Numerical solutions for G^2 Hermite interpolation problem with spirals

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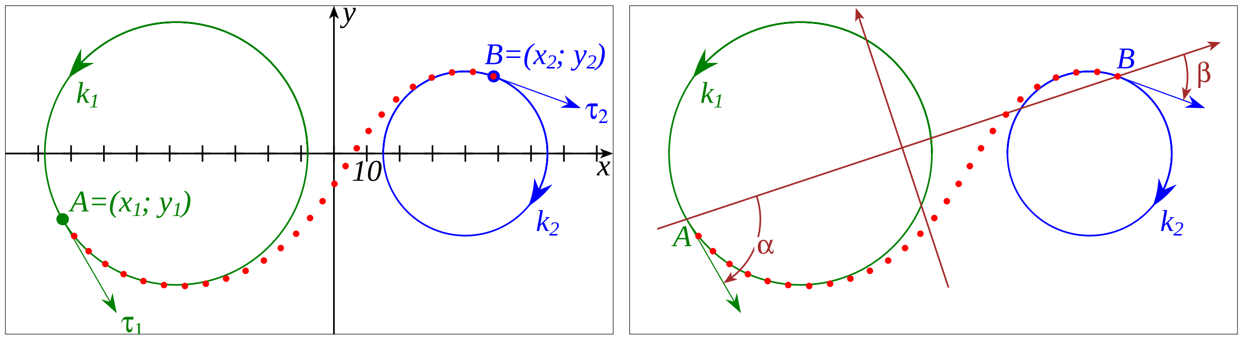
This project is intended to provide a numerical solution (or several solutions) of the
two-point G^2 Hermite interpolation problem with spirals .
I. e., the *transition curve*, joining two given points A and B, is constructed, matching given tangents and curvatures at A and B (Figure 1). *Spirality* means the monotonicity of the Chesaro equation of the curve: function

$$k(s) \equiv \tau'_s(s)$$

(k being curvature, s arc length, and τ the direction of the tangent to the curve), is monotonous.



VogtData

Figure 1. Example of 2-point G^2 Hermite data (boundary conditions): point $A = (x_1, y_1)$, tangent τ_1 and curvature k_1 at A, point $B = (x_2, y_2)$, tangent τ_2 and curvature k_2 at B.

Existence of solutions. . . .

Normalized position. . . .

Brief description of the method . . . [1] (ref. to arxiv)

1. Running the program

The program is written in PostScrpit language. You need some PostScript (GhostScript) interpreter to be inslalled. Under Linux it is usually `gv` or `gs`. Under Windows it is `gsview` or console application `gswin32c`.

You have to edit a few lines in the file `G2spiral.ps` to include your own boundary conditions (user data), and run the program as

```
gv G2spiral.ps
```

Example of user input to construct the spiral in Figure 1 (red dotted curve):

```
<< /UserG2Data [/XYTK8 -82.64 -20. 300  0.025  48.55 23.5 -20.0 -0.04 ] >>
               /method   x1    y1  tau1    k1     x2   y2  tau2   k2
```

You can also keep the user data in a separate file, say, `UserData.ps`, and run the program as

```
gv     -arg="-sFname=/home/ak/G2spiral/UserData.ps" G2spiral.ps
gs -dNOSAFER -sFname=/home/ak/G2spiral/UserData.ps  G2spiral.ps
```

(comments to NOSAFER . . .)
Conversion to pdf by ImageMagick `convert` utility (requires GhostScript):

```
convert G2spiral.ps G2spiral.pdf
```

(see `doc/Example-Converted_to_pdf.pdf`).

2. Setting user data

A few comments on PS syntax. Only data types, used in setting user data, are briefly commented below.

integer, real: as in all other languages: `1  1.  -3.14159`;

bool: `true  false`

string: string `"some text"` should be written in parenthesis as `(some text)`;

name: name, starts with `/`, e. g. `/OutputFile`

array: `[el_0  el_1  el_2 ... ]` Array elements may be of different types.

dictionary: `<< /key1 value1   /key2 value2  /key3 value3 ... >>`

Comment starts with `%`-sign:

```
% user data to approximate an arc of logarithmic spiral
% r(phi) = exp(phi*cot(nu)), 0 <= phi <= Phi
 [/LogSpiral2  80 180]         % nu = 80,  Phi = 180
```

The angles are given in degrees.

Example of a file with user data. User data is stored as a dictionary like

```
<<
  /UserG2Data  [/XYTK8 -82.64 -20. 300  0.025  48.55 23.5 -20.0 -0.04 ]
  /UserPhiData [...]
  /Margin      30
  /OutputFile  (D:/tmp/VogtSpiral.txt)    % ouputs the curve as X Y pairs
>>
```

Only the first key-value pair, `/UserG2Data [ array ]`, is obligatory. Its possible versions are described below. The 1-st element of the array is some `/method`, followed by 2–8 numeric arguments.

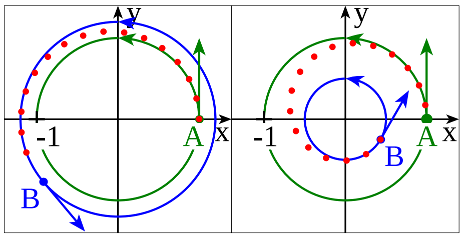
1. `[/XYTK8  x1  y1  tau1   k1    x2   y2  tau2   k2]`
   Direct setting of boundary conditions. 8 arguments are $x_1, y_1, \tau_1, k_1, \quad x_2, y_2, \tau_2, k_2$.

2. `[/Norm4 alpha beta k1  k2]`??? (check!)
   Data normalized to the unit chord $(x_1, y_1) = (-1, 0)$, $\tau_1 = \alpha$, $(x_2, y_2) = (1, 0)$, $\tau_2 = \beta$.
   4 arguments, following the method, are α, β, k_1, k_2. ???

3. `[/Conc2 r2 phi2]`
   Concentric boundary conditions, $r_1 = 1$, $\varphi_1 = 0$, $\pi < \varphi_2 \leqslant 2\pi$, $r_2 \neq 1$:

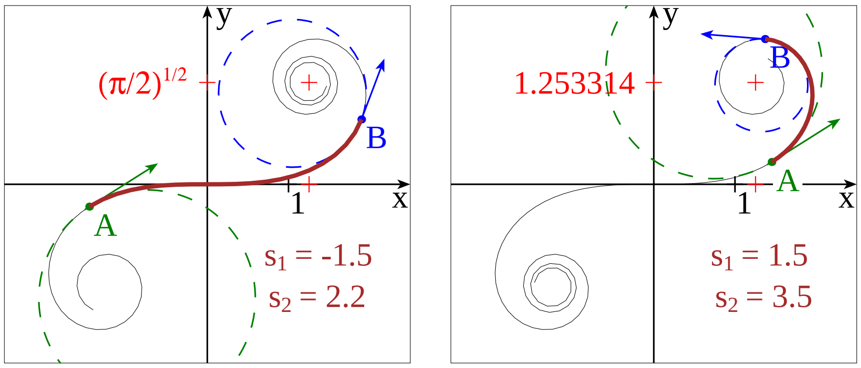
$$A = (x_1, y_1) = (1, 0), \qquad \tau_1 = \frac{\pi}{2}, \qquad k_1 = 1,$$

$$B = (x_2, y_2) = \begin{pmatrix} r_2 \cos\varphi_2 \\ r_2 \sin\varphi_2 \end{pmatrix}, \quad \tau_2 = \varphi_2 + \frac{\pi}{2}, \qquad k_2 = \frac{1}{r_2}.$$



4. `[/Ell2 a b]`
   Boundary coonditions are those of...

5. `[/Cornu3 s1 s2 f]`
   Boundary coonditions are borrowed from Cornu spiral with $k(s) = \dfrac{s}{f^2}$?, $s_1 \leqslant s \leqslant s_2$;
   f is the form parameter (scaling factor): distance ... Both the arc of the Cornu spiral and the approximating curve will be shown in the output.



   Add figure.

6. `[/LogSpir2 nu phi2]`
   Boundary coonditions are borrowed from...

7. `[/ABpp4 alpha beta p1 p2]`
   (bilens parameters; for internal use).

The entry `/UserPhiData [...]`serves to select solutions from the whole family of solutions, thus controlling the number of solutions shown. If absent, default `/UserPhiData [0.]`is assumed. Possible values are:

`/UserPhiData N`, N integer. Shows $2N + 1$ solutions. For $N = 3$:
   $\{-\Phi_{max}, -\Phi_2, -\Phi_1, \Phi_0 = 0., \Phi_1, \Phi_2, \Phi_{max}\}$.

`/UserPhiData [ ]` (empty array)– as the previous one with some automatic selection of N. This is the default behavior if this entry is absent in the user data dictionary.

`/UserPhiData step`, step is real, in degrees (e. g., `5.` , not `5`). Shows solutions for $\Phi \in \{0., \pm\text{step}, \pm 2 \cdot \text{step}, \pm 3 \cdot \text{step}, \ldots\}$.

`/UserPhiData [Phi_min Phi_max N]`, N integer. Shows N solutions within the range $[\Phi_{min}; \Phi_{max}]$.

`/UserPhiData [Phi1 Phi2 Phi3 ... PhiN]` (reals) – list of desired family parameters.

Other options are listed below (add comments!):

```
/Margin  real    % margins in mm                                A4 paper is 210. x
/Margin  int     % margins in pt: /Margin 72 corresponds to 1 inch, A4 paper is 595 x 8
/OutputFile    string  %  e.g. (/home/ak/tmp/mySpiral_XY.txt)
/BlackWhite    false
/BaseLineSkip  20
/ShowAll    true
/LimitCrvLength 5000.
```

About PostScript error messages. . . .

Error messages due to PostScript syntax violation. . .

Errors due to absence of library files. . .

Errors due to forbidden output to disk. . .

Output. Output curve file is formatted as `[ x1 y1  x2 y2 ... xN yN ]`for every solution found:

```
% Phi: -45.0  Length: 4.08248
[
 -0.993767 -0.0829621 -0.987961 -0.0814893 -0.976391 -0.0786075 -0.953409 -0.0731016
 -0.908072 -0.0630687 -0.819742 -0.0465941 -0.651231 -0.0257036 -0.491357 -0.0172414
 -0.337668 -0.017637 -0.0404825 -0.0335358 0.25381 -0.0531306 0.402472 -0.0584971
 0.552631 -0.0578547 0.703972 -0.0489771 0.855578 -0.029798 1.00592 0.00144225
 1.15291 0.046022 1.29402 0.104561 1.42648 0.176884 1.54757 0.261951
 1.65486 0.357907 1.74654 0.46223 1.82151 0.571974 1.87954 0.684055
 1.92118 0.795525 1.94762 0.903802 1.96055 1.00681 1.96188 1.10303
 1.95365 1.1915 1.93781 1.27174 1.91616 1.34367 1.86147 1.46366
 1.79837 1.55647
]
```

References

[1] *Kurnosenko A.I.* Two-point G^2 Hermite interpolation with spirals by inversion of hyperbola // Comp. Aided Geom. Design. 2010. V. 27. P. 474–481.