1 The Concept

We use a simple disk + bulge galaxy model (draw by galsim) to model the effects of weak lensing. We would like to see how much simple empirical priors improve out fits.

2 The Model

The model is a simple disk+bulge galaxy, with the following parameters:

- R_{disk}, disk half-light radius
- F_{disk}, disk flux
- g_1^{disk}, disk intrinsic alignment
- g_2^{disk}, disk intrinsic alignment
- R_{bulge}, bulge half-light radius
- F_{bulge}, bulge flux
- g_1^{bulge}, bulge intrinsic alignment
- g_2^{bulge}, bulge intrinsic alignment
- g_1^{shear}, lensing shear
- g_2^{shear}, lensing shear
- mu, lensing magnification (this parameter is optional)

g_1^{disk}, g_2^{disk}, g_1^{bulge}, and g_2^{shear} all model intrinsic alignment, but are parameterized like reduced shear in weak lensing formalism because it's easy to apply with galsim. The bulge is drawn with a De Vaucouleurs profile, and the disk is drawn with an $n = 1$ Sérsic profile. The image is convolved with a $\sigma = 0.25$ arcsecond Gaussian PSF.

2.1 The color "prior"

One important aspect of the model is that the disk and bulge can optionally be drawn with different $r - g$ colors such that $(g-r)_{disk} \approx (g-r)_{bulge} - 2$. (n.b. I have to check to ensure that these magnitudes are being calculated correctly.) We have been treating the color difference between the disk and bulge like a prior in that we are interested in how much switching it on affects our fits, but our current treatment presumes perfect knowledge of the color difference between disk and bulge.

3 Priors

Besides the "prior" on disk and bulge color, we are applying two empirically motivated priors.

3.1 The Kormendy Relation

The Kormendy relation essentially a projection of the fundamental plane. It states that for elliptical galaxies, $R_e \propto \langle I \rangle^{0.83}$. We take, as a prior on the bulge radius and flux, a distribution that is normally distributed around the line in $\log I$ - $\log R$ space defined by the Kormendy relation.

3.2 Orientation Prior

Assuming that the disk and bulge are oriented randomly in 3D space, they take on a non-uniform distribution of inclination angles. By assuming a uniform distribution of 3D orientations, we arrive at a prior proportional to $\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$ for the disk and bulge. If i is the inclination angle ($i = 0$ for a face-on disk), then for an ideal (circular, perfectly thin) disk, the shear γ is given by

$$\gamma = \frac{a-b}{a+b} = \frac{1-\cos(i)}{1+\cos(i)} \Rightarrow i = \arccos\left(\frac{1-\gamma}{1+\gamma}\right)$$

where a and b are the semi-major and semi-minor axes, respectively.

The prior pdf on γ is

$$p(\gamma) = p(i)\frac{\partial i}{\partial \gamma}$$

where $p(i)$ is the prior pdf on i. It can be shown that $p(i) \propto \sin(i)$. We can write $p(i)$ as

$$p(i) \propto \sin(i) = \sin\left(\arccos\left(\frac{1+\gamma}{1-\gamma}\right)\right) = \sqrt{1 - \left(\frac{1-\gamma}{1+\gamma}\right)^2} = \frac{2\sqrt{\gamma}}{1+\gamma}$$

And we can write $\frac{\partial i}{\partial \gamma}$ as

$$\frac{\partial i}{\partial \gamma} = \frac{\sqrt{\frac{\gamma}{(1+\gamma)^2}}}{\gamma} = \frac{1}{\sqrt{\gamma}(1+\gamma)}$$

Thus,

$$p(\gamma) \propto \frac{2\sqrt{\gamma}}{1+\gamma}\frac{1}{\sqrt{\gamma}(1+\gamma)} \propto \frac{1}{(1+\gamma)^2}$$

4 Doing fits

At first, we tried using a simple custom quadratic MLM estimator (What's the technical terminology?). Due to difficulties in fitting parameters with complex constraints, we switched to MCMC fitting with emcee. The chain is run with flat priors, then the ones mentioned above are applied after the chain has finished with importance sampling.

4.1 Convergence tests

We shoot for a burn-in time a few times larger the longest autocorrelation time out of each parameter, and we run the chain for about an order of magnitude longer than the longest autocorrelation time.

Ideally, we would like an acceptance fraction of 50% for each chain, but we often get closer to 30%.