

# MECH 360 Notes

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# 1 Pure Bending

## 1.1 Unsymmetric Bending Analysis

### Main Ideas:

1. Moment is a vector - resolving vectors
2. "Principal" Centroidal axes - Mohr's circle
3. Superposition - adding stresses

This leads to showing how one unsymmetric bending problem is two symmetric bending problems.

**Principal Centroidal Axes:** Where the product of inertia is zero.

### What is Product of Inertia?

Product of Inertia is a measure of body symmetry.

When you talk about symmetry, we talk about planes of symmetry.

If a body is symmetrical about a plane, let's say xy-plane, then

$$I_{xy} = 0$$

Recall,

$$I_{xy} = \int xy dA,$$

If

$$I_{zy} = 0, I_y \neq 0, I_z \neq 0,$$

then  $y$  and  $z$  are the principal centroidal axes (PCA). PCA's are mutually orthogonal. Finding principal planes is analogous to finding principal stresses on the Mohr's circle.

The area moment of inertia is a tensor too. We apply the Mohr's transformations to find PCA.

Any given section possess *principal centroidal axes* even if it is unsymmetric. Principal centroidal axes can be determined:

1. analytically
2. or using Mohr's circle.

If  $\mathbf{M}$  is along the principal centroidal axis, the N.A. will be along the axis of  $\mathbf{M}$ , then the equations for symmetric members can be used to compute the stresses. The principle of superposition is used to determine stresses in general for unsymmetric cases.

Given some a couple moment  $\mathbf{M}$ , we have

$$M_z = M \cos \theta, \quad M_y = M \sin \theta,$$

then using superposition,

$$\sigma_x(y, z) = \frac{-M_z y}{I_z} + \frac{+M_y z}{I_y}.$$

Points along the N.A. have no stress, thus let  $\sigma_x = 0$ , and using  $M_z = M \cos \theta$ ,  $M_y = M \sin \theta$ , we get

$$y = \underbrace{\left( \frac{I_z}{I_y} \tan \theta \right)}_a z,$$

representing a line  $y(z)$  with slope  $a$ . Letting  $\phi$  be the angle between the N.A. and the z-axis gives

$$\tan \phi = \frac{I_z}{I_y} \tan \theta.$$

Note,  $\phi$  depends on the loading axis. Also,

$$\tan \phi = \frac{y_n}{z_n},$$

where  $(y_n, z_n)$  is some point on the NA.

**Solving Steps:** If  $\underline{M}$  is not in the plane of symmetry, we have an unsymmetric bending problem.

1. Find PCA - (using Mohr)
2. Resolve moment vector about PCA
3. Find/apply  $\sigma = \frac{-My}{I}$  for each component
4. Superposition - (assuming linearly elastic material)

**Steps solving an example:**

1. PCA: found by inspection
2. Decompose moment into cartesian components
3. Stress due to asymmetric bending is the sum of the stress due to each moment component

## 1.2 Eccentric Loading

Using superposition

$$\sigma_x(y, z) = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}.$$