# MECH 360 Notes

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# Contents

# 1 Transformation of Stress

Given the 3-D stress tensor

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yy} & \sigma_{yx} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}, \tag{1.0.1}$$

the principal stress tensor is given by

$$\underline{\underline{\sigma_p}} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}. \tag{1.0.2}$$

**Invarient Properties:**

- trace($\underline{\underline{\sigma}}$) = trace($\underline{\underline{\sigma_p}}$)
- det($\underline{\underline{\sigma}}$) = det($\underline{\underline{\sigma_p}}$)

# 2 Pure Bending

## 2.1 Unsymmetric Bending Analysis

**Main Ideas:**

1. Moment is a vector - resolving vectors
2. "Principal" Centroidal axes - Mohr's circle
3. Superposition - adding stresses

This leads two showing how one unsymmetric bending problem is two symmetric bending problems.

**Principal Centroidal Axes:** Where the product of inertia is zero.

Recall,

$$I_{xy} = \int xy dA,$$

If

$$I_{zy} = 0, I_y \neq 0, I_z \neq 0,$$

then $y$ and $z$ are the principal centroidal axes (PCA). PCA's are mutually orthogonal. Finding principal planes is analogous to finding principal stresses on the Mohr's circle.

The area moment of inertia is a tensor too. We apply the Mohr's transformations to find PCA.

Any given section possess *principal centroidal axes* even if it is unsymmetric. Principal centroidal axes can be determined

1. analytically

2. or using Mohr's circle.

If **M** is along the principal centroidal axis, the N.A. will be along the axis of **M**, then the equations for symmetric members can be used to compute the stresses. The principal of superposition is used to determine stresses in general for unsymmetric cases.

Given some a couple momment **M**, we have

$$M_z = M\cos\theta, \quad M_y = M\sin\theta,$$

then using superposition,

$$\sigma_x(y,z) = \frac{-M_z y}{I_z} + \frac{+M_y z}{I_y}.$$

Points along the N.A. have no stress, thus let $\sigma_x = 0$, and using $M_z = M\cos\theta, \quad M_y = M\sin\theta$, we get

$$y = \underbrace{\left(\frac{I_z}{I_y}\tan\theta\right)}_{a} z,$$

representing a line $y(z)$ with slope $a$. Letting $\phi$ be the angle between the N.A. and the z-axis gives

$$\tan\phi = \frac{I_z}{I_y}\tan\theta.$$

'' Note, $\phi$ depends on the loading axis. Also,

$$\tan\phi = \frac{y_n}{z_n},$$

where $(y_n, z_n)$ is some point on the NA.

**Solving Steps:** If $\underline{M}$ is not in the plane of symmetry, we have an unsymmetric bending problem.

1. Find PCA - (using Mohr)
2. Resolve moment vector about PCA
3. Find/apply $\sigma = \frac{-My}{i}$ for each component
4. Superposition - (assuming linearly elastic material)

**Steps solving an example:**

1. PCA: found by inspection
2. Decompose moment into cartesian components
3. Stress due to asymmetric bending is the sum of the stress due to each moment component

## 2.2 Eccentric Loading

Using superposition

$$\sigma_x(y,z) = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}.$$