PROGRAMMING ASSIGNMENT #6
CS 2223 B-TERM 2019
BACKTRACKING
AND
THE n-QUEENS PROBLEM

SIXTY POINTS
DUE: WEDNESDAY, DECEMBER 11, 2019 MIDNIGHT
(LATE SUBMISSIONS ACCEPTED UNTIL 6PM DECEMBER 13 WITHOUT PENALTY)

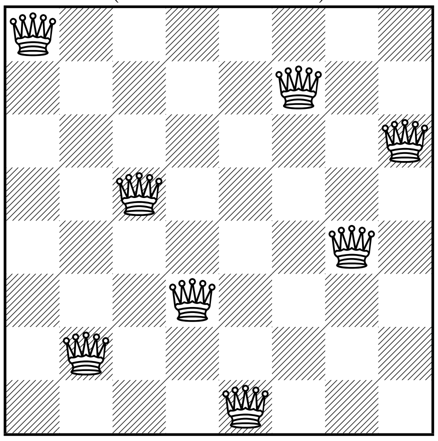
We crown the semester with the n-Queens Problem.

The challenge is to place n Queens on an $n \times n$ board (rectangular array?), so that no two attack each other, i.e. no two Queens may be on the same rank (row), file (column), or diagonal (????).

1. (24 Points) isLegalPosition(board,n)

   Write a function isLegalPosition(board,n) that takes a (possibly partial) position and n as arguments and returns TRUE if and only if no two Queens attack each other.

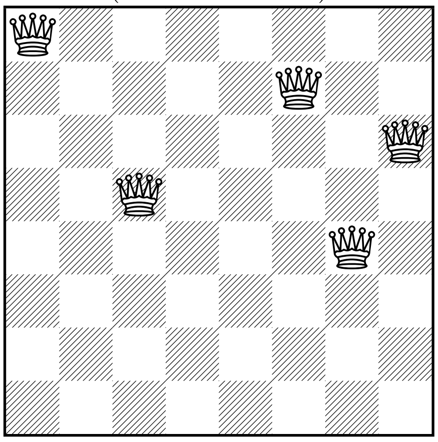
Here is a solution to the 8-Queens Problem:
(1 6 8 3 7 4 2 5)



Thus, isLegalPosition((1 6 8 3 7 4 2 5),8) should return TRUE.

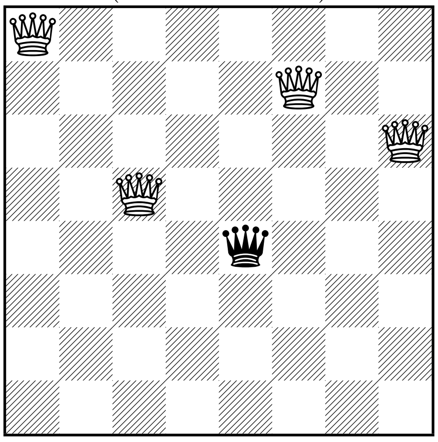
Because we are implementing a backtracking algorithm, we will restrict ourselves to positions which fill from the top of the board. We will insist then that the first $k \leq n$ positions be filled, i.e. non-zero, but the remaining $n - k$ positions may be zeroes. So the partial solution:

(1 6 8 3 7 0 0 0)



should also have isLegalPosition((1 6 8 3 7 0 0 0),8) return TRUE, while

(1 6 8 3 5 0 0 0)



should cause isLegalPosition((1 6 8 3 5 0 0 0),8) to return FALSE.

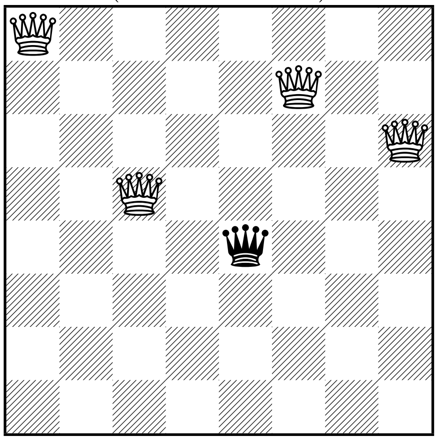
Why?

Do you see an elegant way to check that?

2. (18 Points) NEXTLEGALPOSITION(BOARD,n)

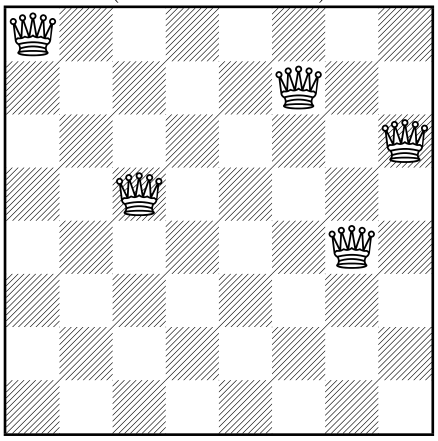
   From any (possibly partial) position, we need to be able to find the *next* legal position. There are, perhaps, three cases here. First, the next legal position from an illegal partial position; second, the next legal position from a *legal* partial position, and third, the next legal position after a full-fledged solution. We will fill our board from the top down and from left to right, so the next legal position after (illegal) partial position:

(1 6 8 3 5 0 0 0)



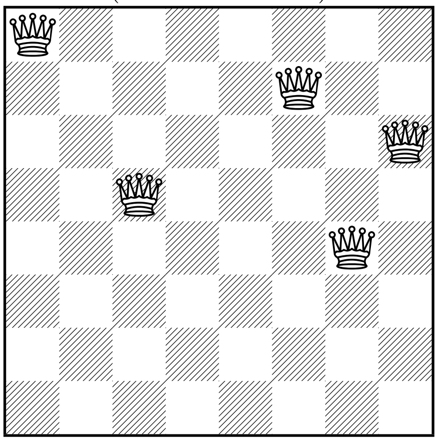
is

(1 6 8 3 7 0 0 0)



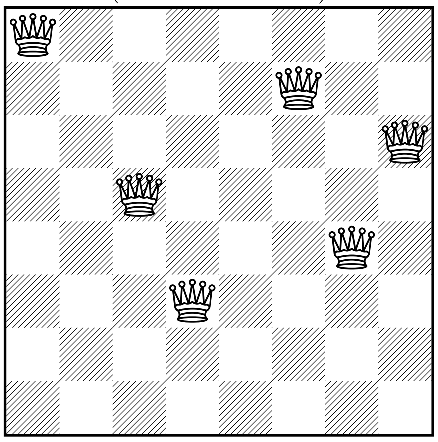
And the next legal position after (legal!) partial position:

(1 6 8 3 7 0 0 0)



is

(1 6 8 3 7 4 0 0)

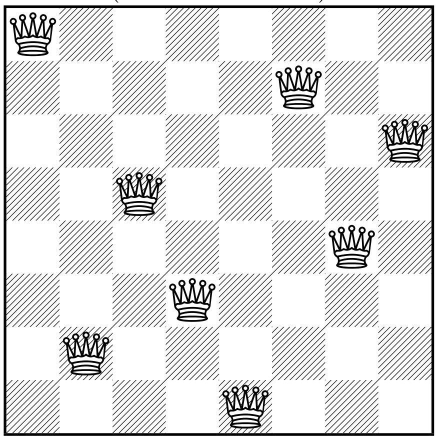


Will the next legal position *from* a legal position *always* add a Queen to the next rank?

Why? / Why not?

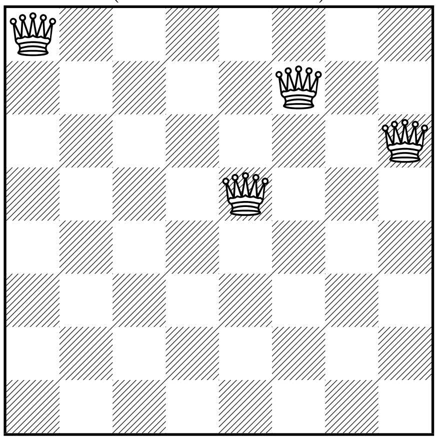
Lastly, the next legal position after our solution:

(1 6 8 3 7 4 2 5)



is

(1 6 8 5 0 0 0 0)



(Understanding this is understanding the backtracking–and then the forwarding–we are doing. This is the crux of the method.)

Write a function NextLegalPosition(board,n) that takes a (possibly partial) position and n as arguments and returns a board/position/array that represents the next legal position, or $(0_1, 0_2, \ldots, 0_n)$ if no legal position succeeds "board".

Hint: It may be useful to write another function Successor(board, n) that returns the next position to "board", whether legal or not.

3. (12 Points) Find the "first" solution to the n-Queens Problem for $n = 4...100$.

   With isLegalPosition(board,n) and NextLegalPosition(board,n) in your hip pocket, write a program which solves the n-Queens problem for all values between 4 and 100, inclusive.

   Your output should give a single solution to each instance of the problem, and it should be the *first* solution lexicographically.

   We saw that the 4-Queens problem has solutions (2, 4, 1, 3) and (3, 1, 4, 2) as its distinct solutions. Your output should be the first of these.

   Is our solution to the 8-Queens problem the first one?

4. (6 Points) Find all solutions to the n-Queens Problem for a particular n.

   With isLegalPosition(board,n) and NextLegalPosition(board,n) in your hip pocket, write a program which finds all solutions to the n-Queens problem for a single instance of the problem with $4 \leq n \leq 20$, input by the user. Your output should consist of a list of solutions in lexicographical order. And keep a running count/index!

   (What constitutes a "solution"?)

For parts 2-4, you can get some gains in efficiency by modifying isLegalPosition(board,n). We will *build* positions from *legal* positions. This means that only the last Queen, the last non-zero entry, can cause a position to be illegal. Do you see?

Why are you going to want increased efficiency?

You may find the Wikipedia entry on the "8-Queens puzzle" to be helpful... maybe even interesting.