

## Abstract

A number of large current and forthcoming astronomical surveys (for instance DES, LSST, WFIRST) plan to use the spatially coherent distortion of galaxy ellipticities by gravitational lensing to map the distribution of dark matter and energy in the universe. Here we explore for the first time (?) the possibility of using physically motivated models of galaxies to improve shape measurement. We use Markov-chain Monte Carlo in conjunction with state-of-the-art lensing simulation software (GalSim) to show that well-understood scaling relations between observable properties of galaxies can be used to remove intrinsic alignment contamination. Specifically, we investigate statistical power of the Kormendy Relation, the color difference between disks and bulges, and the connection between the 3-D orientation of galactic disks and their measured shapes for the purposes of galaxy shape measurement. ( Once the simulations are complete, we will use software (Balrog) and statistical methods that have been developed by this group for similar purposes to deploy our new intrinsic alignment mitigation on Dark Energy Survey data. )

## 1 The Concept

We use a simple disk + bulge galaxy model (draw by galsim) to model the effects of weak lensing. We would like to see how much simple empirical priors improve our fits.

## 2 The Model

The model is a simple disk+bulge galaxy, with the following parameters:

- $R_{disk}$ , disk half-light radius
- $F_{disk}$ , disk flux
- $g_1^{disk}$ , disk intrinsic alignment
- $g_2^{disk}$ , disk intrinsic alignment
- $R_{bulge}$ , bulge half-light radius
- $F_{bulge}$ , bulge flux
- $g_1^{bulge}$ , bulge intrinsic alignment
- $g_2^{bulge}$ , bulge intrinsic alignment
- $g_1^{shear}$ , lensing shear
- $g_2^{shear}$ , lensing shear
- $\mu$ , lensing magnification

$g_1^{disk}$ ,  $g_2^{disk}$ ,  $g_1^{bulge}$ , and  $g_2^{shear}$  all model intrinsic alignment, but are parameterized like reduced shear in weak lensing formalism because it's easy to apply with galsim. The bulge is drawn with a De Vaucouleurs profile, and the disk is drawn with an  $n = 1$  Sérsic profile. The image is convolved with a  $\sigma = 0.25$  arcsecond Gaussian PSF.

## 2.1 The color “prior”

One important aspect of the model is that the disk and bulge can optionally be drawn with different  $r - g$  colors such that  $(g - r)_{disk} \approx (g - r)_{bulge} - 2$ . We have been treating the color difference between the disk and bulge like a prior in that we are interested in how much switching it on affects our fits, but our current treatment presumes perfect knowledge of the color difference between disk and bulge.

If

$$(g - r)_{disk} = (g - r)_{bulge} - \Delta,$$

then

$$2.5 \log \left( \frac{F_g^{\text{Disk}} F_r^{\text{Bulge}}}{F_r^{\text{Disk}} F_g^{\text{Bulge}}} \right) = \Delta.$$

If we define  $\gamma$  such that

$$F_r^{\text{Disk}} = \gamma F_r^{\text{Bulge}} \tag{1}$$

$$F_g^{\text{Disk}} = F_g^{\text{Bulge}} / \gamma \tag{2}$$

$$F_r^{\text{Bulge}} = F_r^{\text{Disk}} / \gamma \tag{3}$$

$$F_g^{\text{Bulge}} = \gamma F_g^{\text{Disk}}, \tag{4}$$

then

$$\gamma^4 = 10^{\Delta/2.5} \Rightarrow \gamma = 10^{\Delta/10}. \tag{5}$$

For  $\Delta = 2$ ,  $\gamma = 10^{1/2} \approx 1.58$ . To generate dual band, images we use the fluxes defined in (1) - (4) and the  $\gamma$  from (5). Then we jointly scale the flux of both images so that the total flux is as expected, e.g.

$$F_r^{\text{Disk}} + F_g^{\text{Disk}} + F_r^{\text{Bulge}} + F_g^{\text{Bulge}} = F^{\text{Disk}} + F^{\text{Bulge}}.$$

## 3 Priors

Besides the “prior” on disk and bulge color, we are applying two empirically motivated priors.

### 3.1 The Kormendy Relation

The Kormendy relation essentially a projection of the fundamental plane. It states that for elliptical galaxies,  $R_e \propto \langle I \rangle_e^{0.83}$ , where  $R_e$  is the effective radius and  $\langle I \rangle_e$  is the mean surface brightness within  $R_e$ . We take, as a prior on the bulge radius and flux, a distribution that is normally distributed around the line in  $\log I - \log R$  space defined by the Kormendy relation.

### 3.2 Orientation Prior

Assuming that the disk is oriented randomly in 3D space, they take on a non-uniform distribution of inclination angles. By assuming a uniform distribution of 3D orientations, we arrive at a prior proportional to  $\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$ . If  $\phi$  is the inclination angle ( $\phi = 0$  for a face-on disk), then for an ideal (circular, perfectly thin) disk, the “shear”  $\gamma$  is given by

$$\gamma = \frac{a - b}{a + b} = \frac{1 - \cos(\phi)}{1 + \cos(\phi)} \Rightarrow \phi = \arccos \left( \frac{1 - \gamma}{1 + \gamma} \right)$$

where  $a$  and  $b$  are the semi-major and semi-minor axes, respectively.

The prior pdf on  $\gamma$  is

$$p(\gamma) = p(\phi) \frac{\partial \phi}{\partial \gamma}$$

where  $p(\phi)$  is the prior pdf on  $\phi$ . It can be shown that  $p(\phi) \propto \sin(\phi)$ . We can write  $p(\phi)$  as

$$p(\phi) \propto \sin(\phi) = \sin\left(\arccos\left(\frac{1+\gamma}{1-\gamma}\right)\right) = \sqrt{1 - \left(\frac{1-\gamma}{1+\gamma}\right)^2} = \frac{2\sqrt{\gamma}}{1+\gamma}$$

And we can write  $\frac{\partial\phi}{\partial\gamma}$  as

$$\frac{\partial\phi}{\partial\gamma} = \frac{\sqrt{\frac{\gamma}{(1+\gamma)^2}}}{\gamma} = \frac{1}{\sqrt{\gamma}(1+\gamma)}$$

Thus,

$$p(\gamma) \propto \frac{2\sqrt{\gamma}}{1+\gamma} \frac{1}{\sqrt{\gamma}(1+\gamma)} \propto \frac{1}{(1+\gamma)^2}$$

The dual band images are given equal Gaussian noise such that if they are added, the composite image has the desired SNR.

## 4 Doing fits

At first, we tried using a simple custom quadratic MLM estimator ([What's the technical terminology?](#)). Due to difficulties in fitting parameters with complex constraints, we switched to MCMC fitting with [emcee](#). The chain is run with flat priors, then the ones mentioned above are applied after the chain has finished with importance sampling.

For a 4-parameter fit (just the disk or just the bulge), `nsample=1500`, `nburnin=300`, `nwalker=50` seems to be about right.

### 4.1 Convergence tests

We shoot for a burn-in time a few times larger the longest autocorrelation time out of each parameter, and we run the chain for about an order of magnitude longer than the longest autocorrelation time.

Ideally, we would like an acceptance fraction of 50% for each chain, but we often get closer to 30%.

### 4.2 Signal to Noise Ratio

Simulated images are generated at a user-specified SNR. This is accomplished with `galsim.noise.addNoiseSNR`, which uses the definition of SNR from Great08. Briefly,

$$\sigma_I^2 = \sum_{\text{pixels } i} \frac{I_i^2}{\text{SNR}^2}.$$

See [Galsim docs](#) for more info.