MECH 360 Notes

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1 Transformation of Stress

Given the 3-D stress tensor

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yy} & \sigma_{yx} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}, \tag{1.0.1}$$

the principal stress tensor is given by

$$\underline{\underline{\sigma_p}} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}. \tag{1.0.2}$$

Invarient Properties:

- $\text{trace}(\underline{\underline{\sigma}}) = \text{trace}(\underline{\underline{\sigma_p}})$
- $\det(\underline{\underline{\sigma}}) = \det(\underline{\underline{\sigma_p}})$

2 Pure Bending

2.1 Unsymmetric Bending Analysis

Main Ideas:

1. Moment is a vector - resolving vectors
2. "Principal" Centroidal axes - Mohr's circle
3. Superposition - adding stresses

This leads two showing how one unsymmetric bending problem is two symmetric bending problems.

Principal Centroidal Axes: Where the product of inertia is zero.

What is Product of Inertia?
Product of Inertia is a measure of body symmetry.
When you talk about symmetry, we talk about planes of symmetry.
If a body is symmetrical about a plane, let's say xy-plane, then

$$I_{xy} = 0$$

.
Recall,

$$I_{xy} = \int xydA,$$

If

$$I_{zy} = 0, I_y \neq 0, I_z \neq 0,$$

then y and z are the principal centroidal axes (PCA). PCA's are mutually orthogonal. Finding principal planes is analogous to finding principal stresses on the Mohr's circle.

The area moment of inertia is a tensor too. We apply the Mohr's transformations to find PCA.

Any given section possess *principal centroidal axes* even if it is unsymmetric. Principal centroidal axes can be determined:

1. analytically
2. or using Mohr's circle.

If $\mathbf{M}$ is along the principal centroidal axis, the N.A. will be along the axis of $\mathbf{M}$, then the equations for symmetric members can be used to compute the stresses. The principal of superposition is used to determine stresses in general for unsymmetric cases.

Given some a couple momment $\mathbf{M}$, we have

$$M_z = M\cos\theta, \quad M_y = M\sin\theta,$$

then using superposition,

$$\sigma_x(y,z) = \frac{-M_z y}{I_z} + \frac{+M_y z}{I_y}.$$

Points along the N.A. have no stress, thus let $\sigma_x = 0$, and using $M_z = M\cos\theta, \quad M_y = M\sin\theta$, we get

$$y = \underbrace{\left(\frac{I_z}{I_y}\tan\theta\right)}_{a} z,$$

representing a line $y(z)$ with slope a. Letting ϕ be the angle between the N.A. and the z-axis gives

$$\tan\phi = \frac{I_z}{I_y}\tan\theta.$$

'' Note, ϕ depends on the loading axis. Also,

$$\tan\phi = \frac{y_n}{z_n},$$

where (y_n, z_n) is some point on the NA.

Solving Steps: If $\underline{M}$ is not in the plane of symmetry, we have an unsymmetric bending problem.

1. Find PCA - (using Mohr)
2. Resolve moment vector about PCA
3. Find/apply $\sigma = \frac{-My}{i}$ for each component
4. Superposition - (assuming linearly elastic material)

Steps solving an example:

1. PCA: found by inspection
2. Decompose moment into cartesian components
3. Stress due to asymmetric bending is the sum of the stress due to each moment component

2.2 Eccentric Loading

Using superposition

$$\sigma_x(y,z) = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}.$$