### Global Value Numbering in Factor

#### Alex Vondrak

ajvondrak@csupomona.edu

September 1, 2011

PAGE 3			
DEPARTMENT	COURSE	DESCRIPTION	PREREQS
COMPUTER SCIENCE		INTERMEDIATE COMPILER DESIGN, WITH A FOCUS ON DEPENDENCY RESOLUTION.	CPSC 432
	-	and the same of the same of the	

### **Factor**

Factor (http://factorcode.org/)

- Started development September 2003—a baby among languages
- Stack-based
- Object-oriented
- Dynamically typed
- Extensive standard library
- High-level, yet fully compiled

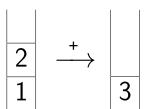
Won't really have time to discuss the language in depth

### Stacks as an Evaluation Model

```
Example (Code)
1 2 +
```

Example (Execution)

```
push(1);
push(2);
y = pop(); // y = 2;
x = pop(); // x = 1;
push(x + y); // push(3);
```



- Compiler
  - Structure
  - Optimizations
- Value Numbering
  - Local Value Numbering
  - Global Value Numbering

# Organization

### Non-optimizing base compiler

- VM written in C++
- Responsible for basic runtime services
  - Garbage collection
  - Method dispatch
  - Polymorphic inline caches
  - ...
- Single pass—outputs assembly stubs for primitives

### Optimizing compiler

- Written in Factor code
  - Possible by bootstrapping
- Optimizes in passes across two intermediate representations (IRs)
  - High-level IR (compiler.tree)
  - Low-level IR (compiler.cfg)

### High-level IR

- Tree of node objects
- Very simple virtual instruction set
  - #introduce, #return
  - #push & #call
  - #renaming—#copy & #shuffle
  - #declare & #terminate
  - #branch—#if & #dispatch
  - #phi
  - #recursive, #enter-recursive, #call-recursive, #return-recursive
  - #alien-node, #alien-invoke, #alien-indirect, #alien-assembly, #alien-callback
- Input/output values of stack given unique names

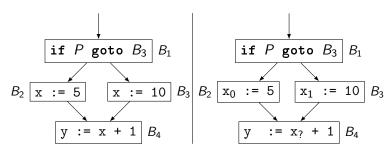
## High-level IR

```
1 2 +
```

```
Example
₩{
    T{ #push { literal 1 } { out-d { 6256273 } } }
    T{ #push { literal 2 } { out-d { 6256274 } } }
    T{ #call
        { word + }
        { in-d V{ 6256273 6256274 } }
        { out-d { 6256275 } }
    }
    T{ #return { in-d V{ 6256275 } } }
```

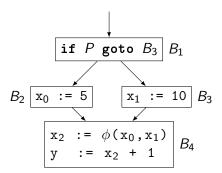
### Low-level IR

- Control flow graph (CFG)
  - Basic blocks = maximal sequence of "straight-line" code
  - Directed edges = transfer of control flow
- insn objects modeled closely after assembly-like instructions
- Static single assignment (SSA) form



### Low-level IR

- Control flow graph (CFG)
  - Basic blocks = maximal sequence of "straight-line" code
  - Directed edges = transfer of control flow
- insn objects modeled closely after assembly-like instructions
- Static single assignment (SSA) form



## Optimizations—High-level IR

```
: optimize-tree ( nodes -- nodes' )
     analyze-recursive
     normalize
     propagate
      cleanup
     dup run-escape-analysis? [
          escape-analysis
          unbox-tuples
      ] when
     apply-identities
      compute-def-use
     remove-dead-code
     ?check
      compute-def-use
     optimize-modular-arithmetic
     finalize
   with-scope ;
```

### Optimizations—Low-level IR

```
: optimize-cfg ( cfg -- cfg' )
   optimize-tail-calls
   delete-useless-conditionals
   split-branches
   join-blocks
   normalize-height
   construct-ssa
   alias-analysis
   value-numbering
   copy-propagation
   eliminate-dead-code:
```

- - Structure
  - Optimizations
- Value Numbering
  - Local Value Numbering
  - Global Value Numbering

## Value Numbering

### Idea: assign each variable a value number

- Equal value numbers ⇒ equal at runtime
- Turn recomputations into ##copy instructions, saving time

### General problem is undecidable

- Seek conservative solution
- Discover Herbrand equivalences
- Consider two values congruent if
  - They're computed by the same operator
  - Their operands are congruent

- Thought to be invented by Balke in the 1960s
- Largely credited to Cocke & Schwartz in the 1970s
- Current implementation Factor uses

Pro: Easy to understand, implement, and extend

Con: Is local and pessimistic, discovering fewer congruences

#### Implementation

Expressions are constructed from instructions

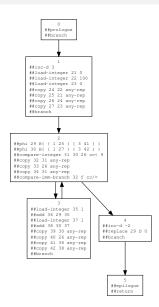
### Example

```
T{ ##add { dst 1 } { src1 2 } { src2 3 } } >expr { ##add 2 3 }
```

- Expression graph = 3 global hash tables
  - vregs>vns
  - exprs>vns
  - vns>insns
- If possible, instructions are simplified using data from expression graph

```
Example (In Factor)
0 100 [ 1 fixnum+fast ] times
Example (In Java)
```

```
int i = 0;
for (int j = 0; j < 100; j++) {
  i += 1;
```



#### Basic Block 1

```
vregs>vns = H{ }
exprs>vns = H{ }
```

```
##inc-d 3
##load-integer 21 0
##load-integer 22 100
##load-integer 23 0
##copy 24 22 any-rep
##copy 25 21 any-rep
##copy 26 24 any-rep
##copy 27 23 any-rep
##branch
```

(no-op)

```
vregs>vns = H{ { 21 21 } }
exprs>vns = H\{ \{ 0 21 \} \}
```

```
##inc-d 3
##load-integer 21 0
##load-integer 22 100
##load-integer 23 0
##copy 24 22 any-rep
##copy 25 21 any-rep
##copy 26 24 any-rep
##copy 27 23 any-rep
##branch
```

```
(no-op)
> expr = 0
```

```
vregs>vns = H{ { 21 21 } { 22 22 } }
exprs>vns = H\{ \{ 0 21 \} \{ 100 22 \} \}
```

```
##inc-d 3
##load-integer 21 0
##load-integer 22 100
##load-integer 23 0
##copy 24 22 any-rep
##copy 25 21 any-rep
##copy 26 24 any-rep
##copy 27 23 any-rep
##branch
```

```
(no-op)
> expr = 0
> expr = 100
```

```
vregs>vns = H{ { 21 21 } { 22 22 } { 23 21 } }
exprs>vns = H\{ \{ 0 21 \} \{ 100 22 \} \}
```

```
##inc-d 3
##load-integer 21 0
##load-integer 22 100
##load-integer 23 0
##copy 24 22 any-rep
##copy 25 21 any-rep
##copy 26 24 any-rep
##copy 27 23 any-rep
##branch
```

```
(no-op)
> expr = 0
> expr = 100
> expr = 0
```

```
vregs>vns = H{ { 21 21 } { 22 22 } { 23 21 } { 24 22 } ... }
exprs>vns = H\{ \{ 0 21 \} \{ 100 22 \} \}
```

```
##inc-d 3
##load-integer 21 0
##load-integer 22 100
##copy 23 21 any-rep
##copy 24 22 any-rep
##copy 25 21 any-rep
##copy 26 24 any-rep
##copy 27 23 any-rep
##branch
```

```
(no-op)
> expr = 0
> expr = 100
> expr = 0
. . .
. . .
```

```
vregs>vns = H{ }
exprs>vns = H{ }
```

```
##phi 29 H{ { 1 25 } { 3 41 } }
##phi 30 H{ { 1 27 } { 3 42 } }
##compare-integer 31 30 26 cc< 9
##copy 32 31 any-rep
##copy 33 26 any-rep
##copy 34 31 any-rep
##compare-imm-branch 32 f cc/=
```

```
(no-op)
(no-op)
```

```
vregs>vns = H{ { 30 30 } { 26 26 } { 31 31 } }
exprs>vns = H{ { { ##compare-integer 30 26 cc< } 31 } }
```

```
##phi 29 H{ { 1 25 } { 3 41 } }
##phi 30 H{ { 1 27 } { 3 42 } }
##compare-integer 31 30 26 cc< 9
##copy 32 31 any-rep
##copy 33 26 any-rep
##copy 34 31 any-rep
##compare-imm-branch 32 f cc/=
```

```
(no-op)
(no-op)
>expr = ...
```

```
vregs>vns = H{ { 30 30 } { 26 26 } { 31 31 } ... }
exprs>vns = H{ { { ##compare-integer 30 26 cc< } 31 } }
```

```
##phi 29 H{ { 1 25 } { 3 41 } }
##phi 30 H{ { 1 27 } { 3 42 } }
##compare-integer 31 30 26 cc< 9
##copy 32 31 any-rep
##copy 33 26 any-rep
##copy 34 31 any-rep
##compare-imm-branch 32 f cc/=
```

```
(no-op)
(no-op)
>expr = ...
```

```
vregs>vns = H{ { 30 30 } { 26 26 } { 31 31 } ... }
exprs>vns = H{ { { ##compare-integer 30 26 cc< } 31 } }
```

```
##phi 29 H{ { 1 25 } { 3 41 } }
##phi 30 H{ { 1 27 } { 3 42 } }
##compare-integer 31 30 26 cc< 9
##copy 32 31 any-rep
##copy 33 26 any-rep
##copy 34 31 any-rep
##compare-integer-branch 30 26 cc<
```

```
(no-op)
(no-op)
>expr = ...
```

```
vregs>vns = H{ }
exprs>vns = H{ }
```

```
##load-integer 35 1
##add 36 29 35
##load-integer 37 1
##add 38 30 37
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

#### Basic Block 3

```
vregs>vns = H\{ \{ 35 35 \} \}
exprs>vns = H\{ \{ 1 35 \} \}
```

```
##load-integer 35 1
##add 36 29 35
##load-integer 37 1
##add 38 30 37
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

> expr = 1

```
vregs>vns = H{ { 35 35 } { 29 29 } { 36 36 } }
exprs>vns = H{ { 1 35 } { { ##add-imm 29 1 } 36 } }
```

```
##load-integer 35 1
##add 36 29 35
##load-integer 37 1
##add 38 30 37
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

```
> expr = 1
>expr = { ##add-imm 29 1 }
```

```
vregs>vns = H{ { 35 35 } { 29 29 } { 36 36 } { 37 35 } }
exprs>vns = H{ { 1 35 } { { ##add-imm 29 1 } 36 } }
```

```
##load-integer 35 1
##add-imm 36 29 1
##load-integer 37 1
##add 38 30 37
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

```
> expr = 1
>expr = { ##add-imm 29 1 }
> expr = 1
```

```
vregs>vns = H{ { 35 35 } { 29 29 } { 36 36 } { 37 35 } ... }
exprs>vns = H{ { 1 35 } { { ##add-imm 29 1 } 36 } ... }
```

```
##load-integer 35 1
##add-imm 36 29 1
##copy 37 35 any-rep
##add 38 30 37
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

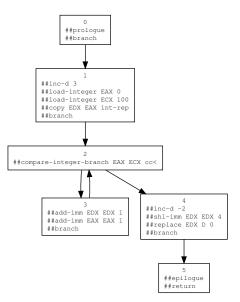
```
> expr = 1
>expr = { ##add-imm 29 1 }
> expr = 1
>expr = { ##add-imm 30 1 }
```

```
vregs>vns = H{ { 35 35 } { 29 29 } { 36 36 } { 37 35 } ... }
exprs>vns = H{ { 1 35 } { { ##add-imm 29 1 } 36 } ... }
```

```
##load-integer 35 1
##add-imm 36 29 1
##copy 37 35 any-rep
##add-imm 38 30 1
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

```
> expr = 1
>expr = { ##add-imm 29 1 }
> expr = 1
>expr = { ##add-imm 30 1 }
. . .
```

## Local Value Numbering Results



## Same Example, Global Algorithm

### Changes I made:

- Global—don't wipe out hash tables after each block
- Optimistic—previously unseen values considered redundant
- Fixed point iteration—optimism may be wrong first time around
- Offline replacements—only on final pass

### Definition (Notation)

$$\langle n \rangle = \{x, y, z\}$$
 (expr)

versus

```
vregs>vns = H{ { x n } { y n } { z n } }
exprs>vns = H{ { expr n } }
```

## Global Value Numbering

Iteration 1, Basic Block 1

```
##inc-d 3
##load-integer 21 0
##load-integer 22 100
##load-integer 23 0
##copy 24 22 any-rep
##copy 25 21 any-rep
##copy 26 24 any-rep
##copy 27 23 any-rep
##branch
```

 $\langle \mathbf{f} \rangle = U$  (everything)

## Global Value Numbering

Iteration 1, Basic Block 1

```
##inc-d 3
##load-integer 21 0
##load-integer 22 100
##load-integer 23 0
##copy 24 22 any-rep
##copy 25 21 any-rep
##copy 26 24 any-rep
##copy 27 23 any-rep
##branch
```

$$\langle 21 \rangle = \{21\} \tag{0}$$

```
##inc-d 3
##load-integer 21 0
##load-integer 22 100
##load-integer 23 0
##copy 24 22 any-rep
##copy 25 21 any-rep
##copy 26 24 any-rep
##copy 27 23 any-rep
##branch
```

$$\langle 21 \rangle = \{21\} \tag{0}$$

$$\langle 22 \rangle = \{22\} \tag{100}$$

Value Numbering

```
##inc-d 3
##load-integer 21 0
##load-integer 22 100
##load-integer 23 0
##copy 24 22 any-rep
##copy 25 21 any-rep
##copy 26 24 any-rep
##copy 27 23 any-rep
##branch
```

$$\langle 21 \rangle = \{21, 23\}$$
 (0)  $\langle 22 \rangle = \{22\}$  (100)

```
##inc-d 3
##load-integer 21 0
##load-integer 22 100
##load-integer 23 0
##copy 24 22 any-rep
##copy 25 21 any-rep
##copy 26 24 any-rep
##copy 27 23 any-rep
##branch
```

$$\langle 21 \rangle = \{21, 23, 25, 27\}$$
 (0)  
 $\langle 22 \rangle = \{22, 24, 26\}$  (100)

```
##phi 29 H{ { 1 25 } { 3 41 } }
##phi 30 H{ { 1 27 } { 3 42 } }
##compare-integer 31 30 26 cc< 9
##copy 32 31 any-rep
##copy 33 26 any-rep
##copy 34 31 any-rep
##compare-imm-branch 32 f cc/=</pre>
```

```
\langle 21 \rangle = \{21, 23, 25, 27\} (0)
\langle 22 \rangle = \{22, 24, 26\} (100)
```

```
##phi 29 H{ { 1 25 } { 3 41 } }
##phi 30 H{ { 1 27 } { 3 42 } }
##compare-integer 31 30 26 cc< 9
##copy 32 31 any-rep
##copy 34 31 any-rep
##compare-imm-branch 32 f cc/=</pre>
```

$$\langle 21 \rangle = \{21, 23, 25, 27, 29\}$$
 (0)  
 $\langle 22 \rangle = \{22, 24, 26\}$  (100)

```
##phi 29 H{ { 1 25 } { 3 41 } }

##phi 30 H{ { 1 27 } { 3 42 } }

##compare-integer 31 30 26 cc< 9

##copy 32 31 any-rep

##copy 33 26 any-rep

##copy 34 31 any-rep

##compare-imm-branch 32 f cc/=
```

```
\langle 21 \rangle = \{21, 23, 25, 27, 29, 30\} (0)
\langle 22 \rangle = \{22, 24, 26\} (100)
```

```
##phi 29 H{ { 1 25 } { 3 41 } }
##phi 30 H{ { 1 27 } { 3 42 } }
##compare-integer 31 30 26 cc< 9
##copy 32 31 any-rep
##copy 33 26 any-rep
##copy 34 31 any-rep
##compare-imm-branch 32 f cc/=</pre>
```

```
\langle 21 \rangle = \{21, 23, 25, 27, 29, 30\} (0)

\langle 22 \rangle = \{22, 24, 26\} (100)

\langle 31 \rangle = \{31\} (t)
```

```
##phi 29 H{ { 1 25 } { 3 41 } }

##phi 30 H{ { 1 27 } { 3 42 } }

##compare-integer 31 30 26 cc< 9

##copy 32 31 any-rep

##copy 33 26 any-rep

##copy 34 31 any-rep

##compare-imm-branch 32 f cc/=
```

```
\langle 21 \rangle = \{21, 23, 25, 27, 29, 30\} (0)

\langle 22 \rangle = \{22, 24, 26, 33\} (100)

\langle 31 \rangle = \{31, 32, 34\} (t)
```

```
##load-integer 35 1
##add 36 29 35
##load-integer 37 1
##add 38 30 37
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

```
\langle 21 \rangle = \{21, 23, 25, 27, 29, 30\} (0)

\langle 22 \rangle = \{22, 24, 26, 33\} (100)

\langle 31 \rangle = \{31, 32, 34\} (t)
```

```
##load-integer 35 1
##add 36 29 35
##load-integer 37 1
##add 38 30 37
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

```
\langle 21 \rangle = \{21, 23, 25, 27, 29, 30\} (0)

\langle 22 \rangle = \{22, 24, 26, 33\} (100)

\langle 31 \rangle = \{31, 32, 34\} (t)

\langle 35 \rangle = \{35\} (1)
```

```
##load-integer 35 1
##add 36 29 35
##load-integer 37 1
##add 38 30 37
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

```
 \begin{aligned} \langle 21 \rangle &= \{21,23,25,27,29,30\} & \text{ (0)} \\ \langle 22 \rangle &= \{22,24,26,33\} & \text{ (100)} \\ \langle 31 \rangle &= \{31,32,34\} & \text{ (t)} \\ \langle 35 \rangle &= \{35,36\} & \text{ (1)} \end{aligned}
```

```
##load-integer 35 1
##add 36 29 35
##load-integer 37 1
##add 38 30 37
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

```
 \begin{aligned} \langle 21 \rangle &= \{21,23,25,27,29,30\} & (0) \\ \langle 22 \rangle &= \{22,24,26,33\} & (100) \\ \langle 31 \rangle &= \{31,32,34\} & (t) \\ \langle 35 \rangle &= \{35,36,37\} & (1) \end{aligned}
```

```
##load-integer 35 1
##add 36 29 35
##load-integer 37 1
##add 38 30 37
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

```
 \langle 21 \rangle = \{21, 23, 25, 27, 29, 30, 39\}  (0)  \langle 22 \rangle = \{22, 24, 26, 33, 40\}  (100)  \langle 31 \rangle = \{31, 32, 34\}  (t)  \langle 35 \rangle = \{35, 36, 37, 38, 41, 42\}  (1)
```

```
##inc-d 3
##load-integer 21 0
##load-integer 22 100
##load-integer 23 0
##copy 24 22 any-rep
##copy 25 21 any-rep
##copy 26 24 any-rep
##copy 27 23 any-rep
##branch
```

```
 \langle 21 \rangle = \{21, 23, 25, 27, 29, 30, 39\}  (—)  \langle 22 \rangle = \{22, 24, 26, 33, 40\}  (—)  \langle 31 \rangle = \{31, 32, 34\}  (—)  \langle 35 \rangle = \{35, 36, 37, 38, 41, 42\}  (—)
```

```
##inc-d 3
##load-integer 21 0
##load-integer 22 100
##load-integer 23 0
##copy 24 22 any-rep
##copy 25 21 any-rep
##copy 26 24 any-rep
##copy 27 23 any-rep
##branch
```

```
 \begin{array}{lll} \langle 21 \rangle = \{21,23,25,27,29,30,39\} & \text{(0)} \\ \langle 22 \rangle = \{22,24,26,33,40\} & \text{(100)} \\ \langle 31 \rangle = \{31,32,34\} & \text{(-)} \\ \langle 35 \rangle = \{35,36,37,38,41,42\} & \text{(-)} \\ \end{array}
```

```
##phi 29 H{ { 1 25 } { 3 41 } }
##phi 30 H{ { 1 27 } { 3 42 } }
##compare-integer 31 30 26 cc< 9
##copy 32 31 any-rep
##copy 33 26 any-rep
##copy 34 31 any-rep
##compare-imm-branch 32 f cc/=</pre>
```

```
\langle 21 \rangle = \{21, 23, 25, 27, 29, 30, 39\} (0)

\langle 22 \rangle = \{22, 24, 26, 33, 40\} (100)

\langle 31 \rangle = \{31, 32, 34\} (—)

\langle 35 \rangle = \{35, 36, 37, 38, 41, 42\} (—)
```

```
##phi 29 H{ { 1 25 } { 3 41 } }
##phi 30 H{ { 1 27 } { 3 42 } }
##compare-integer 31 30 26 cc< 9
##copy 32 31 any-rep
##copy 34 31 any-rep
##copy 34 31 any-rep
##compare-imm-branch 32 f cc/=</pre>
```

```
\langle 21 \rangle = \{21, 23, 25, 27, 30, 39\} (0)

\langle 22 \rangle = \{22, 24, 26, 33, 40\} (100)

\langle 29 \rangle = \{29\} (##phi 2 21 35)

\langle 31 \rangle = \{31, 32, 34\} (—)

\langle 35 \rangle = \{35, 36, 37, 38, 41, 42\} (—)
```

```
##phi 29 H{ { 1 25 } { 3 41 } }
##phi 30 H{ { 1 27 } { 3 42 } }
##compare-integer 31 30 26 cc< 9
##copy 32 31 any-rep
##copy 33 26 any-rep
##copy 34 31 any-rep
##compare-imm-branch 32 f cc/=</pre>
```

```
\langle 21 \rangle = \{21, 23, 25, 27, 39\} (0)

\langle 22 \rangle = \{22, 24, 26, 33, 40\} (100)

\langle 29 \rangle = \{29, 30\} (##phi 2 21 35)

\langle 31 \rangle = \{31, 32, 34\} (—)

\langle 35 \rangle = \{35, 36, 37, 38, 41, 42\} (—)
```

```
##phi 29 H{ { 1 25 } { 3 41 } }
##phi 30 H{ { 1 27 } { 3 42 } }
##compare-integer 31 30 26 cc< 9
##copy 32 31 any-rep
##copy 33 26 any-rep
##copy 34 31 any-rep
##compare-imm-branch 32 f cc/=
```

```
\langle 21 \rangle = \{21, 23, 25, 27, 39\}
                                                  (0)
  \langle 22 \rangle = \{22, 24, 26, 33, 40\}
                                               (100)
  \langle 29 \rangle = \{29, 30\} (##phi 2 21 35)
  \langle 31 \rangle = \{31, 32, 34\}
(##compare-integer-imm 29 100 cc<)
  \langle 35 \rangle = \{35, 36, 37, 38, 41, 42\}
```

```
##load-integer 35 1
##add 36 29 35
##load-integer 37 1
##add 38 30 37
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

```
 \begin{array}{lll} \langle 21 \rangle = \{21,23,25,27,39\} & \text{(0)} \\ \langle 22 \rangle = \{22,24,26,33,40\} & \text{(100)} \\ \langle 29 \rangle = \{29,30\} & \text{(##phi 2 21 35)} \\ \langle 31 \rangle = \{31,32,34\} \\ \text{(##compare-integer-imm 29 100 cc<)} \\ \langle 35 \rangle = \{35,36,37,38,41,42\} & \text{($-$)} \\ \end{array}
```

```
##load-integer 35 1
##add 36 29 35
##load-integer 37 1
##add 38 30 37
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

```
##load-integer 35 1
##add 36 29 35
##load-integer 37 1
##add 38 30 37
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

```
 \langle 21 \rangle = \{21, 23, 25, 27, 39\}  (0)  \langle 22 \rangle = \{22, 24, 26, 33, 40\}  (100)  \langle 29 \rangle = \{29, 30\}  (##phi 2 21 35)  \langle 31 \rangle = \{31, 32, 34\}  (##compare-integer-imm 29 100 cc<)  \langle 35 \rangle = \{35, 37, 38, 41, 42\}  (1)  \langle 36 \rangle = \{36\}  (##add-imm 29 1)
```

```
##load-integer 35 1
##add 36 29 35
##load-integer 37 1
##add 38 30 37
##copy 39 30 any-rep
##copy 40 26 any-rep
##copy 41 36 any-rep
##copy 42 38 any-rep
##branch
```

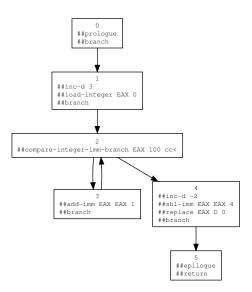
```
 \begin{array}{lll} \langle 21 \rangle = \{21,23,25,27\} & (0) \\ \langle 22 \rangle = \{22,24,26,33,40\} & (100) \\ \langle 29 \rangle = \{29,30,39\} & (\texttt{##phi 2 21 35}) \\ \langle 31 \rangle = \{31,32,34\} & (\texttt{##compare-integer-imm 29 100 cc<}) \\ \langle 35 \rangle = \{35,37\} & (1) \\ \langle 36 \rangle = \{36,38,41,42\} & (\texttt{##add-imm 29 1}) \\ \end{array}
```

Iteration 3

No value numbers change, only the expressions:

Final pass simplifies instructions based on this information

# Global Value Numbering Results



- - Structure
  - Optimizations
- Value Numbering
  - Local Value Numbering
  - Global Value Numbering
- Results