# MECH 360 Notes

By AJ Wong and Eugene Lee

October 8, 2024

# Contents

# 1   Transformation of Stress

Given the 3-D stress tensor

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yy} & \sigma_{yx} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}, \tag{1.0.1}$$

the principal stress tensor is given by

$$\underline{\underline{\sigma_p}} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}. \tag{1.0.2}$$

**Invarient Properties:**

- $\text{trace}(\underline{\underline{\sigma}}) = \text{trace}(\underline{\underline{\sigma_p}})$

- $\det(\underline{\underline{\sigma}}) = \det(\underline{\underline{\sigma_p}})$

# 2   Pure Bending

## 2.1   Unsymmetric Bending Analysis

**Main Ideas:**

1. Moment is a vector - resolving vectors

2. "Principal" Centroidal axes - Mohr's circle

3. Superposition - adding stresses

This leads two showing how one unsymmetric bending problem is two symmetric bending problems.

**Principal Centroidal Axes:** Where the product of inertia is zero.

**What is Product of Inertia?**
Product of Inertia is a measure of body symmetry.
When you talk about symmetry, we talk about planes of symmetry.
If a body is symmetrical about a plane, let's say xy-plane, then

$$I_{xy} = 0$$

.
Recall,

$$I_{xy} = \int xy \, dA,$$

If

$$I_{zy} = 0, I_y \neq 0, I_z \neq 0,$$

then $y$ and $z$ are the principal centroidal axes (PCA). PCA's are mutually orthogonal. Finding principal planes is analogous to finding principal stresses on the Mohr's circle.

The area moment of inertia is a tensor too. We apply the Mohr's transformations to find PCA.

Any given section possess *principal centroidal axes* even if it is unsymmetric. Principal centroidal axes can be determined:

  1. analytically

  2. or using Mohr's circle.

If $\mathbf{M}$ is along the principal centroidal axis, the N.A. will be along the axis of $\mathbf{M}$, then the equations for symmetric members can be used to compute the stresses. The principal of superposition is used to determine stresses in general for unsymmetric cases.

Given some a couple momment $\mathbf{M}$, we have

$$M_z = M \cos\theta, \quad M_y = M \sin\theta,$$

then using superposition,

$$\sigma_x(y, z) = \frac{-M_z y}{I_z} + \frac{+M_y z}{I_y}.$$

Points along the N.A. have no stress, thus let $\sigma_x = 0$, and using $M_z = M \cos\theta, \quad M_y = M \sin\theta$, we get

$$y = \underbrace{\left( \frac{I_z}{I_y} \tan\theta \right)}_{a} z,$$

representing a line $y(z)$ with slope $a$. Letting $\phi$ be the angle between the N.A. and the z-axis gives

$$\tan\phi = \frac{I_z}{I_y} \tan\theta.$$

'' Note, $\phi$ depends on the loading axis. Also,

$$\tan\phi = \frac{y_n}{z_n},$$

where $(y_n, z_n)$ is some point on the NA.

**Solving Steps:** If $\underline{M}$ is not in the plane of symmetry, we have an unsymmetric bending problem.

  1. Find PCA - (using Mohr)

  2. Resolve moment vector about PCA

  3. Find/apply $\sigma = \frac{-My}{i}$ for each component

  4. Superposition - (assuming linearly elastic material)

**Steps solving an example:**

  1. PCA: found by inspection

  2. Decompose moment into cartesian components

  3. Stress due to asymmetric bending is the sum of the stress due to each moment component

## 2.2   Eccentric Loading

Using superposition

$$\sigma_x(y, z) = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}.$$

## 2.3   Shear Flow

q is shear flow.
Q is first moment of area, units $m^3$ of some partial area of cross-section (dependent on question) about centroid.
I would like to emphasize that Q is for PART of the cross-sectional area.
I is the normal moment of inertia calculated by $bh^3 \overline{_{12 and}} Ad^2$.

For a beam in bending, there is shear flow.

I think we've just been ignoring it.

The steps to doing a shear flow problem are:

1. Find moment of Inertia of entire crosssection

2. Identify the area that you will find Q for. I think the area depends on what the nail of interest is in.

3.