j,k} = \begin{cases} \infty & \text{if } y_j \geqslant y_k & \text{(projectile past target)} \\ 0 & \text{if } y_j < y_k \text{ and } x_j = x_k & \text{(projectile nails target)} \\ \sqrt{(x_k - x_j)^2 + (y_k - y_j)^2} & \text{if } y_j < y_k \text{ and } x_j \neq x_k & \text{(projectile-to-target distance)} \end{cases}$$

$$\delta_L = \min_{i \in L}\left(\mu_{k \in L \neg \{i\}} \left(\min_{j \in L \neg \{k\}} (\pi_{j,k}) \right) \right)$$

Note: computing min or μ over an empty set results in 0, and computing μ over any set that contains ∞ results in ∞.

Input/Output Format and Samples

(see next page on reverse side)

Input Format

Each input line contains a nonempty list of pins (without duplicates) constituting a leave $L \subseteq \{1, 2, 3, \ldots, 10\}$.

Output Format

For each input line containing leave L, compute and print δ_L accurate to three decimal digits, as shown in the sample output below. Print `Infinity` in place of ∞.

Input Sample

```
1
1 2
1 5
2 3
2 10
4 7 10
1 2 3 5
1 7 10
7 10
```

Output Sample

```
spare difficulty is 0.000
spare difficulty is 1.000
spare difficulty is 0.000
spare difficulty is Infinity
spare difficulty is 2.646
spare difficulty is 1.823
spare difficulty is 0.667
spare difficulty is 3.000
spare difficulty is Infinity
```