GLOBAL EDGE

Intelligence Of Things™

Big O and Complexity

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- Calculation of time complexity
  - Non-recursive function
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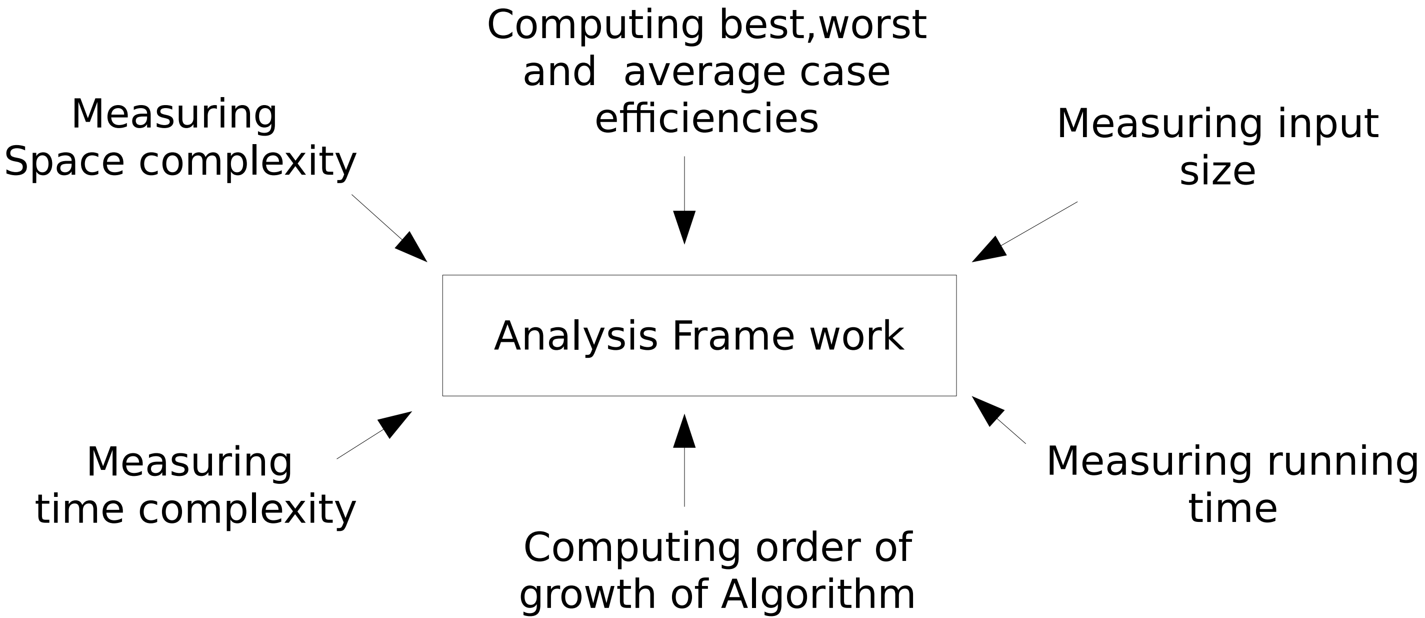
Analysis framework



Order of Growth

Value	logn	n	nlogn	n^2	n^3	2^n	n!
1	0	1	0	1	1	2	1
2	1	2	2	4	8	4	2
4	2	4	8	16	64	16	24
8	3	8	24	64	512	256	40320
16	4	16	64	256	4096	65536	2.6*10^48
10	3.2	10	3.3*10	10^2	10^3	10^3	3.6*10^6
10^2	6.6	10^2	6.6*10^2	10^4	10^6	1.3*10^30	9.3*10^157
10^3	10	10^3	1.0*10^4	10^6	10^9	high	high

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Asymptotic Notation

What is Asymptotic Notation ?

- Asymptotic Notations of a function is the study of how the value of the function varies for large values of n, where n is the size of input.

- Using Asymptotic notation we could easily find the efficiency of the algorithm.

Types of Asymptotic notation

Three different type of asymptotic notation

- O (Big Oh)
- Ω (Big Omega)
- Θ (Big Theta)

O (Big Oh)

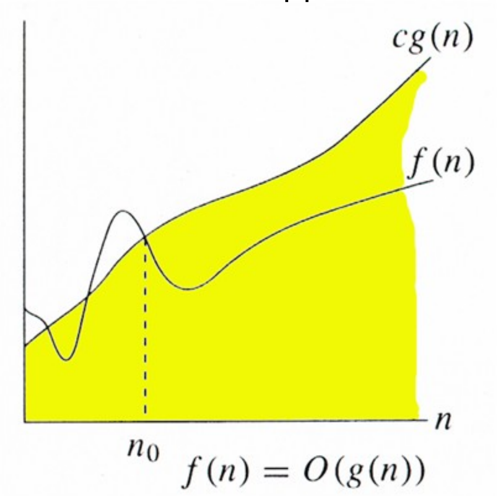
- Upper bound of algorithm's running time.
- Worst case running time for an algorithm
- Longest amount of time the algorithm takes to compute.

$$f(n) \leq c*g(n) \text{ for all } n \geq n0.$$

or

$$f(n) \in O(g(n))$$

Upper bound



Examples for Big- O

1. fn() = 100n + 5

   Let c * g(n) = 100n + n (replacing constant 5 with n)

   ie c * g(n) = 101n

   since f(n) <= c * g(n)

   100n + 5 <= 101n for all n >= 5

   so c = 101   g(n) = n   n0 = 5

   Hence $f(n) \leq c * g(n)$ for all $n \geq n0$

   $f(n) \in O(g(n))$

Cont....

1. $f(n) = 3n + 2$

   Ans: $3n + 2 <= 4n$

2. $f(n) = 10n^3 + 8$

   Ans: $10n^3 <= 11n^3$

3. $f(n) = 10n^2 + 4n + 2$

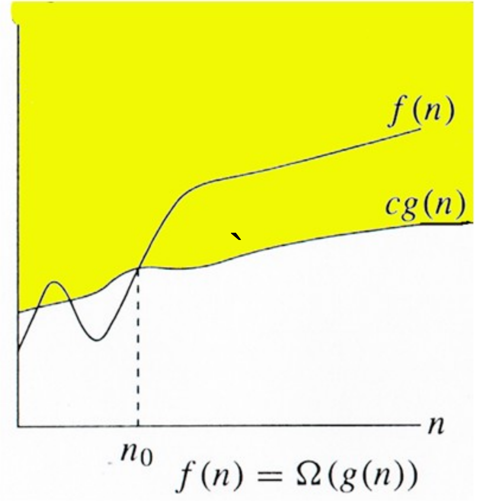
   Ans: $10n^2 + 4n + 2 <= 11n^2$

Ω (Big Omega)

- Lower bound of algorithm's running time.
- Best case running time for an algorithm.
- Gives minimum amount of time the algorithm takes to compute.

f(n) > = c * g(n) for all n >= n0

f(n) Ω (g(n))



Examples for Big- Ω

1. fn() = 100n + 5

Let c * g(n) = 100n (Discarding constant 5)

ie c * g(n) = 100n

since f(n) >= c * g(n)

100n + 5 >= 100n for all n >= 0

so c = 100   g(n) = n   n0 = 0

Hence f(n) >=   c * g(n) for all n ≥ n0

f(n) Є Ω(g(n))

Cont....

1. $f(n) = 50n + 6$

   Ans: $50n$

2. $f(n) = 17n^3 + 5$

   Ans: $17n^3$

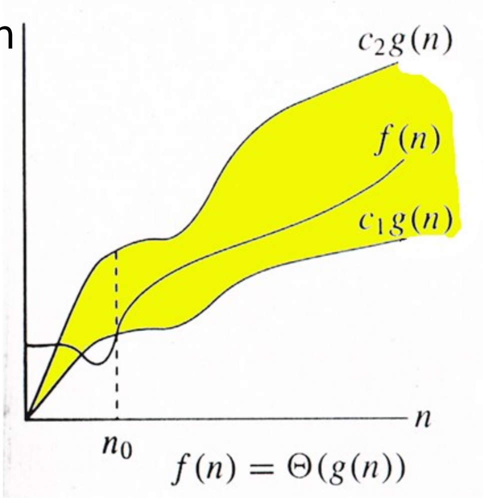
Θ (Big Theta)

- Gives average amount of time the algorithm takes to compute.

$$c1 * g(n) <= f(n) <= c2 * g(n)$$

$$\text{for all } n >= n0$$

f(n) Θ (g(n))



Examples for Big- Θ

1. fn() = 10n^3 + 5

   Let c2 * g(n) = 10n^3 + n (replacing constant 5 with n)

   ie c2 * g(n) = 11n^3

   Let c1 * g(n) = 10n^3

   since c1 * g(n) <= f(n) <= c2 * g(n)

   10n^3 <= 10n^3 +5 <= 11n^3

- so c1 = 10n^3, c2 = 11n^3, g(n) = n^3, n0 = 2

f(n) ∈ Θ(g(n))

General plan for Analyzing the Time Efficiency of Non-recursive Algorithm

1. Based on input determine the number of parameters to be considered.

2. Identify the algorithms **basic operation**.

3. Check the basic operation depends only on the size of the input.

4. Obtain the total number times to basic operation is executed.

5. An useful formula to compute is $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Matrix multiplication

```
for i ← 0 to n - 1 do
    for j ← 0 to n - 1 do
        c[j] ← 0
        for k ← 0 to n - 1 do
            c[i,j] ← c[i,j] + A[i,k] * B[k,j]
```

Basic operation

$T(n) = \Theta(n^3)$

General plan for Analyzing the Time Efficiency of recursive Algorithm

1. Based on input determine the number of parameters to be considered.

2. Identify the algorithms ***basic operation***.

3. Check the basic operation depends only on the size of the input.

4. Set up a ***recurrence relation*** with an appropriate ***initial condition***, for number of times the basic operation is executed.

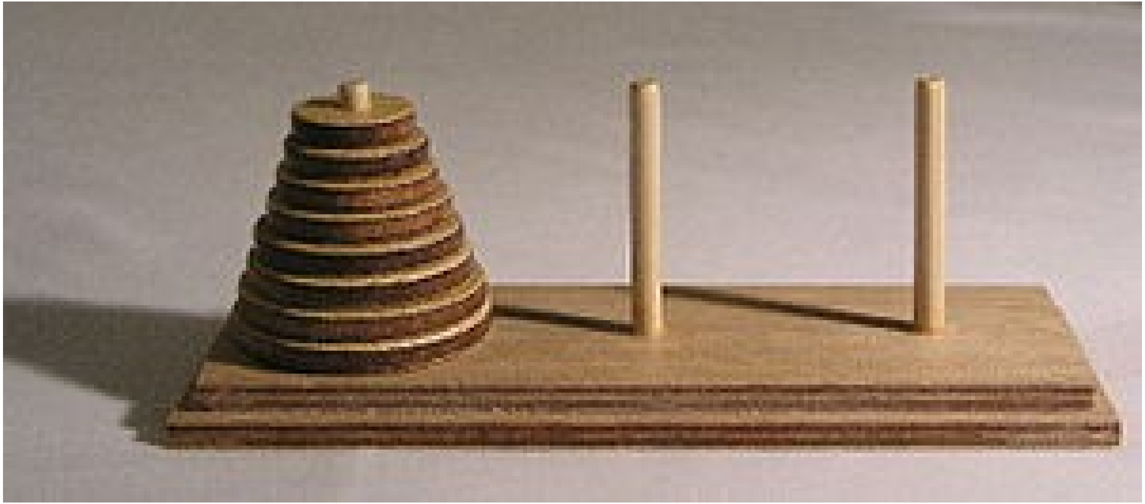
Example: 1

Recursive algorithm to find factorial of a number

```
Factorial(int n){
    if (n == 0)
            return 1;
    return (Factorial(n – 1) * n);
}
```

Example: 2

Tower of Hanoi



Algorithm

```
    Tower_honai(n, source, temp, dest)

Step 1:   If( n == 1)
          move(n'st, source, dest)
          return

Step 2:   Tower_honai(n - 1, source, dest, temp)

Step 3:   move(nth, source, dest)

Step 3:   Tower_honai(n - 1, temp, dest, source)
```

The Algorithm - details

```
BubbleSort(A[0..n-1])
        for i  <- 0  to n–2 do
            for j  <- 0 to n-2-i do
                if A[j+1] < A[j]
                swap A[j] and A[j+1]
```

The Algorithm - details

```
SelectionSort(A[0..n-1])

for i  <- 0  to n–2 do
      min <- i
     for j  <- i+1 to n-1 do
         if A[j] > A[min]  min <- j
         swap A[i] and A[min]
```

The Algorithm - details

```
InsertionSort(A[0..n-1])
for i  <- 0  to n–2 do {
    v <- a[i];
    j<- i – 1;
    while( j >= 0 and A[j] > v) do{
        A[j+1] <- A[j];
        j<- j – 1;
    }
    A[j+1] <- v;
}
```

Merge sort

```
mergesort (A , p , r )        - T(n)
     if( p > r)   return
    q = (p+r)/2
    mergesort(B , p , q)  – T(n/2)
    mergesort(C , q+1 , r)– T(n/2)
    merge(A, B, C)          – (n)
```

Efficiency

$$T(n) = T(n/2) + T(n/2) + (n)$$

$$T(n) = 2T(n/2) + (n)$$

$$T(n) = (n \text{ Log } (n))$$

Comparison between various sorting algorithms

Sorting Algorithm	Best	Average	worst
Bubble sort	Ω(n^2)	θ(n^2)	O(n^2)
Insertion sort	Ω(n)	θ(n^2)	O(n^2)
Selection sort	Ω(n^2)	θ(n^2)	O(n^2)
Heap sort	Ω(n*log(n))	θ(n*log(n))	O(n*log(n))
Merge sort	Ω(n*log(n))	θ(n*log(n))	O(n*log(n))
Quick sort	Ω(n*log(n))	θ(n*log(n))	O(n^2)

References

- The Design and Analysis of Algorithms by ***Anany Levitin***

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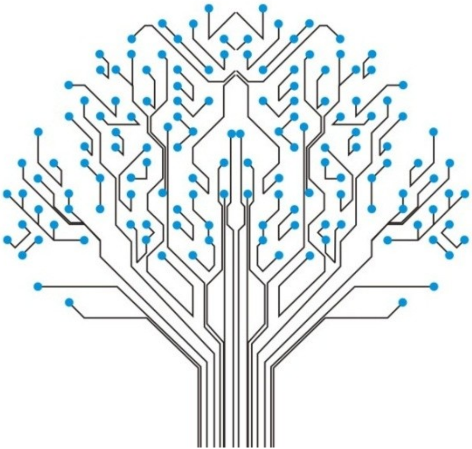
Thank you



Fairness

Learning

Responsibility

Innovation

Respect