MECH 360 Notes

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1 Transformation of Stress

Given the 3-D stress tensor

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yy} & \sigma_{yx} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}, \tag{1.0.1}$$

the principal stress tensor is given by

$$\underline{\underline{\sigma_p}} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}. \tag{1.0.2}$$

Invarient Properties:

- $\text{trace}(\underline{\underline{\sigma}}) = \text{trace}(\underline{\underline{\sigma_p}})$
- $\det(\underline{\underline{\sigma}}) = \det(\underline{\underline{\sigma_p}})$

2 Pure Bending

2.1 Unsymmetric Bending Analysis

Main Ideas:

1. Moment is a vector - resolving vectors
2. "Principal" Centroidal axes - Mohr's circle
3. Superposition - adding stresses

This leads two showing how one unsymmetric bending problem is two symmetric bending problems.

Principal Centroidal Axes: Where the product of inertia is zero.

What is Product of Inertia?
Product of Inertia is a measure of body symmetry.
When you talk about symmetry, we talk about planes of symmetry.
If a body is symmetrical about a plane, let's say xy-plane, then

$$I_{xy} = 0$$

.
Recall,

$$I_{xy} = \int xy dA,$$

If

$$I_{zy} = 0, I_y \neq 0, I_z \neq 0,$$

then y and z are the principal centroidal axes (PCA). PCA's are mutually orthogonal. Finding principal planes is analogous to finding principal stresses on the Mohr's circle.

The area moment of inertia is a tensor too. We apply the Mohr's transformations to find PCA.

Any given section possess *principal centroidal axes* even if it is unsymmetric. Principal centroidal axes can be determined:

1. analytically
2. or using Mohr's circle.

If $\mathbf{M}$ is along the principal centroidal axis, the N.A. will be along the axis of $\mathbf{M}$, then the equations for symmetric members can be used to compute the stresses. The principal of superposition is used to determine stresses in general for unsymmetric cases.

Given some a couple momment $\mathbf{M}$, we have

$$M_z = M\cos\theta, \quad M_y = M\sin\theta,$$

then using superposition,

$$\sigma_x(y,z) = \frac{-M_z y}{I_z} + \frac{+M_y z}{I_y}.$$

Note, we are using a right-handed frame to achieve these signs.

Points along the N.A. have no stress, thus let $\sigma_x = 0$, and using $M_z = M\cos\theta, \quad M_y = M\sin\theta$, we get

$$y = \underbrace{\left(\frac{I_z}{I_y}\tan\theta\right)}_{a} z,$$

representing a line $y(z)$ with slope a. Letting ϕ be the angle between the N.A. and the z-axis gives

$$\tan\phi = \frac{I_z}{I_y}\tan\theta.$$

'' Note, ϕ depends on the loading axis. Also,

$$\tan\phi = \frac{y_n}{z_n},$$

where (y_n, z_n) is some point on the NA.

Solving Steps: If $\underline{M}$ is not in the plane of symmetry, we have an unsymmetric bending problem.

1. Find PCA - (using Mohr)
2. Resolve moment vector about PCA
3. Find/apply $\sigma = \frac{-My}{i}$ for each component
4. Superposition - (assuming linearly elastic material)

Steps solving an example:

1. PCA: found by inspection
2. Decompose moment into cartesian components
3. Stress due to asymmetric bending is the sum of the stress due to each moment component

Example 8 Steps: For the given section, write the steps to find the bending stress and maximum bending stress.

1. Know/compute $\underline{\underline{I}}$. If x-y are principal axes, $I_{xy} \neq 0$, which is an easy problem. To find I_{ij} where i and j represent x, y, or z, we find the centroid of each simple section, then apply the parallel axis theorem.
2. Draw Mohr's Circle to find principal moments of inertia and principal axes. In principal centroidal axis $\underline{\underline{I_p}} = \begin{bmatrix} I_u & 0 \\ 0 & I_v \end{bmatrix}$. Note, $I_{uv} = 0$.
3. Superposition: $\sigma(u, v) = -\frac{M_v u}{I_v} + \frac{M_u v}{I_u}$ on an element at (u, v). Since we can observe the direction of the moment and find a point $A : (u_A, v_A)$ that is farthest away from the neutral axis and causes each term in each direction to be of the same sign. Then, the max bending stress will be given by $|\sigma_{\max}| = \sigma(u_A, v_A)$. If unsure, you can try multiple points (u_i, v_i) and take $\max\{\sigma(u_i, v_i)\}$.

2.2 Eccentric Loading

Using superposition

$$\sigma_x(y, z) = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}.$$

3 Shear Stresses in Beams and Thin-Walled Members

Consider the case when M not constant, $V = \frac{dM}{dx} \neq 0$. This is not pure bending since there is a varying bending moment.

- Shear stresses: $\tau = \frac{V}{A}$ is actually wrong
- Shear Stress Formula:

What induces shear stresses?

...

Most importantly: Always ask what is the shear plane? Visualize shear action, and calculate shear area.

3.1 Shear Flow

- q is shear flow
- Q is first moment of area, units m^3 of some partial area of cross-section (dependent on question) about centroid

I would like to emphasize that Q is for PART of the cross-sectional area.

- I is the moment of inertia calculated given by

  $$\frac{bh^3}{12}$$

  and Ad^2.

For a beam in bending, there is shear flow.

I think we've just been ignoring it.

Steps for shear flow problem:

1. Find moment of Inertia of entire crosssection
2. Identify the area that you will find Q for. I think the area depends on what the nail of interest is in.
3.