Stat 515 exams 2019-2022

1 Final, Spring 2022

- **Rules. A calculator is allowed.** You may refer to notes written on both sides of a single 8.5×11 inch sheet of paper.

- **Style.** To receive full credit, the reasoning leading to a solution must be clear and complete. A correct answer given without a complete and correct argument will be worth little or no credit.

  Organize your work. Messy and scattered work will receive very little credit.

  There is no need to compute expressions involving binomial or multinomial coefficients or permutations. For example, $\sum_{k=10}^{17} \binom{19}{k} (\frac{1}{4})^k (\frac{3}{4})^{19-k}$ and P_3^5 are acceptable answers. However, please simplify your answers when possible.

- **Extra Paper.** Do not write outside the boxes. If you need more space to write your solutions, get more paper from the proctor. If you decide to use extra paper, please write that you have done so on the front of the exam, please staple the extra sheets to the rest of exam, and please notify the proctor when you hand in the exam.

- A machine produces ball bearings. Assume that the diameters of the bearings produced are independent, identically distributed random variables with mean 2 cm and standard deviation of 0.01 cm.

  5 Use the central limit theorem to estimate the probability that the average diameter of 100 randomly selected bearings exceeds 2.0015 cm.

  5 Use the central limit theorem to estimate an interval centered around 2 cm that includes, with probability 0.95, the average diameter of 100 randomly selected bearings.

5 The weights of a certain population of fish are normally distributed with mean 20 kg and standard deviation 4 kg. Four fish are selected uniformly at random from the population. What is the probability that the sample average of the weights of the four fish will differ from the population mean by more than 2 kg?

5 Let Y be a random variable with the density function

$$f_Y(y) = \begin{cases} 2(1-y) & \text{if } 0 \le y \le 1, \\ 0 & \text{elsewhere.} \end{cases}$$

Let $U = 5Y - 9$. Find the density f_U of U.

10 Let Y have the cumulative distribution function

$$F(y) = \begin{cases} 0 & \text{if } y < 0, \\ 1 - \exp(-y^6) & \text{if } y \ge 0. \end{cases}$$

Let $U \sim \text{uniform}(0,1)$ be uniformly distributed on the interval $(0,1)$. Find a transformation $G : \mathbb{R} \to \mathbb{R}$ so that $G(U)$ has the same distribution function F as Y.

10 Let $X_1, \ldots, X_n$ be independent, identically distributed random variables having the Gamma(α, β) distribution. Recall that the moment generating function of the Gamma(α, β) distribution is

$$m(t) = \frac{1}{(1-\beta t)^\alpha} \text{ for } t < \frac{1}{\beta}.$$

Show that

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim \text{Gamma}\left(n\alpha, \frac{\beta}{n}\right).$$

2 Exam 2, Spring 2022

- **Rules.** No calculator is allowed. You may refer to notes written on both sides of a single 8.5 × 11 inch sheet of paper.

- **Style.** To receive full credit, the reasoning leading to a solution must be clear and complete. A correct answer given without a complete and correct argument will be worth little or no credit.

  Organize your work. Messy and scattered work will receive very little credit.

There is no need to compute expressions involving binomial or multinomial coefficients or permutations. For example, $\sum_{k=10}^{17} \binom{19}{k} \left(\frac{1}{4}\right)^k \left(\frac{3}{4}\right)^{19-k}$ and P_3^5 are acceptable answers. However, please simplify your answers when possible.

- **Extra Paper.** Do not write outside the boxes. If you need more space to write your solutions, get more paper from the proctor. If you decide to use extra paper, please write that you have done so on the front of the exam, please staple the extra sheets to the rest of exam, and please notify the proctor when you hand in the exam.

- Let Y be a continuous random variable with cumulative distribution function

$$F_Y(y) = \begin{cases} 0 & \text{if } y < 0, \\ 1 - \exp(-y^2) & \text{if } y \geq 0. \end{cases}$$

5 What is the density f_Y of Y?

5 What is $\mathbb{P}\left[Y \geq \sqrt{\ln(3)} \middle| \sqrt{\ln(2)} < Y < \sqrt{\ln(4)}\right]$?

5 Let $b > 0$, and define

$$f(y) = \begin{cases} 0 & \text{if } y < b, \\ \frac{C}{y^2} & \text{if } y \geq b. \end{cases}$$

For what value of C is f the density of a random variable?

- Let X be a random variable with moment generating function

$$m_X(t) = \frac{1}{6}\exp(-t) + \frac{2}{3}\exp(3t) + \frac{1}{6}\exp(t).$$

5 What is the probability mass function p_X of X?

5 Calculate $\mathbb{E}[X^3]$.

10 Let X and Y be independent, identically distributed random variables that each have the cumulative distribution function

$$F(z) = \begin{cases} 0 & \text{if } z < 0, \\ 1 - \exp(-z) & \text{if } z \geq 0. \end{cases}$$

Let $M = \min\{X, Y\}$ be the minimum of X and Y. What is the cumulative distribution function F_M of M?

- Let X and Y be jointly continuous random variables. Suppose that X is uniformly distributed on $(0, 1)$. Suppose that for $0 < x < 1$, the conditional probability density of Y given $X = x$ is

$$f(y|x) = \begin{cases} \frac{1}{x} & \text{if } 0 \leq y \leq x, \\ 0 & \text{otherwise.} \end{cases}$$

2 Draw a picture of the region $\{(x,y) \in \mathbb{R}^2 : f(x,y) > 0\}$ in $\mathbb{R}^2$ where the joint density $f(x,y)$ of X and Y is positive.

2 For which values of y is the marginal density f_Y of Y positive?

6 What is the marginal density f_Y of Y?

- Let X and Y be jointly continuous random variables with the density

$$f(x,y) = \begin{cases} 6x^2y & \text{if } 0 \leq x \leq y \text{ and } x+y \leq 2, \\ 0 & \text{elsewhere.} \end{cases}$$

2 Draw a picture of the region $\{(x,y) \in \mathbb{R}^2 : f(x,y) > 0\}$ where $f(x,y)$ is positive.

2 Draw a picture of the region $\{(x,y) \in \mathbb{R}^2 : y-x \geq 1\}$ where $y-x \geq 1$.

3 Set up, but do not evaluate, a double integral for calculating $\mathbb{P}[Y - X \geq 1]$.

3 Set up, but do not evaluate, a double integral for calculating $\mathbb{E}[Y-X]$.

3 Exam 1 Spring 2022

- **Rules.** No calculator is allowed. You may refer to notes written on both sides of a single 8.5×11 inch sheet of paper.

- **Style.** To receive full credit, the reasoning leading to a solution must be clear and complete. A correct answer given without a complete and correct argument will be worth little or no credit.

  Organize your work. Messy and scattered work will receive very little credit.

  There is no need to compute expressions involving binomial or multinomial coefficients or permutations. For example, $\sum_{k=10}^{17} \binom{19}{k}(\frac{1}{4})^k(\frac{3}{4})^{19-k}$ and P_3^5 are acceptable answers. However, please simplify your answers when possible.

- **Extra Paper.** Do not write outside the boxes. If you need more space to write your solutions, get more paper from the proctor. If you decide to use extra paper, please write that you have done so on the front of the exam, please staple the extra sheets to the rest of exam, and please notify the proctor when you hand in the exam.

20 A box contains 4 red balls, 6 yellow balls, and 2 blue balls. You select a sequence of 4 balls uniformly at random without replacement.

6 What is the probability that you select a red ball first, then a yellow ball, then another red ball, and finally a blue ball?

6 Suppose that you will win \$6 if the first ball you select is red and \$12 if the first ball you select is yellow. You will win −\$24 if the first ball you select is blue. (Of course, this is equivalent to losing \$24.) Let Z be the amount of money that you will win. What is $\mathbb{E}[Z]$?

8 What is the probability that you select at least one ball of each color?

10 Show that if A and B are independent events, then A and $\bar{B}$ are independent events.

10 Suppose that C and D are independent events with $\mathbb{P}[C] = \frac{1}{3}$ and $\mathbb{P}[D] = \frac{1}{2}$. What is $\mathbb{P}[C|C \cup D]$?

10 Three companies produce bolts. It is known that 5% of bolts made by company A are defective, 2% of bolts made by company B are defective, and 1% of bolts made by company C are defective. You buy a bolt from a store that gets 50% of its bolts from company A and 25% from each of companies B and C. What is the conditional probability that your bolt was made by company A given that it is not defective?

10 A box of 8 nuts is known to contain 4 defective nuts. You buy 3 randomly selected nuts from the box, and you use them to build a bridge. Let Y be the number of defective nuts among the 3 that you buy.

2 What is the distribution of Y?

8 If some of your nuts are defective, they will fail. The cost to repair the bridge after the failure of the defective nuts is $C = 7Y^2 + Y + 5$. What is $\mathbb{E}[C]$?

20 Suppose that 10% of the applicants for a certain job possess advanced training. Applicants are interviewed sequentially and are selected uniformly at random from a very large pool. Assume for simplicity that applicants are selected from the pool with replacement.

10 What is the probability that the first applicant with advanced training is found within the first three interviews?

10 What is the expected number of applicants who need to be interviewed in order to find the first one with advanced training?

4 Final Fall 2021

Instructions:

- The exam is a total of 100 points.
- You can use your calculator and 1 double-sided cheat sheet with formulas.

- The last page of the exam consists **a table for the z-score**. You can rip it off and use the back for scratch paper.
- You need to provide proper **explanation and details** for your answers to get full credit.

25 Suppose Y is an exponential random variable with mean equal to $1/2$.

  5 What is the CDF $F_Y(y) = P(Y \leq y)$ of the random variable Y?

  5 Consider the random variable $Z = e^Y$. What is the range of Z?

  7.5 Compute the CDF $F_Z(z) = P(Z \leq z)$ of the random variable $Z = e^Y$. (Indicate the range clearly).

  7.5 Compute the PDF $f_Z(z)$ of the the random variable $Z = e^Y$. (Indicate the range clearly).

25 Suppose that Y is a random variable with pdf

$$f_Y(y) = \frac{3}{4}(1 - y^2) \quad -1 \leq y \leq 1\,,$$

and 0 otherwise.

  12.5 Find the PDF $f_Z(z)$ of the random variable $Z = 2Y - 3$. Indicate clearly the range of Z.

  12.5 Find the PDF $f_Z(z)$ of the random variable $Z = Y^2$. Indicate clearly the range of Z.

25 At a certain car insurance company in Massachusetts, the expected yearly claim per customer is \$240 with a standard deviation of \$800. (A "claim" refers here to the amount paid to the customer by the insurance to reimburse some damage.)

  12.5 Each customer is asked to pay an amount of \$270 per year to subscribe to the car insurance. The insurance company has 10,000 customers in Massachusetts. (It is safe to assume that the customers are independent of each other).

  Use the central limit theorem to approximate the probability that the company has to pay more money in claims in Massachusetts than what it receives from its customers?

  12.5 In 2020, the 1,000 customers of the company in Hampshire County had an average claim of $\bar{X} = \frac{1}{1000}(X_1 + \cdots + X_{1000}) = \$252$ while the 2,000 customers of the company in Hampden County had an average claim of $\bar{Y} = \frac{1}{2000}(Y_1 + \cdots + Y_{2000}) = \$238$.
  From these numbers I claim that the drivers in Hampden County drivers are better drivers than the drivers in Hampshire county.
  Use the central limit theorem to justify or disprove this claim. (It is safe to assume that $\overline{X}$ and $\overline{Y}$ are independent samples).

s

25 At the game of craps, a game is won with probability $\frac{244}{495}$ (in which case the casino pays \$1 to the player) and a game is lost with probability $\frac{251}{495}$ (in which case the player pays \$1 to the casino).

5 Suppose that a player plays 100 games of craps and let X denote the number of games that the player wins. What are the mean $E[X]$ and the variance $V[X]$ of the random variable X?

10 Use the central limit theorem to compute, approximately, the probability that the player earn any money after playing 100 games of craps.

10 Use the central limit theorem to compute, approximately, the probability that after 100 games, the player ends up with at least \$10 more in her pocket than when she started.

5 Exam 2, Fall 2021

Instructions:

- The exam is a total of 100 points.
- You can use your calculator and 1 double-sided cheat sheet with formulas.
- The last page of the exam consists **a table for the z-score**. You can rip it off and use the back for scratch paper.
- You need to provide proper **explanation and details** for your answers to get full credit.

20 Consider the random variable Y with CDF

$$F(y) = \begin{cases} 0 & \text{if } y \leq -1 \\ \frac{(y+1)^2}{4} & \text{if } -1 \leq y \leq 1 \\ 1 & \text{if } y \geq 1 \end{cases}.$$

6 Find the median m of Y, i.e. m such that $P(Y \leq m) = \frac{1}{2}$.

7 Compute $P(Y \leq 1/2 | Y > 0)$.

7 Compute $E[Y]$.

20 The lifetime X (measured in years) of the iPhone 27 is a random variable that follows a Gamma distribution with parameters $\alpha = 1$ and $\beta = 3$ (also known as an exponential distribution with parameter $\beta = 3$).

5 What is the expected lifetime $E[X]$ and its variance $V[X]$?

5 Find the probability that the lifetime of your device is at least 2 times its expected lifetime.

5 A nice friend of yours gives you her used iPhone she had for the past year. What is the probability that this device lasts another two years?

5 Suppose you have two iPhones which are assumed to be independent of each other and with lifetime X_1 and X_2, respectively. What is the probability that their combined lifetime $X_1 + X_2$ does not exceed 4 years.

20 The random variables Y_1 and Y_2 have the joint pdf

$$f(y_1, y_2) = \begin{cases} 6(1-y_2) & 0 \leq y_1 \leq y_2 \leq 1 \\ 0 & \text{elsewhere} \end{cases} .$$

7 Find the conditional pdf $f(y_1|y_2)$ and compute $E(Y_1|Y_2 = y_2)$.

7 Compute $E(Y_1 - 3Y_2)$.

6 Compute $\mathrm{Cov}(Y_1, Y_2)$. Are Y_1 and Y_2 positively correlated?

20 Three fair coins are tossed independently and successively. Let X denote the numbers of heads obtained in the three tosses. Let Y denote the winnings on the following side bet. If the first head appear on the first toss, you win \$1, if the first heads appears on the second toss you win \$2, and if the first heads appears on the third toss, you win \$3. If no heads appear you lose \$10 (that is your winning is equal to -10).

5 What are the possible values for X and Y? Make a table for the joint pdf of X and Y.

5 What is the probability that you win \$1 or less and that fewer than three heads are tossed?

5 What is the conditional pdf for $X|Y = 1$, that is of the number of heads given that you won \$1?

5 What is the marginal pdf of your winnings Y and your expected winning $E[Y]$?

20 The height, measured in inches of adult american men is described by a normal random variable with mean $\mu = 69.3$ and $\sigma^2 = 7.84$. **Use the attached table on page 13 for this question.**

5 What is the probability that the height of a randomly chosen man is between 70 and 75 inches?

5 Find an interval of the form $[\mu - a, \mu + a]$ such that 80% of men have a height in this interval.

5 LeBron James measures 6'8" (that is 80 inches). What is the probability that a randomly chosen man is taller than LeBron James?

5 Two randomly and independently chosen men have respective heights Y_1 and Y_2. Compute

2.5 The variance of the average of the heights $V[\frac{Y_1+Y_2}{2}]$.

2.5 The variance of the difference between their heights, $V[Y_1 - Y_2]$.

6 Exam 1, Fall 2021

- **Rules.** No calculator is allowed. You may refer to a single double-sided sheet of notes.

- **Style.** To receive full credit, the reasoning leading to a solution must be clear and complete. A correct answer given without a complete and correct argument will be worth little or no credit.

  Organize your work. Messy and scattered work will receive very little credit.

  There is no need to reduce or compute expressions involving binomial or multinomial coefficients or permutations. For example, $\sum_{k=10}^{17} \binom{19}{k}(\frac{1}{4})^k(\frac{3}{4})^{19-k}$ and P_3^5 are acceptable answers. However, please simplify your answers when possible.

- **Extra Paper.** Do not write outside the boxes. If you need more space to write your solutions, get more paper from the proctor. If you decide to use extra paper, please write that you have done so on the front of the exam, please staple the extra sheets to the rest of exam, and please notify the proctor when you hand in the exam.

20 My house has 2 smoke detectors, detector A and detector B. Smoke in the kitchen will be detected by detector A with probability .9 and by detector B with probability .8. The detectors are assumed to work independently of each other.

5 What is the probability that the smoke is detected by at least one smoke detector?

5 What is the probability that the smoke is detected by exactly one smoke detector?

5 What is the probability that the smoke is detected by detector A given that it was not detected by detector B?

5 What is the probability that the smoke is detected by detector A given that it was detected?

15 A hand of 6 cards is dealt from a regular deck of 52 cards containing 4 suits of 13 cards (clubs, spades, diamonds, hearts).
Note: You can leave your answers in terms of binomial coefficients, etc...

- 5 What is the probability that exactly 5 cards out the 6 cards are spades?
- 5 What is the probability there are 4 cards of one suit and 2 cards of another suit?
- 5 What is the conditional probability that the last three out of the six cards contain no spades given that the first three cards were all spades?

15 There are two cookie jars in my kitchen: Jar A contains 7 ginger snaps and 4 chocolate chip cookies while Jar B contains 8 ginger snaps and 11 chocolate chip cookies. Last night my daughter picked a jar at random and then picked a cookie at random from that jar and brought me a ginger snap. What is the probability that it came from Jar A?

20 The Boston Red Sox are matched against the Chicago Cubs in a series of games. The games are assumed to be independent and the Boston Red Sox will win any single game with probability .55
Note: You can leave your answer in terms of binomial coefficients, etc... no need for a numerical answer

- 5 Find the probability that the Red Sox will win no more than 2 out of the first 6 games.
- 5 What is the expected number of victories for the Red Sox between the 4^{th} game and the 10^{th} game? (both included).
- 5 What is the expected number of games played until Red Sox first win? (The series of games played is considered to be arbitrary long).
- 5 Suppose that the teams are matched in a playoff where games are played until one team accumulates 3 wins (also known as best of 5 playoff). What are the probabilities that
  - 2.5 The Cubs win the playoff in 3 games?
  - 2.5 The Red Sox win the playoff in 5 games?

15 The number of earthquakes in Oklahoma in a given week is described by a Poisson random variable Y with parameter $\lambda = 2$.

- 5 What is the probability that no earthquake will occur next week?
- 5 What is the probability that at least 3 earthquakes occur next week?
- 5 The cost C of the damages due to the earthquakes is estimated to be $C = 5Y + \frac{1}{2}Y^2$. Compute the expected value $E[C]$.

15 If you play a certain game you will win \$1 with probability p=.45 and lose \$1 with probability q=(1-p)=.55. Consider the following strategy: If you win the first game you quit and stop playing. If you lose the first game then you play two more games for a total of three games. Let X denote the total gain you make following this strategy (X negative means you lose money).

4 What are the possible values for the random variable X?

4 Show that with this strategy $P(X > 0) > \frac{1}{2}$

4 Compute the probability distribution of X.

3 Compute $E[X]$

7 Exam 1, Spring 2020

In problems that require reasoning, algebraic calculation, or the use of your graphing calculator, it is not sufficient just to write the answers. You must explain how you arrived at your answers and show all your algebraic calculations.

1. [10 pts] Articles coming through an inspection line are visually inspected by two successive inspectors. When a defective article comes through the inspection line, the probability that it gets by the first inspector is 10%. The second inspector will miss five out of ten of the defective items that get past the first inspector. What is the probability that a defective item gets by both inspectors?
2. [15 pts] Five identical bowls are labeled $1, 2, 3, 4, 5$. Bowl # i contains i white and $5 - i$ red balls, with $i = 1, 2, ..., 5$. A bowl is randomly selected and then two balls are randomly selected (without replacement) from the contents of the bowl. What is the probability that both balls selected are white?
3. [15 pts] A diagnostic test for a disease is such that it (correctly) detects the disease in 99% of the individuals who actually have the disease. Also, if a person does not have the disease, the test will report that she/he does not have it with probability .99. Only 0.1% of the population has the disease in question. If a person is chosen at random from the population and the diagnostic test indicates that this person has the disease, what is the conditional probability that the person actually has the disease?
4. [15 pts] Let Y be a discrete random variable with probability distribution $P(Y = y) = p(y)$, expected value $E(Y)$ and variance $V(Y)$. If a and b are constants, use the definition of $E(Y)$, namely $E(Y) = \sum_y y\, p(y)$, to prove that
(a) [5 pts] $E(aY + b) = aE(Y) + b$
(b) [5 pts] $V(Y) = E(Y^2) - (E(Y))^2$
(a) [5 pts] $V(aY + b) = a^2 V(Y)$
5. [15pts] An oil exploration firm is formed with enough capital to finance ten explorations. The probability of a particular exploration being successful is 10%. Assume the explorations are independent.

(a) [5 pts] What is the distribution of the number of successful explorations? Justify your answer.
(b) [5 pts] Find the mean and standard deviation of the number of successful explorations.
(c) [5 pts] Suppose the firm has a fixed cost of $20,000 in preparing equipment prior to doing its first exploration. If each successful exploration costs $30,000 and each unsuccessful exploration costs $15,000, find the expected total cost to the firm for its ten explorations.
[**Note:** You do not need to carry out all calculations until you reach a final number: you can just write your answers using sums, exponentials, powers, factorials, etc.]
6. [15 pts] A certified public accountant (CPA) has found over the years that nine out of ten company accounts contain substantial errors. Assume the CPA audits – independently and in sequence – a series of company accounts.
(a) [5 pts] What is the distribution of the first account found to contain substantial errors? Justify your answer.
(b) [5 pts] What is the probability that the first account containing substantial errors is the third one to be audited?
(c) [5 pts] What is the probability that neither of the first two accounts (to be audited) will contain substantial errors?
[**Note:** You do not need to carry out all calculations until you reach a final number: you can just write your answers using sums, exponentials, powers, factorials, etc.]
7. [15 pts] The number of typing errors made by a typist has a Poisson distribution with an average of four errors per page. If more than four errors appear on a given page, the typist must retype the whole page.
(a) [5 pts] What is the probability that a randomly selected page has exactly 3 errors.
(b) [5 pts] What is the probability that a randomly selected page does not need to be retyped.
(c) [5 pts] Can you guess or prove (either one is OK) what is the most likely number of errors per page? Show how you arrived to your answer.
[**Note:** You do not need to carry out all calculations until you reach a final number: you can just write your answers using sums, exponentials, powers, factorials, etc.]

8 Exam 2, Spring 2019

1. [10pts](a) Decide if each of the following functions defines a probability density function (provide full justification):

$$f(x) = \begin{cases} 6x(1-x) & \text{if} \quad 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}, \quad g(x) = \begin{cases} x - \frac{1}{2} & \text{if} \quad 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}.$$

(b) A random variable X has the probability density function

$$f(x) = \begin{cases} e^x & , \quad x < 0 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find the mean $\mathbf{E}(X)$, and the variance $\mathrm{Var}(X)$ of the random variable X.

2. **[20pts]** A certain brand of tires has average life $50K$ miles when tested in standardized conditions. Assume that based on the current technology used in the manufacturing process the standard deviation from the average life is estimated to be $\sigma = 2K$ miles.

(a) Use Tchebysheff's Theorem to determine whether it is highly likely (probability of occurrence $> 95\%$) that a tire will last between $40K$ and $60K$ miles?

(b) What is the desired standard deviation if the percentage in (a) needs to improve to <u>at least</u> 99%?

3. **[20pts]** The weekly production in a factory depends on the number of orders from customers which can vary significantly.

(a) The number of orders in a factory during a specific week of the year is normally distributed random variable with mean 50. If the standard deviation of that week's orders is known to equal to 10, then what can be said about the probability that the orders will be between 40 and 60 items?

(b) What is the probability that the number of items ordered is 30% more than the mean?

(c) Can you redo the problem in (a) by just using Tchebysheff's Theorem? Compare the findings to part (a). Which result is more accurate and why?

4. [20 pts] The random variables X and Y denote the lengths of life, in hundreds of hours, for components of types I (for X) and II (for Y), respectively, in an electronic system. Their joint density is given by

$$f(x,y) = e^{-(x+y)}\,, \quad \text{if} \quad x > 0, y > 0 \qquad \text{and} \qquad f(x,y) = 0 \quad \text{otherwise}\,.$$

(a) Show that f is indeed a joint probability density function.

(b) Find the $f_X(x)$ and $f_Y(y)$ marginal densities.

(c) Are the X and Y random variables dependent or independent? Justify your answer.

(d) Find the probability that a component of type II will have a life length in excess of 200 hours.

5. [20 pts] Let X denote the number of customers that arrive in a store tomorrow. If the day is rainy X is Poisson with the expected (mean) number of customers to be 20. If the day is sunny X is also Poisson with the expected number of customers to be 40. The probability for rain tomorrow is 30%.

(a) What is the probability the store will have more than 35 customers?

[**Hint:** Recall that if $Y \sim \text{Poisson}(\lambda)$, then $P(Y = k) = e^{-\lambda}\frac{\lambda^k}{k!}$ and $E(Y) = \lambda$.]

(b) What is the expected number of customers tomorrow?

[**Note:** You do not need to carry out all calculations: you can just write your answers using sums, exponentials, powers, etc.]

6. [10pts] Suppose that X is the total time between a customer's arrival in the store and exit from the check-out window, Y is the time spent in line during checkout, and the joint densities of these variables is given by

$$f(x, y) = \begin{cases} e^{-x}, & 0 \leq y \leq x < \infty, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) [5 pts] Find the average time a customer spends shopping. That is, find $E(X - Y)$.
(b) [5 pts] Find the average proportion of time the customer spends at checkout. That is, find $E(\frac{Y}{X})$.