n = 100 $sam_mean = 52.80$ # Significance value alpha=0.05 #standard Error  $SE = pop_std/n**0.5$  $Z = (sam_mean - pop_mean)/SE$ print("Z score is:",Z) print("Critical region is ", norm.ppf(alpha/2), -norm.ppf(alpha/2)) print("\n Z score is less than critical value so we accept null hypothesis") Z score is: 1.7777777777715 Critical region is -1.9599639845400545 1.9599639845400545 Z score is less than critical value so we accept null hypothesis **Problem Statement 3:** A certain chemical pollutant in the Genesee River has been constant for several years with mean  $\mu$  = 34 ppm (parts per million) and standard deviation  $\sigma$  = 8 ppm. A group of factory representatives whose companies discharge liquids into the river is now claiming that they have lowered the average with improved filtration devices. A group of environmentalists will test to see if this is true at the 1% level of significance. Assume \ that their sample of size 50 gives a mean of 32.5 ppm. Perform a hypothesis test at the 1% level of significance and state your decision In [7]: pop\_mean = 34  $pop_std = 8$ n = 50 $sam_mean = 32.5$ # Significance value alpha=0.01 #standard Error  $SE = pop_std/n**0.5$  $Z = (sam_mean - pop_mean)/SE$ print("Z score is: ",Z) print("Critical region is ", norm.ppf(alpha/2), -norm.ppf(alpha/2)) print("\n Z score is less than critical value so we accept null hypothesis") Z score is: -1.3258252147247767 Critical region is -2.575829303548901 2.575829303548901 Z score is less than critical value so we accept null hypothesis **Problem Statement 4:** Based on population figures and other general information on the U.S. population, suppose it has been estimated that, on average, a family of four in the U.S. spends about \$1135 annually on dental expenditures. Suppose further that a regional dental association wants to test to determine if this figure is accurate for their area of country. To test this, 22 families of 4 are randomly selected from the population in that area of the country and a log is kept of the family's dental expenditure for one year. The resulting data are given below. Assuming, that dental expenditure is normally distributed in the population, use the data and an alpha of 0.5 to test the dental association's hypothesis. 1008, 812, 1117, 1323, 1308, 1415, 831, 1021, 1287, 851, 930, 730, 699, 872, 913, 944, 954, 987, 1695, 995, 1003, 994 In [8]: data=[1008, 812, 1117, 1323, 1308, 1415, 831, 1021, 1287, 851, 930, 730, 699, 872, 913, 944, 954, 987, 1695, 995, 1003, 994] pop\_mean =1135 sam\_std = np.std(data)  $sam_mean = np.sum(data,axis=0)/len(data)$  $SE = sam_std/n**0.5$ alpha = 0.5test = (sam\_mean-pop\_mean)/SE print("t\_Score is", test) print("Critical Region is ", stats.t.ppf((alpha/2), df=21), stats.t.ppf(1-(alpha/2), df=21)) print("\n t score is greater than critical value so we reject null hypothesis") t\_Score is -2.070747228595759 Critical Region is -0.6863519891164291 0.6863519891164291 t score is greater than critical value so we reject null hypothesis **Problem Statement 5:** In a report prepared by the Economic Research Department of a major bank the Department manager maintains that the average annual family income on Metropolis is 48,432. What doyou conclude about the validity of the report if a random sample of 400 families shows and average of the report in the report of the reporwith a standard deviation of 2000? In [11]: pop\_mean = 48432  $pop_std = 2000$ n = 400 $sam_mean = 48574$  $SE = pop_std/n**0.5$  $Z = (sam_mean - pop_mean)/SE$ alpha=0.05 print("Z score is: ",Z) print("Critical region is ", norm.ppf(alpha/2), -norm.ppf(alpha/2)) print("\n Z score is less than critical value so we accept null hypothesis") Z score is: 1.42 Critical region is -1.9599639845400545 1.9599639845400545 Z score is less than critical value so we accept null hypothesis **Problem Statement 6:** Suppose that in past years the average price per square foot for warehouses in the United States has been 32.28. A national real estate investor wants to determine whether that figure has changed now. The investor hires a researcher who randomly samples 19 warehouses that are for sale across the United States and finds that the mean price per square foot is 31.67, with a standard deviation of 1.29. assume that the prices of warehouse footage are normally distributed in population. If the researcher uses a 5% level of significance, what statistical conclusion can be reached? What are the hypotheses? In [16]: pop\_mean =32.28 n=19  $sam_mean = 31.67$  $sam_std = 1.29$ alpha = 0.05 $SE=sam_std/(n**0.5)$ t=(sam\_mean-pop\_mean)/SE print("Null hypothesis:  $H0:\mu = 32.28$ ") print("Alternative hypothesis: H1:μ ≠ 32.28") print("t\_score is", round((t),1)) print("Critical region is ", round(stats.t.ppf((alpha/2), df=18), 1) , -round(stats.t.ppf((alpha/2), df=18 a/2), df=18), 1))print("\n t score is within critical value so we accept null hypothesis") Null hypothesis:  $H0:\mu = 32.28$ Alternative hypothesis:  $H1:\mu \neq 32.28$ t\_score is -2.1 Critical region is -2.1 2.1 t score is within critical value so we accept null hypothesis **Problem Statement 7:** Fill in the blank spaces in the table and draw your conclusions from it. **image.png** In [25]: # Calculate Beta at Mu1 = 52 n = 10Sig = 2.5Mu1 = 52a = (48.5 - Mu1)/(Sig/math.sqrt(n))b = (51.5 - Mu1)/(Sig/math.sqrt(n))# As b > a, so our z score lies in between these two b < z < a. # probability at these z score P11 = 0P12 = 0.2643Beta11 = P12 - P11print("Beta at Mu1 = 52 is : ", Beta11) # Calculate Beta at Mu2 = 50.5 n1 = 10Sig = 2.5Mu2 = 50.5c = (48.5 - Mu2)/(Sig/math.sqrt(n1))d = (51.5 - Mu2)/(Sig/math.sqrt(n1))# z score lies in between these two c < z < d. # probability at these z score P13 = 0.0057P14 = 0.8962# now Beta = p13 + (1 - p14)Beta12 = P13 +(1 - P14)print("Beta at Mu2 = 50.5 is : ", Beta12)Beta at Mu1 = 52 is : 0.2643 Beta at Mu2 = 50.5 is : 0.1095 In [26]: # Calculate Beta at Mu1 = 52 n2 = 10Sig = 2.5Mu1 = 52a = (48.0 - Mu1)/(Sig/math.sqrt(n2))b = (51.0 - Mu1)/(Sig/math.sqrt(n2))# As p22 > p21, so our z score lies in between these two b < z < a. # Find probability at these z score P21 = 0P22 = 0.1038# now Beta = p22 - p21Beta21 = P22 - P21print("Beta at Mu2 = 52 is : ", Beta21) # Calculate Beta at Mu2 = 50.5 n2 = 10Sig = 2.5Mu2 = 50.5c = (48 - Mu2)/(Sig/math.sqrt(n2))d = (51 - Mu2)/(Sig/math.sqrt(n2))# so our z score lies in between these two c < z < d. # Find probability at these z score P23 = 0.0008P24 = 0.7357# now Beta = p13 + (1 - p14)Beta22 = P23 + (1 - P24) $print(f"Beta at Mu2 = 50.5 is : {Beta22}")$ Beta at Mu2 = 52 is : 0.1038 Beta at Mu2 = 50.5 is : 0.2651 In [27]: # Calculate Beta at Mu1 = 52 n3 = 16Sig = 2.5Mu1 = 52a = (48.81 - Mu1)/(Sig/math.sqrt(n3))b = (51.9 - Mu1)/(Sig/math.sqrt(n3))# so our z score lies in between these two a < z < b. # Find probability at these z score P31 = 0.4364P32 = 0# now Beta = p31 - p32 Beta31 = P31 - P32print("Beta at Mu2 = 52 is : ", Beta31) # Calculate Beta at Mu2 = 50.5 n3 = 16Sig = 2.5Mu2 = 50.5c = (48.81 - Mu2)/(Sig/math.sqrt(n3))d = (51.9 - Mu2)/(Sig/math.sqrt(n3))# so our z score lies in between these two c < z < 1- d. # Find probability at these z score P33 = 0.0032P34 = 0.9875# now Beta = P33 + 1- P34 Beta32 = P33 + 1 - P34print("Beta at Mu2 = 50.5 is : ", Beta32)Beta at Mu2 = 52 is : 0.4364 Beta at Mu2 = 50.5 is : 0.0157000000000000047In [28]: # Calculate Beta at Mu1 = 52 n4 = 16Sig = 2.5Mu1 = 52a = (48.42 - Mu1)/(Sig/math.sqrt(n4))b = (51.58 - Mu1)/(Sig/math.sqrt(n4))# As P42> P41, so our z score lies in between these two b < z < a. # Find probability at these z score P41 = 0.0P42 = 0.2514# now Beta = p31 - p32Beta41 = P42 - P41 $print(f"Beta at Mu2 = 52 is : {Beta41}")$ # Calculate Beta at Mu2 = 50.5 n4 = 16Sig = 2.5Mu2 = 50.5c = (48.42 - Mu2)/(Sig/math.sqrt(n4))d = (51.58 - Mu2)/(Sig/math.sqrt(n4))# so our z score lies in between these two c < z < 1- d. # Find probability at these z score P43 = 0.0035P44 = 0.9875# now Beta = P43 + 1- P44 Beta42 = P43 + (1 - P44)print("Beta at Mu2 = 50.5 is : ", Beta42)Beta at Mu2 = 52 is : 0.2514 Beta at Mu2 = 50.5 is : 0.01599999999999955**Problem Statement 8:** Find the t-score for a sample size of 16 taken from a population with mean 10 when the sample mean is 12 and the sample standard deviation is 1.5. In [30]: n = 16 $pop_mean = 10$ sample\_mean =12 sample\_std =1.5  $SE = sample_std/(n**0.5)$ t = (sample\_mean-pop\_mean)/SE print("t\_score is ",round((t),1)) t\_score is 5.3 **Problem Statement 9:** Find the t-score below which we can expect 99% of sample means will fall if samples of size 16 are taken from a normally distributed population. In [31]: n=16 alpha=(1-0.99)/2print("t\_score is ", stats.t.ppf(1-alpha,df=15)) t\_score is 2.946712883338615 **Problem Statement 10:** If a random sample of size 25 drawn from a normal population gives a mean of 60 and a standard deviation of 4, find the range of t-scores where we can expect to find the middle 95% of all sample means. Compute the probability that (-t0.05 <*t*<*t*0.10). In [4]: n=25 std=4 mean=60 alpha=(1-0.95)/2t\_score=stats.t.ppf(1-alpha,df=24) print("Range is: ", mean+t\_score\*(std/(n\*\*0.5)), mean-t\_score\*(std/(n\*\*0.5))) Range is : 61.651118849302414 58.348881150697586 In [33]: p=stats.t.cdf(0.1, df=24)-stats.t.cdf(-0.05, df=24)print("probability that (-t0.05 < t < t0.10) is ",p) probability that (-t0.05 < t < t0.10) is 0.05914441613731247 **Problem Statement 11:** Two-tailed test for difference between two population means is there evidence to conclude that the number of people travelling from Bangalore to Chennai is different from the number of people travelling from Bangalore to Hosur in a week, given the following: Population 1: Bangalore to Chennai  $n1 = 1200 \times 1 = 452 \times 1 = 212$  Population 2: Bangalore to Hosur  $n2 = 800 \times 2 = 1200 \times 1 = 1200$ 523 s2 = 185In [15]: n1 = 1200x1 = 452s1 = 212n2 = 800x2 = 523s2 = 185std1=s1\*\*2 std2=s2\*\*2 alpha=0.05 se=((std1/n1)+(std2/n2))\*\*0.5 $z_score=(x1-x2)/se$ print("Z\_Score is ",z\_score) print("Critical region is ", norm.ppf(alpha/2), -norm.ppf(alpha/2)) print("\nFrom above we can conclude that we reject null hypothesis since it lies within crit ical region at alpha=5%") Z\_Score is -7.926428526759299 Critical region is -1.9599639845400545 1.9599639845400545 From above we can conclude that we reject null hypothesis since it lies within critical regio n at alpha=5% **Problem Statement 12:** Is there evidence to conclude that the number of people preferring Duracell battery is different from the number of people preferring Energizer battery, given the following: Population 1: Duracell  $n1 = 100 \times 1 = 308 \times 1 = 84$  Population 2: Energizer n2 $= 100 \times 2 = 254 \text{ s}2 = 67$ In [20]: n1 = 100 x1 = 308s1 = 84n2 = 100x2 = 254s2 = 67s\_1=s1\*\*2 s\_2=s2\*\*2 alpha=0.05  $SE=((s_1/n1)+(s_2/n2))**0.5$  $z_score=(x1-x2)/SE$ print("Z\_Score is ",z\_score) print("Critical region is ",norm.ppf(alpha/2),-norm.ppf(alpha/2)) print("\nFrom above we can conclude that we reject null hypothesis since it lies within crit ical region.So, number of people preferring Duracell battery is different from the number of people preferring Energizer battery.") Z\_Score is 5.025702668336442 Critical region is -1.9599639845400545 1.9599639845400545 From above we can conclude that we reject null hypothesis since it lies within critical regio n.So, number of people preferring Duracell battery is different from the number of people pre ferring Energizer battery. **Problem Statement 13:** Pooled estimate of the population variance Does the data provide sufficient evidence to conclude that average percentage increase in the price of sugar differs when it is sold at two different prices? Population 1: Price of sugar = Rs. 27.50 n1 =  $14 \times 1$ = 0.317% s1 = 0.12% Population 2: Price of sugar = Rs. 20.00 n2 = 9 x2 = 0.21% s2 = 0.11%In [19]: | n1 = 14 x1 = 0.317s1 = 0.12n2 = 9x2 = 0.21s2 = 0.11s\_1=s1\*\*2 s\_2=s2\*\*2  $s=((n1-1)*s_1)+((n2-1)*s_2)$ n=(n1+n2-2)se=(s/n)\*\*0.5 $n_1=((1/n1)+(1/n2))**0.5$  $t_score=(x1-x2)/se*n_1$ print("t\_score is ",t\_score) print("Critical Region is ", stats.t.ppf(1-0.05, df=n)) print("\n We accept null hypothesis at alpha=5%. So, average price do not increase") t\_score is 0.3931089218182991 Critical Region is 1.7207429028118775 We accept null hypothesis at alpha=5%. So, average price do not increase **Problem Statement 14:** The manufacturers of compact disk players want to test whether a small price reduction is enough to increase sales of their product. Is there evidence that the small price reduction is enough to increase sales of compact disk players? Population 1: Before reduction  $n1 = 15 \times 1 = Rs$ . 6598 s1 = Rs. 844 Population 2: After reduction  $n2 = 12 \times 2 = Rs$ . 6870 s2 = Rs. 669 In [21]: n1 = 15x1 = 6598s1 = 844n2 = 12x2 = 6870s2 = 669s\_1=s1\*\*2 s\_2=s2\*\*2  $s=((n1-1)*s_1)+((n2-1)*s_2)$ n=(n1+n2-2)se=(s/n)\*\*0.5 $n_1=((1/n1)+(1/n2))**0.5$  $t_score=(x1-x2)/se*n_1$ print("t\_score is ",t\_score) print("Critical Region is ", stats.t.ppf(0.05, df=n)) print("\nWe accept null hypothesis at alpha=5%.So average price remains same") t\_score is -0.1364745051598569 Critical Region is -1.708140761251899 We accept null hypothesis at alpha=5%. So average price remains same **Problem Statement 15:** Comparisons of two population proportions when the hypothesized difference is zero Carry out a two-tailed test of the equality of banks' share of the car loan market in 1980 and 1995. Population 1: 1980 n1 = 1000 x1 = 53 p 1 = 0.53 Population 2: 1985  $n2 = 100 \times 2 = 43 p = 0.53$ In [23]: n1 = 1000x1 = 53p1 = 0.53n2 = 100x2 = 43p2 = 0.53p=(x1+x2)/(n1+n2)n=(1/n1)+(1/n2) $p_1=p^*(1-p)$  $Z=(p1-p2)/((p_1*n)**0.5)$ print("Z\_score is ",Z) print("Critical region is ",norm.ppf(0.05)) print("\n We reject null hypothesis as Z score is above critical region.") Z\_score is 0.0 Critical region is -1.6448536269514729 We reject null hypothesis as Z score is above critical region. **Problem Statement 16:** Carry out a one-tailed test to determine whether the population proportion of traveler's check buyers who buy at least \$2500 in checks when sweepstakes prizes are offered as at least 10% higher than the proportion of such buyers when no sweepstakes are on. Population 1: With sweepstakes  $n1 = 300 \times 1 = 120 p = 0.40$  Population 2: No sweepstakes  $n2 = 700 \times 2 = 140 p = 0.40$ 0.20 In [25]: n1 = 300x1 = 120p1 = 0.40n2 = 700x2 = 140p2 = 0.20p=(x1+x2)/(n1+n2)n=(1/n1)+(1/n2) $p_1=p^*(1-p)$  $Z=(p1-p2-0.1)/((p_1*n)**0.5)$ print("Z\_score is", Z) print("Critical region is ", -norm.ppf(0.05)) print("\nWe reject null hypothesis at alpha=5%") Z\_score is 3.303749523611152 Critical region is 1.6448536269514729 We reject null hypothesis at alpha=5% **Problem Statement 17:** A die is thrown 132 times with the following results: Number turned up: 1, 2, 3, 4, 5, 6 Frequency: 16, 20, 25, 14, 29, 28 Is the die unbiased? Consider the degrees of freedom as p - 1. In [29]: obs= [16, 20, 25, 14, 29, 28] exp= [22,22,22,22,22] result=stats.chisquare(obs,exp) print("Chi square value is", result[0]) print("p-value is", result[1]) print('Dice is unbiased.') Chi square value is 9.0 p-value is 0.1090641579497725 Dice is unbiased. **Problem Statement 18:** In a certain town, there are about one million eligible voters. A simple random sample of 10,000 eligible voters was chosen to study the relationship between gender and participation in the last election. The results are summarized in the following 2X2 (read two by two) contingency table: **download.png** We would want to check whether being a man or a woman (columns) is independent of having voted in the last election (rows). In other words, is "gender and voting independent"? In [37]: observed\_voted\_men=2792 observed\_voted\_women=3591 observed\_not\_voted\_men=1486 observed\_not\_voted\_women=2131 total\_voted=2792+3591 total\_not\_voted=1486+2131 total\_men=2792+1486 total\_women=3591+2131 expected\_voted\_men=(total\_voted\*total\_men)/10000 expected\_voted\_women=(total\_voted\*total\_women)/10000 expected\_not\_voted\_men=(total\_not\_voted\*total\_men)/10000 expected\_not\_voted\_women=(total\_not\_voted\*total\_women)/10000 chisquare1=(((observed\_voted\_women-expected\_voted\_women)\*\*2)/expected\_voted\_women) chisquare2=(((observed\_voted\_men-expected\_voted\_men)\*\*2)/expected\_voted\_men) chisquare3=(((observed\_not\_voted\_men-expected\_not\_voted\_men)\*\*2)/expected\_not\_voted\_men) chisquare4=(((observed\_not\_voted\_women-expected\_not\_voted\_women)\*\*2)/expected\_not\_voted\_wome chisquare=chisquare1+chisquare2+chisquare3+chisquare4 print("Chi Square value is ", chisquare) print("Critical region with alpha=0.05 is 3.84") print("We reject null hypothesis.It is not gender and voting independent") Chi Square value is 6.660455899328067 Critical region with alpha=0.05 is 3.84 We reject null hypothesis. It is not gender and voting independent **Problem Statement 19:** A sample of 100 voters are asked which of four candidates they would vote for in an election. The number supporting each candidate is given below:image.pngDo the data suggest that all candidates are equally popular? [Chi-Square = 14.96,with 3 df, p 0.05.] **image.png** In [38]: #print(np.mean([41,19,24,16])) obs=[41, 19, 24, 16] exp=[25, 25, 25, 25] result=stats.chisquare(obs,exp) print("Chi Square value is ",result[0]) print("Critical region with 3df and alpha=0.05 is 7.82") print("We reject null hypothesis. All candidates are not equally popular") Critical region with 3df and alpha=0.05 is 7.82 We reject null hypothesis. All candidates are not equally popular **Problem Statement 20:** Children of three ages are asked to indicate their preference for three photographs of adults. Do the data suggest that there is a significant relationship between age and photograph preference? What is wrong with this study? [Chi-Square = 29.6, with 4 df: p < 0.05]. **image.png** In [39]: obs=([[18,22,20],[2,28,40],[20,10,40]]) result=chi2\_contingency(obs) print("Chi Square value is ",result[0]) print("Critical region with 4df and alpha=0.001 is 18.47") print("We reject null hypothesis. There is significant relationship between age and photograp h preference") Chi Square value is 29.603174603174608 Critical region with 4df and alpha=0.001 is 18.47 We reject null hypothesis. There is significant relationship between age and photograph prefer **Problem Statement 21:** A study of conformity using the Asch paradigm involved two conditions: one where one confederate supported the true judgement and another where no confederate gave the correct response. **image.png** Is there a significant difference between the "support" and "no support" conditions in the frequency with which individuals are likely to conform? [Chi-Square = 19.87, with 1 df: p < 0.05] In [40]: | obs=np.array([[18,40],[32,10]]) result=chi2\_contingency(obs) print("Chi Square value is", result[0]) print("Critical region with 1df and alpha=0.001 is 10.83") print("We reject null hypoythesis.So, there is significant difference between the 'support' a nd 'no support' conditions in the frequency with which individuals are likely to conform") Chi Square value is 18.10344827586207 Critical region with 1df and alpha=0.001 is 10.83 We reject null hypoythesis. So, there is significant difference between the 'support' and 'no s upport' conditions in the frequency with which individuals are likely to conform **Problem Statement 22:** We want to test whether short people differ with respect to their leadership qualities (Genghis Khan, Adolf Hitler and Napoleon were all stature-deprived, and how many midget MP's are there?) The following table shows the frequencies with which 43 short people and 52 tall people were categorized as "leaders", "followers" or as "unclassifiable". Is there a relationship between height and leadership qualities? [Chi-Square = 10.71, with 2 df: p < 0.01]. **image.png** In [41]: obs=([[12,32],[22,14],[9,6]]) result=chi2\_contingency(obs) print("Chi Square value is", result[0]) print("Critical region with 2df and alpha=0.001 is 13.82") print("We accept null hypothesis.there is no relationship between height and leadership qual ities") Chi Square value is 10.712198008709638 Critical region with 2df and alpha=0.001 is 13.82 We accept null hypothesis.there is no relationship between height and leadership qualities **Problem Statement 23:** Each respondent in the Current Population Survey of March 1993 was classified as employed, unemployed, or outside the labor force. The results for men in California age 35-44 can be cross-tabulated by marital status, as follows:

**image.png** 

may assume the table results from a simple random sample.)

In [43]: obs = np.array([[679,103,114], [63,10,20],[42,18,25]])

Critical region with 4df and alpha=0.001 is 18.47

print("Critical region with 4df and alpha=0.001 is 18.47")

print("Chi Square value is ",result[0])

Chi Square value is 31.61310319407798

result=chi2\_contingency(obs)

s and employment status")

ployment status

Men of different marital status seem to have different distributions of labor force status. Or is this just chance variation? (you

print("We reject null hypothesis at alpha=0.001, there is relationship between martial statu

We reject null hypothesis at alpha=0.001, there is relationship between martial status and em

**Problem Statement 1:** 

1. H0:  $\mu$  = 25, H1:  $\mu \neq$  25

2. *H*0:  $\sigma$  > 10, *H*1:  $\sigma$  = 10

3. *H*0: x = 50, *H*1:  $x \neq 50$ 

4. H0: p = 0.1, H1: p = 0.5

5. H0: s = 30. H1: s > 30

In [2]: from scipy.stats import norm import numpy as np

import scipy.stats as stats

from scipy.stats import chi2\_contingency

import math

 $pop_mean = 52$  $pop_std = 4.50$ 

**Problem Statement 2:** 

In each of the following situations, state whether it is a correctly stated hypothesis testing problem and why?

No, because null hypothesis has an eqaulity claim and alternate hypothesis has inequality.

No, because hypothesis are always statements about population or distribution and not about sample

The college bookstore tells prospective students that the average cost of its textbooks is Rs. 52 with a standard deviation of Rs. 4.50. A group of smart statistics students thinks that the average cost is higher. To test the bookstore's claim against their alternative, the students will select a random sample of size 100. Assume that the mean from their random sample is Rs.

No, because hypothesis is stated in terms of statistics and not sample data

52.80. Perform a hypothesis test at the 5% level of significance and state your decision.

No, because values in both hypothesis is different and has equal sign.

Yes, because values in both statement are about population or distribution, have equal values and has inequality in