BAPS SWAMINARAYAN VIDYAMANDIR GONDAL

Subject : Mathematics
Standard : 12
Total Mark : 132

2025 Matrices Determinants PYQs

Paper Set : 1
Date : 01-07-2025
Time : 1H:20M

Mathematics - Section A (MCQ)

(1) If A, B and $\left(\operatorname{adj}\left(A^{-1}\right)+\operatorname{adj}\left(B^{-1}\right)\right)$ are non-singular matrices of same order, then the inverse of $A\left(\operatorname{adj}\left(A^{-1}\right)+\operatorname{adj}\left(B^{-1}\right)\right)^{-1} B$, is equal to [JEE MAIN 2025]

(A) $AB^{-1}+A^{-1}B$
(B) $\operatorname{adj}\left(B^{-1}\right)+\operatorname{adj}\left(A^{-1}\right)$
(C) $\frac{1}{|AB|}(\operatorname{adj}(B)+\operatorname{adj}(A))$
(D) $\frac{AB^{-1}}{|A|}+\frac{BA^{-1}}{|B|}$

(2) The system of equations $x+y+z=6$, $x+2y+5z=9$, $x+5y+\lambda z=\mu$ has no solution if [JEE MAIN 2025]

(A) $\lambda=17, \mu\neq 18$
(B) $\lambda\neq 17, \mu\neq 18$
(C) $\lambda=15, \mu\neq 17$
(D) $\lambda=17, \mu=18$

(3) Let $A=[a_{ij}]$ be a 3×3 matrix such that $A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}$ and $A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$, then a_{23} equals: [JEE MAIN 2025]

(A) -1
(B) 0
(C) 2
(D) 1

(4) Let $A=\begin{bmatrix}\alpha & -1\\ 6 & \beta\end{bmatrix}$, $\alpha>0$, such that $\det(A)=0$ and $\alpha+\beta=1$. If I denotes 2×2 identity matrix, then the matrix $(I+A)^8$ is: [JEE MAIN 2025]

(A) $\begin{bmatrix}4 & -1\\ 6 & -1\end{bmatrix}$
(B) $\begin{bmatrix}257 & -64\\ 514 & -127\end{bmatrix}$
(C) $\begin{bmatrix}1025 & -511\\ 2024 & -1024\end{bmatrix}$
(D) $\begin{bmatrix}766 & -255\\ 1530 & -509\end{bmatrix}$

(5) Let the system of equations: $2x+3y+5z=9$, $7x+3y-2z=8$, $12x+3y-(4+\lambda)z=16-\mu$ have infinitely many solutions. Then the radius of the circle centred at (λ,μ) and touching the line $4x=3y$ is [JEE MAIN 2025]

(A) $\frac{17}{5}$
(B) $\frac{7}{5}$
(C) 7
(D) $\frac{21}{5}$

Mathematics - Section B (NUMERIC)

(6) Let A be a square matrix of order 3 such that $\det(A)=-2$ and $\det(3\operatorname{adj}(-6\operatorname{adj}(3A)))=2^{m+n}\cdot 3^{mn}$, $m>n$. Then $4m+2n$ is equal to ______ [JEE MAIN 2025]

(7) For a 3×3 matrix M, let trace (M) denote the sum of all the diagonal elements of M. Let A be a 3×3 matrix such that $|A|=\frac{1}{2}$ and trace $(A)=3$. If $B=\operatorname{adj}(\operatorname{adj}(2A))$, then the value of $|B|+$ trace (B) equals: [JEE MAIN 2025]

(8) If the system of linear equations : $x+y+2z=6$, $2x+3y+az=a+1$, $-x-3y+bz=2b$ where $a,b\in R$, has infinitely many solutions, then $7a+3b$ is equal to : [JEE MAIN 2025]

(9) If the system of equations $(\lambda-1)x+(\lambda-4)y+\lambda z=5$, $\lambda x+(\lambda-1)y+(\lambda-4)z=7$, $(\lambda+1)x+(\lambda+2)y-(\lambda+2)z=9$ has infinitely many solutions, then $\lambda^2+\lambda$ is equal to [JEE MAIN 2025]

(10) If the system of equations $2x-y+z=4$, $5x+\lambda y+3z=12{,}100x-47y+\mu z=212$ has infinitely many solutions, then $\mu-2\lambda$ is equal to [JEE MAIN 2025]

(11) Let be a 3×3 matrix such that $X^TAX=O$ for all nonzero 3×1 matrices $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$. If $A\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}1\\4\\-5\end{bmatrix}$, $A\begin{bmatrix}1\\2\\1\end{bmatrix}=\begin{bmatrix}0\\4\\-8\end{bmatrix}$, and $\det(\operatorname{adj}(2(A+I)))=2^\alpha 3^\beta 5^\gamma$, $\alpha,\beta,\gamma,\in N$, then $\alpha^2+\beta^2+\gamma^2$ is [JEE MAIN 2025]

(12) If the system of equations $x+2y-3z=2$, $2x+\lambda y+5z=5$, $14x+3y+\mu z=33$ has infinitely many solutions, then $\lambda+\mu$ is equal to: [JEE MAIN 2025]

(13) For some a,b, let

$$f(x)=\begin{vmatrix}a+\frac{\sin x}{x} & 1 & b\\ a & 1+\frac{\sin x}{x} & b\\ a & 1 & b+\frac{\sin x}{x}\end{vmatrix},\quad x\neq 0,$$

$\lim_{x\to 0} f(x)=\lambda+\mu a+vb$. Then $(\lambda+\mu+v)^2$ is equal to: [JEE MAIN 2025]

(14) Let M denote the set of all real matrices of order 3×3 and let $S=\{-3,-2,-1,1,2\}$. Let
$S_1=\left\{A=[a_{ij}]\in M : A=A^T \text{ and } a_{ij}\in S, \forall i,j\right\}$
$S_2=\left\{A=[a_{ij}]\in M : A=-A^T \text{ and } a_{ij}\in S, \forall i,j\right\}$
$S_3=$
$\{A=[a_{ij}]\in M : a_{11}+a_{22}+a_{33}=0 \text{ and } a_{ij}\in S, \forall i,j\}$
If $n(S_1\cup S_2\cup S_3)=125\alpha$, then α equals. [JEE MAIN 2025]

(15) Let $A=\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}$ and $P=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, $\theta>0$. If $B=PAP^T$, $C=P^TB^{10}P$ and the sum of the diagonal elements of C is $\frac{m}{n}$, where $\gcd(m,n)=1$, then $m+n$ is : [JEE MAIN 2025]

(16) Let M and m respectively be the maximum and the minimum values of

$$f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 4x\\ \sin^2 x & 1+\cos^2 x & 4\sin 4x\\ \sin^2 x & \cos^2 x & 1+4\sin 4x\end{vmatrix}, x\in R$$

Then M^4-m^4 is equal to :_____ [JEE MAIN 2025]

(17) Let $A=[a_{ij}]=\begin{bmatrix}\log_5 128 & \log_4 5\\ \log_5 8 & \log_4 25\end{bmatrix}$. If A_{ij} is the cofactor of a_{ij}, $C_{ij}=\sum_{k=1}^{2} a_{ik}A_{jk}$, $1\leq i,j\leq 2$, and $C=[C_{ij}]$, then $8|C|$ is equal to : [JEE MAIN 2025]

(18) Let $S=\left\{m \in Z: A^{m^2}+A^m=3I-A^{-6}\right\}$, where $A=\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$. Then $n(S)$ is equal to [JEE MAIN 2025]

(19) Let $\alpha, \beta(\alpha \neq \beta)$ be the values of m, for which the equations $x+y+z=1; x+2y+4z=m$ and $x+4y+10z=m^2$ have infinitely many solutions. Then the value of $\sum_{n=1}^{10}\left(n^\alpha+n^\beta\right)$ is equal to : [JEE MAIN 2025]

(20) Let $A=[a_{ij}]$ be a matrix of order 3×3, with $a_{ij}=(\sqrt{2})^{i+j}$. If the sum of all the elements in the third row of A^2 is $\alpha+\beta\sqrt{2}, \alpha, \beta \in Z$, then $\alpha+\beta$ is equal to [JEE MAIN 2025]

(21) Let integers $a, b \in[-3,3]$ be such that $a+b \neq 0$. Then the number of all possible ordered pairs (a, b), for which $\left|\frac{z-a}{z+b}\right|=1$ and $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=1, z \in C$, where ω and ω^2 are the roots of $x^2+x+1=0$, is equal to________ [JEE MAIN 2025]

(22) If the system of linear equations $3x+y+\beta z=3$, $2x+\alpha y-z=-3$, $x+2y+z=4$ has infinitely many solutions, then the value of $22\beta-9\alpha$ is : [JEE MAIN 2025]

(23) Let $a \in R$ and A be a matrix of order 3×3 such that $\det(A)=-4$ and $A+I=\begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$, where I is the identity matrix of order 3×3. If $\det((a+1)\operatorname{adj}((a-1)A))$ is $2^m 3^n, m, n \in\{0,1,2, \ldots \ldots 20\}$, then $m+n$ is equal to : [JEE MAIN 2025]

(24) If the system of equation $2x+\lambda y+3z=5$, $3x+2y-z=7$, $4x+5y+\mu z=9$ has infinitely many solutions, then $\left(\lambda^2+\mu^2\right)$ is equal to : [JEE MAIN 2025]

(25) Let A be a 3×3 real matrix such that $A^2(A-2I)-4(A-I)=O$, where I and O are the identity and null matrices, respectively. If $A^5=\alpha A^2+\beta A+\gamma I$, where α, β and γ are real constants, then $\alpha+\beta+\gamma$ is equal to: [JEE MAIN 2025]

(26) Let A be a matrix of order 3×3 and $|A|=5$. If $|2\operatorname{adj}(3A\operatorname{adj}(2A))|=2^\alpha \cdot 3^\beta \cdot 5^\gamma \alpha, \beta, \gamma \in N$ then $\alpha+\beta+\gamma$ is equal to [JEE MAIN 2025]

(27) Let I be the identity matrix of order 3×3 and for the matrix $A=\begin{bmatrix} \lambda & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -1 & 2 \end{bmatrix}, |A|=-1$. Let B be the inverse of the matrix $\operatorname{adj}\left(A\operatorname{adj}\left(A^2\right)\right)$. Then $|(\lambda B+1)|$ is equal to ______. [JEE MAIN 2025]

(28) Let $A=\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$. If for some $\theta \in(0, \pi)$, $A^2=A^T$, then the sum of the diagonal elements of the matrix $(A+I)^3+(A-I)^3-6A$ is equal to ______. [JEE MAIN 2025]

(29) Let the matrix $A=\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ satisfy $A^n=A^{n-2}+A^2-I$ for $n \geq 3$. Then the sum of all the elements of A^{50} is :- [JEE MAIN 2025]

(30) Let A be a 3×3 matrix such that $|\operatorname{adj}(\operatorname{adj}(\operatorname{adj} A))|=81$. If $S=\left\{n \in Z:(|\operatorname{adj}(\operatorname{adj} A)|)^{\frac{(n-1)^2}{2}}=|A|^{\left(3n^2-5n-4\right)}\right\}$, then $\sum_{n \in S}\left|A^{\left(n^2+n\right)}\right|$ is equal to [JEE MAIN 2025]

(31) Let the system of equations $x+5y-z=1$, $4x+3y-3z=7$, $24x+y+\lambda z=\mu$, $\lambda, \mu \in R$, have infinitely many solutions. Then the number of the solutions of this system, If x, y, z are integers and satisfy $7 \leq x+y+z \leq 77$, is [JEE MAIN 2025]

(32) Let α be a solution of $x^2+x+1=0$, and for some a and b in R, $\begin{bmatrix} 4 & a & b \end{bmatrix}\begin{bmatrix} 1 & 16 & 13 \\ -1 & -1 & 2 \\ -2 & -14 & -8 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$. If $\frac{4}{\alpha^4}+\frac{m}{\alpha^a}+\frac{n}{\alpha^b}=3$, then $m+n$ is equal to [JEE MAIN 2025]

(33) Let $A=\begin{bmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{bmatrix}$ If $\det(\operatorname{adj}(\operatorname{adj}(3A)))=2^m \cdot 3^n, m, n \in N$, then $m+n$ is equal to [JEE MAIN 2025]

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Mathematics - Section A (MCQ)

(1) જો A, B અને $\left(\text{adj}\left(A^{-1}\right)+\text{adj}\left(B^{-1}\right)\right)$ એ સમાન કક્ષાના શૂન્યતર શ્રેણિક છે તો $A\left(\text{adj}\left(A^{-1}\right)+\text{adj}\left(B^{-1}\right)\right)^{-1} B$ નો વ્યસ્ત શ્રેણિક મેળવો. [JEE MAIN 2025]

(A) $AB^{-1}+A^{-1}B$
(B) $\text{adj}\left(B^{-1}\right)+\text{adj}\left(A^{-1}\right)$
(C) $\frac{1}{|AB|}(\text{adj}(B)+\text{adj}(A))$
(D) $\frac{AB^{-1}}{|A|}+\frac{BA^{-1}}{|B|}$

(2) સમીકરણોની સંહતિ $x+y+z=6, x+2y+5z=9$, $x+5y+\lambda z=\mu$ ને એકપણ ઉકેલ નો હોય જો . . . . [JEE MAIN 2025]

(A) $\lambda=17, \mu \neq 18$
(B) $\lambda \neq 17, \mu \neq 18$
(C) $\lambda=15, \mu \neq 17$
(D) $\lambda=17, \mu=18$

(3) અહી $A=[a_{ij}]$ એ 3×3 શ્રેણિક છે કે જેથી

$A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}, A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}$ અને

$A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$, તો a_{23} ની કિમંત મેળવો. [JEE MAIN 2025]

(A) -1
(B) 0
(C) 2
(D) 1

(4) અહી $A=\begin{bmatrix}\alpha & -1\\ 6 & \beta\end{bmatrix}, \alpha>0$, આપેલ છે કે જેથી $\det(A)=0$ અને $\alpha+\beta=1$ છે. જો I એ 2×2 એકમ શ્રેણિક હોય તો શ્રેણિક $(I+A)^8=$ [JEE MAIN 2025]

(A) $\begin{bmatrix}4 & -1\\ 6 & -1\end{bmatrix}$
(B) $\begin{bmatrix}257 & -64\\ 514 & -127\end{bmatrix}$
(C) $\begin{bmatrix}1025 & -511\\ 2024 & -1024\end{bmatrix}$
(D) $\begin{bmatrix}766 & -255\\ 1530 & -509\end{bmatrix}$

(5) અહી સમીકરણ સંહતિ : $2x+3y+5z=9, 7x+3y-2z=8$, $12x+3y-(4+\lambda)z=16-\mu$ ને અનંત ઉકેલ ધરાવે છે. તો વર્તુળ કે જેનું કેન્દ્ર (λ, μ) અને જે રેખા $4x=3y$ ને સ્પર્શે છે તેની ત્રિજ્યા મેળવો. [JEE MAIN 2025]

(A) $\frac{17}{5}$
(B) $\frac{7}{5}$
(C) 7
(D) $\frac{21}{5}$

Mathematics - Section B (NUMERIC)

(6) અહી A 3 કક્ષાવાળો ચોરચ શ્રેણિક છે કે જેથી $\det(A)=-2$ અને $\det(3\,\text{adj}(-6\,\text{adj}(3A)))=2^{m+n}\cdot 3^{\text{mn}}, m>n$ હોય તો $4m+2n$ ની કિમંત મેળવો. [JEE MAIN 2025]

(7) એક 3×3 શ્રેણિક M માટે ધારોકે (M) એ M ના તમામ વિકર્ણી ઘટકોનો સરવાળો દર્શાવે છે. ધારોકે A એવો 3×3 શ્રેણિક છે કે જેથી $|A|=\frac{1}{2}$ તથા trace $(A)=3$. જો $B=\text{adj}(\text{adj}(2A))$ હોય, તો $|B|+\text{trace}(B)$ નું મૂલ્ય = _______ [JEE MAIN 2025]

(8) જો સુરેખ સમીકરણોની સંહતિ $x+y+2z=6$, $2x+3y+az=a+1, -x-3y+bz=2b$ જ્યાં $a, b\in R$, ને અસંખ્ય ઉકેલો હોય, તો $7a+3b=$ _______ [JEE MAIN 2025]

(9) જો સમીકરણની સંહતિ $(\lambda-1)x+(\lambda-4)y+\lambda z=5$, $\lambda x+(\lambda-1)y+(\lambda-4)z=7$, $(\lambda+1)x+(\lambda+2)y-(\lambda+2)z=9$ ને અનંત ઉકેલો હોય તો $\lambda^2+\lambda$ ની કિમંત મેળવો. [JEE MAIN 2025]

(10) જો સમીકરણ સંહિતા $2x-y+z=4$, $5x+\lambda y+3z=12, 100x-47y+\mu z=212$ ને અસંખ્ય ઉકેલો હોય તો $\mu-2\lambda=$ .... [JEE MAIN 2025]

(11) ધારો કે A એવો એક 3×3 શ્રેણિક છે કે જેથી પ્રત્યેક શૂન્યેત્તર 3×1 શ્રેણિકો $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ માટે $X^TAX=0$ થાય. જો

$A\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}1\\4\\-5\end{bmatrix}, A\begin{bmatrix}1\\2\\1\end{bmatrix}=\begin{bmatrix}0\\4\\-8\end{bmatrix}$ અને

$\det(\text{adj}(2(A+I)))=2^{\alpha}3^{\beta}5^{\gamma}, \alpha, \beta, \gamma\in N$ હોય, તો $\alpha^2+\beta^2+\gamma^2=$ _______ [JEE MAIN 2025]

(12) જો સમીકરણ સંહિત $x+2y-3z=2, 2x+\lambda y+5z=5$, $14x+3y+\mu z=33$ ને અસંખ્ય ઉકેલો હોય, તો $\lambda+\mu$ ______ [JEE MAIN 2025]

(13) કોઈક a, b, માટે ધારો કે

$f(x)=\begin{vmatrix}a+\frac{\sin x}{x} & 1 & b\\ a & 1+\frac{\sin x}{x} & b\\ a & 1 & b+\frac{\sin x}{x}\end{vmatrix}, \quad x\neq 0,$

$\lim_{x\to 0} f(x)=\lambda+\mu a+vb$. તો $(\lambda+\mu+v)^2$ __________. [JEE MAIN 2025]

(14) ધારો કે M એ કક્ષા 3×3 વાળા તમામ વાસ્તવિક શ્રેણિકોનો ગણ દર્શાવ છે તથા $S=\{-3,-2,-1,1,2\}$. ધારો કે

$S_1=\left\{A=[a_{ij}]\in M: A=A^T \text{ and } a_{ij}\in S, \forall i, j\right\}$

$S_2=\left\{A=[a_{ij}]\in M: A=-A^T \text{ and } a_{ij}\in S, \forall i, j\right\}$

$S_3=$
$\left\{A=[a_{ij}]\in M: a_{11}+a_{22}+a_{33}=0 \text{ and } a_{ij}\in S, \forall i, j\right\}$

જો $n(S_1\cup S_2\cup S_3)=125\alpha$, હોય તો $\alpha=$_____. [JEE MAIN 2025]

(15) ધારો કે $A=\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}$ અને $P=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}, \theta>0$. જો $B=PAP^T, C=P^TB^{10}P$ અને C ના વિકીર્ણ ઘટકોનો સરવાળો $\frac{m}{n}$, હોય, જ્યાં ગુ.સા.અ. $(m,n)=1$, તો $m+n=$________ [JEE MAIN 2025]

(16) જો M અને m એ અનુક્રમે

$f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 4x\\ \sin^2 x & 1+\cos^2 x & 4\sin 4x\\ \sin^2 x & \cos^2 x & 1+4\sin 4x\end{vmatrix}, x\in R$ ની

મહતમ અને ન્યૂનતમ કિમતો હોય તો M^4-m^4 ની કિમંત મેળવો. [JEE MAIN 2025]

(17) અહી $A=[a_{ij}]=\begin{bmatrix}\log_5 128 & \log_4 5\\ \log_5 8 & \log_4 25\end{bmatrix}$ છે. જો A_{ij} એ a_{ij} નો

સહઅવયજ શ્રેણિક છે. જો $C_{ij}=\sum_{k=1}^{2} a_{ik}A_{jk}, 1 \le i, j \le 2$ અને $C=[C_{ij}]$ આપેલ હોય તો $8|C|$ ની કિંમત મેળવો. [JEE MAIN 2025]

(18) અહી $S=\left\{m \in Z : A^{m^2}+A^m=3I-A^{-6}\right\}$ કે જ્યાં $A=\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$ હોય તો $n(S)$ ની કિમંત મેળવો. [JEE MAIN 2025]

(19) ધારોકે, $\alpha, \beta (\alpha \neq \beta)$ એ m ની એવી કિંમતો છે કે જેના માટે સમીકરણો $x+y+z=1; x+2y+4z=m$ અને $x+4y+10z=m^2$ ને અસંખ્ય ઉકેલો હોય તો $\sum_{n=1}^{10}\left(n^{\alpha}+n^{\beta}\right)$ નું મૂલ્ય______ છે. [JEE MAIN 2025]

(20) ધારો કે $A=(a_{ij})$ એ કક્ષા 3×3 નો એક શ્રેણિક છે, જ્યાં $a_{ij}=(\sqrt{2})^{i+j}$ છે. જો A^2 ની ત્રીજી હારના તમામ ઘટકોનો સરવાળો $\alpha+\beta\sqrt{2}, \alpha+\beta \in Z$ હોય તો $\alpha+\beta=$ _______. [JEE MAIN 2025]

(21) ધારો કે જો પૂર્ણાંકો $a, b \in [-3,3]$ એવાં છે કે જેથી $a+b \neq 0$. તો શક્ય તમામ એવી જોડ (a,b) ની સંખ્યા શોધો, કે જેના માટે $\left|\frac{z-a}{z+b}\right|=1$ અને $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=1, z \in C$, જ્યાં ω અને ω^2 એ $x^2+x+1=0$, નાં બીજ છે. [JEE MAIN 2025]

(22) જો સુરેખ સંહતિઓ $3x+y+\beta z=3, 2x+\alpha y-z=-3$, $x+2y+z=4$ ને અનંત ઉકેલો હોય તો $22\beta-9\alpha$ ની કિમંત મેળવો. [JEE MAIN 2025]

(23) અહી $a \in R$ અને શ્રેણિક A એ 3×3 કક્ષાનો છે કે જેથી $\det(A)=-4$ અને $A+I=\begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$, કે જ્યાં I એ 3×3 કક્ષાનો એકમ શ્રેણિક છે. જો $\det((a+1)\operatorname{adj}((a-1)A))$ એ $2^m 3^n, m, n \in \{0,1,2,\ldots\ldots 20\}$, હોય તો $m+n$ ની કિમંત મેળવો. [JEE MAIN 2025]

(24) જો સમીકરણ સંહતીઓ $2x+\lambda y+3z=5, 3x+2y-z=7$, $4x+5y+\mu z=9$ ને અનંત ઉકેલ હોય તો $\left(\lambda^2+\mu^2\right)$ ની કિમંત મેળવો. [JEE MAIN 2025]

(25) અહી A એ 3×3 કક્ષાનો વાસ્તવિક શ્રેણિક છે કે જેથી $A^2(A-2I)-4(A-I)=O$ છે જ્યાં I અને O અનુક્રમે એકમ અને શૂન્ય શ્રેણિક છે . જો $A^5=\alpha A^2+\beta A+\gamma I$ કે જ્યાં α, β અને γ એ વાસ્તવિક અચળાંક છે તો $\alpha+\beta+\gamma$ ની કિમંત મેળવો. [JEE MAIN 2025]

(26) અહી A એ 3×3 કક્ષાનો શ્રેણી છે અને $|A|=5$ છે. જો $|2\operatorname{adj}(3A\operatorname{adj}(2A))|=2^{\alpha}\cdot 3^{\beta}\cdot 5^{\gamma}\alpha, \beta, \gamma \in N$ હોય તો $\alpha+\beta+\gamma$ મેળવો. [JEE MAIN 2025]

(27) અહી I એ 3×3 કક્ષાનો એકમ શ્રેણિક છે અને શ્રેણિક $A=\begin{bmatrix} \lambda & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -1 & 2 \end{bmatrix}, |A|=-1$ આપેલ છે . જો B એ શ્રેણિક $\operatorname{adj}\left(A\operatorname{adj}\left(A^2\right)\right)$ વ્યસ્ત શ્રેણિક હોય તો $|(\lambda B+1)|$ ની કિમંત મેળવો. [JEE MAIN 2025]

(28) ધારો કે $A=\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$. કોઈક $\theta \in (0,\pi)$ માટ, જો $A^2=A^T$ હોય, તો ક્ષેર્ણિક $(A+I)^3+(A-I)^3-6A$ ના વિકીર્ણ ઘટકીનો સરવાળો _______ છે. [JEE MAIN 2025]

(29) અહી શ્રેણિક $A=\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ એ $n \geq 3$ માટે $A^n=A^{n-2}+A^2-I$ નું સમાધાન કરે છે તો A^{50} ના બધાજ ઘટકનો સરવાળો મેળવો. [JEE MAIN 2025]

(30) અહી A એ 3×3 કક્ષાનો શ્રેણિક છે કે જેથી $|\operatorname{adj}(\operatorname{adj}(\operatorname{adj} A))|=81$ છે. જો $S=\left\{n \in Z : (|\operatorname{adj}(\operatorname{adj} A)|)^{\frac{(n-1)^2}{2}}=|A|^{\left(3n^2-5n-4\right)}\right\}$ હોય તો $\sum_{n \in S}\left|A^{\left(n^2+n\right)}\right|$ ની કિમંત મેળવો. [JEE MAIN 2025]

(31) સમીકરણ સહંતિ $x+5y-z=1, 4x+3y-3z=7$, $24x+y+\lambda z=\mu, \lambda, \mu \in R$ ના ઉકેલની સંખ્યા અનંત હોય તો આ સમીકરણોની સંહતિના ઉકેલ ની સંખ્યા મેળવો કે જેમાં x, y, z એ પૂર્ણાંક હોય અને $7 \le x+y+z \le 77$ નું સમાધાન કરતું હોય . [JEE MAIN 2025]

(32) ધારો કે α એ $x^2+x+1=0$ નું બીજ છે,તથા a અને b એ $R, \begin{bmatrix} 4 & a & b \end{bmatrix}\begin{bmatrix} 1 & 16 & 13 \\ -1 & -1 & 2 \\ -2 & -14 & -8 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ નું સમાધાન કરે છે. જો $\frac{4}{\alpha^4}+\frac{m}{\alpha^a}+\frac{n}{\alpha^b}=3$, હોય, તો $m+n=$_____ [JEE MAIN 2025]

(33) ધારો કે $A=\begin{bmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{bmatrix}$ જો $\det(\operatorname{adj}(\operatorname{adj}(3A)))=2^m\cdot 3^n, m, n \in N$, હોય તો $m+n=$ _______ [JEE MAIN 2025]

BAPS SWAMINARAYAN VIDYAMANDIR GONDAL

Subject : Mathematics
Standard : 12
Total Mark : 132

2025 Matrices Determinants PYQs

(Answer Key)

Paper Set : 1
Date : 01-07-2025
Time : 1H:20M

Mathematics - Section A (MCQ)

1 - C	2 - A	3 - A	4 - D	5 - B

Mathematics - Section B (NUMERIC)

6 - 34	7 - 280	8 - 16	9 - 12	10 - 57	11 - 44	12 - 12	13 - 16	14 - 1613	15 - 65
16 - 1280	17 - 242	18 - 2	19 - 440	20 - 224	21 - 10	22 - 31	23 - 16	24 - 26	25 - 12
26 - 27	27 - 38	28 - 6	29 - 53	30 - 732	31 - 3	32 - 11	33 - 24		

BAPS SWAMINARAYAN VIDYAMANDIR GONDAL

Subject : Mathematics
Standard : 12
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2025 Matrices Determinants PYQs

(Solutions)

Paper Set : 1
Date : 01-07-2025
Time : 1H:20M

Mathematics - Section A (MCQ)

(1) If A, B and $\left(\operatorname{adj}\left(A^{-1}\right)+\operatorname{adj}\left(B^{-1}\right)\right)$ are non-singular matrices of same order, then the inverse of $A\left(\operatorname{adj}\left(A^{-1}\right)+\operatorname{adj}\left(B^{-1}\right)\right)^{-1} B$, is equal to [JEE MAIN 2025]

(A) $AB^{-1}+A^{-1}B$ (B) $\operatorname{adj}\left(B^{-1}\right)+\operatorname{adj}\left(A^{-1}\right)$

(C) $\frac{1}{|AB|}(\operatorname{adj}(B)+\operatorname{adj}(A))$ (D) $\frac{AB^{-1}}{|A|}+\frac{BA^{-1}}{|B|}$

Solution:(Correct Answer:C)

$$\left[A\left(\operatorname{adj}\left(A^{-1}\right)+\operatorname{adj}\left(B^{-1}\right)\right)^{-1}\cdot B\right]^{-1}$$
$$B^{-1}\cdot\left(\operatorname{adj}\left(A^{-1}\right)+\operatorname{adj}\left(B^{-1}\right)\right)\cdot A^{-1}$$
$$B^{-1}\operatorname{adj}\left(A^{-1}\right)A^{-1}+B^{-1}\left(\operatorname{adj}\left(B^{-1}\right)\right)\cdot A^{-1}$$
$$B^{-1}\left|A^{-1}\right|I+\left|B^{-1}\right|IA^{-1}$$
$$\frac{B^{-1}}{|A|}+\frac{A^{-1}}{|B|}$$
$$\Rightarrow\frac{\operatorname{adj}B}{|B||A|}+\frac{\operatorname{adj}A}{|A||B|}$$
$$=\frac{1}{|A||B|}(\operatorname{adjB}+\operatorname{adj}A)$$

(2) The system of equations $x+y+z=6$, $x+2y+5z=9$, $x+5y+\lambda z=\mu$ has no solution if [JEE MAIN 2025]

(A) $\lambda=17, \mu\neq 18$ (B) $\lambda\neq 17, \mu\neq 18$

(C) $\lambda=15, \mu\neq 17$ (D) $\lambda=17, \mu=18$

Solution:(Correct Answer:A)

$$D=\begin{vmatrix}1&1&1\\1&2&5\\1&5&\lambda\end{vmatrix}=0$$
$$\lambda=17$$
$$D_z=\begin{vmatrix}1&1&6\\1&2&9\\1&5&\mu\end{vmatrix}\neq 0$$
$$\mu\neq 18$$

(3) Let $A=[a_{ij}]$ be a 3×3 matrix such that $A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}$ and $A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$, then a_{23} equals: [JEE MAIN 2025]

(A) -1 (B) 0

(C) 2 (D) 1

Solution:(Correct Answer:A)

Let $A=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}$

$$A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}\Rightarrow\begin{bmatrix}a_{12}\\a_{22}\\a_{32}\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}\Rightarrow\begin{matrix}a_{22}=0;a_{12}=0\\a_{32}=1\end{matrix}$$
$$A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}\Rightarrow\begin{matrix}4a_{11}+a_{12}+3a_{13}=0\\4a_{21}+a_{22}+3a_{25}=1\Rightarrow 4a_{21}+3a_{25}\\4a_{31}+a_{32}+3a_{33}=0\end{matrix}$$
$$A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}\Rightarrow\begin{matrix}2a_{11}+a_{12}+2a_{13}=1\\2a_{21}+a_{22}+2a_{23}=0\Rightarrow a_{21}+a_{23}=\\2a_{31}+a_{32}+2a_{33}=0\end{matrix}$$
$$-4a_{23}+3a_{23}=1\Rightarrow a_{23}=-1$$

(4) Let $A=\begin{bmatrix}\alpha&-1\\6&\beta\end{bmatrix}$, $\alpha>0$, such that $\det(A)=0$ and $\alpha+\beta=1$. If I denotes 2×2 identity matrix, then the matrix $(I+A)^8$ is: [JEE MAIN 2025]

(A) $\begin{bmatrix}4&-1\\6&-1\end{bmatrix}$ (B) $\begin{bmatrix}257&-64\\514&-127\end{bmatrix}$

(C) $\begin{bmatrix}1025&-511\\2024&-1024\end{bmatrix}$ **(D)** $\begin{bmatrix}766&-255\\1530&-509\end{bmatrix}$

Solution:(Correct Answer:D)

$$|A|=0$$
$$\alpha\beta+6=0$$
$$\alpha\beta=-6$$
$$\alpha+\beta=1$$
$$\Rightarrow\alpha=3,\beta=-2$$
$$A=\begin{bmatrix}3&-1\\6&-2\end{bmatrix}$$
$$A^2=\begin{bmatrix}3&-1\\6&-2\end{bmatrix}\begin{bmatrix}3&-1\\6&-2\end{bmatrix}=\begin{bmatrix}3&-1\\6&-2\end{bmatrix}$$
$$\therefore A^2=A$$
$$A=A^2=A^3=A^4=A^5$$
$$(I+A)^8$$
$$=I+{}^8C_1A^7+{}^8C_2A^6+\ldots.+{}^8C_8A^8$$
$$=I+A\left({}^8C_1+{}^8C_2+\ldots..+{}^8C_8\right)$$
$$=I+A\left(2^8-1\right)$$
$$=\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}765&-255\\1530&-510\end{bmatrix}$$
$$=\begin{bmatrix}766&-255\\1530&-509\end{bmatrix}$$

(5) Let the system of equations: $2x+3y+5z=9$, $7x+3y-2z=8$, $12x+3y-(4+\lambda)z=16-\mu$ have infinitely many solutions. Then the radius of the circle centred at (λ,μ) and touching the line $4x=3y$ is [JEE MAIN 2025]

(A) $\frac{17}{5}$ **(B)** $\frac{7}{5}$

(C) 7 (D) $\frac{21}{5}$

Solution:(Correct Answer:B)

$$\begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 12 & 3 & -(\lambda+4) \end{vmatrix} = 0$$
$\Rightarrow 12(-21) - 3(-39) - (\lambda+4)(-15) = 0$
$\Rightarrow -252 + 117 + 15(1+4) = 0$
$\Rightarrow 15\lambda + 177 - 252 = 0$
$\Rightarrow 15\lambda - 75 = 0 \Rightarrow \lambda = 5$
$$\begin{vmatrix} 9 & 3 & 5 \\ 8 & 3 & -2 \\ 16-\mu & 3 & -9 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 0 & 7 \\ \mu-8 & 0 & 7 \\ 16-\mu & 3 & -9 \end{vmatrix} = 0$$
$\Rightarrow 7 - 7(\mu - 8) = 0 \Rightarrow 1 - (\mu - 8) = 0 \Rightarrow \mu = 9$
$\Rightarrow$ centre of circle $(5, 9)$
radius $=$ length of $\perp$ from centre $(5,9) = \left|\frac{20-27}{5}\right| = \frac{7}{5}$

Mathematics - Section B (NUMERIC)

(6) Let A be a square matrix of order 3 such that $\det(A) = -2$ and $\det(3\,\text{adj}(-6\,\text{adj}(3A))) = 2^{m+n} \cdot 3^{mn}$, $m > n$. Then $4m + 2n$ is equal to ______ [JEE MAIN 2025]

Solution:
$|A| = -2$
$\det(3\,\text{adj}(-6\,\text{adj}(3A)))$
$= 3^3 \det(\text{adj}(-\text{adj}(3A)))$
$= 3^3(-6)^6(\det(3A))^4$
$= 3^{21} \times 2^{10}$
$m + n = 10$
$mn = 21$
$m = 7; n = 3$

(7) For a 3×3 matrix M, let trace (M) denote the sum of all the diagonal elements of M. Let A be a 3×3 matrix such that $|A| = \frac{1}{2}$ and trace $(A) = 3$. If $B = \text{adj}(\text{adj}(2A))$, then the value of $|B|+$ trace (B) equals: [JEE MAIN 2025]

Solution:
$|A| = \frac{1}{2}, \text{trace}(A) = 3, B = \text{adj}(\text{adj}(2A)) = |2A|^{2-2}(2A)$
$n = 3, B = |2A|(2A) = 2^3 \cdot |A|(2A) = 8A$
$|B| = |8A| = 8^3 \cdot |A| = 2^8 = 256$
$\text{trace}(B) = 8\,\text{trace}(A) = 24$
$|B| + \text{trace}(B) = 280$

(8) If the system of linear equations : $x + y + 2z = 6$, $2x + 3y + az = a + 1$, $-x - 3y + bz = 2b$ where $a, b \in R$, has infinitely many solutions, then $7a + 3b$ is equal to : [JEE MAIN 2025]

Solution:
$$\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & a \\ -1 & -3 & b \end{vmatrix} = 0$$
$\Rightarrow 2a + b - 6 = 0$
$$\Delta_1 = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 3 & a+1 \\ -1 & -3 & 2b \end{vmatrix} = 0$$
$\Rightarrow a + b - 8 = 0$
Solving $(1) + (2)$
$a = -2, b = 10$
$\Rightarrow 7a + 3b = 16$

(9) If the system of equations $(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$, $\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$, $(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$ has infinitely many solutions, then $\lambda^2 + \lambda$ is equal to [JEE MAIN 2025]

Solution:
$(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$
$\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$
$(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$
For infinitely many solutions
$$D = \begin{vmatrix} \lambda-1 & \lambda-4 & \lambda \\ \lambda & \lambda-1 & \lambda-4 \\ \lambda+1 & \lambda+2 & -(\lambda+2) \end{vmatrix} = 0$$
$(\lambda - 3)(2\lambda + 1) = 0$
$$D_x = \begin{vmatrix} 5 & \lambda-4 & \lambda \\ 7 & \lambda-1 & \lambda-4 \\ 9 & \lambda+2 & -(\lambda+2) \end{vmatrix} = 0$$
$2(3 - \lambda)(23 - 2\lambda) = 0$
$\lambda = 3$
$\therefore \lambda^2 + \lambda = 9 + 3 = 12$

(10) If the system of equations $2x - y + z = 4$, $5x + \lambda y + 3z = 12, 100x - 47y + \mu z = 212$ has infinitely many solutions, then $\mu - 2\lambda$ is equal to [JEE MAIN 2025]

Solution:
$$\Delta = 0 \Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ 5 & \lambda & 3 \\ 100 & -47 & \mu \end{vmatrix} = 0$$
$2(\lambda\mu + 141) + (5\mu - 300) - 235 - 100\lambda = 0 \ldots$
$$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 2 & -1 & 4 \\ 5 & \lambda & 12 \\ 100 & -47 & 212 \end{vmatrix} = 0$$
$6\lambda = -12 \Rightarrow \lambda = -2$
Put $\lambda = 2$ in.......(1)
$2(-2\mu + 141) + 5\mu - 300 - 235 + 200 = 0$
$\mu = 53$
$\therefore 57$

(11) Let be a 3×3 matrix such that $X^T AX = O$ for all nonzero 3×1 matrices $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. If
$A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}, A\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$, and $\det(\text{adj}(2(A + I))) = 2^\alpha 3^\beta 5^\gamma, \alpha, \beta, \gamma, \in N$, then $\alpha^2 + \beta^2 + \gamma^2$ is [JEE MAIN 2025]

Solution:
$X^T AX = 0$
(xyz) $\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$
(xyz) $\begin{pmatrix} a_1x + a_2y + a_3z \\ b_1x + b_2y + b_3z \\ c_1x + c_2y + c_3z \end{pmatrix} = 0$
$x(a_1x + a_2y + a_3z) + y(b_1x + b_2y + b_3z)$
$+z(c_1x + c_2y + c_3z) = 0$
$a_1 = 0, b_2 = 0c_3 = 0$
$a_2 + b_1 = 0, a_3 + c_1 = 0, b_3 = c_2 = 0$
$A =$ skew symm matrix
$$A = \begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix}; \quad A = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$$
$$\Rightarrow A = \begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$$
$x + y = 1$
$-x + z = 4$
$y + z = 5$

$$\begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}=\begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$$

$2x+y=0x=-1$

$-x+z=4y=2$

$-y-2z=-8z=3$

$$A=\begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$$

$$2(A+I)=\begin{pmatrix} 2 & -2 & 4 \\ 2 & 2 & 6 \\ -2 & -6 & 2 \end{pmatrix}$$

$2(A+I)=120 \Rightarrow \det|\operatorname{adi}(2(A+I))|$

$=120^2=2^6 \cdot 3^2 \cdot 5^2$

$\alpha=6, \beta=2, \gamma=2$

(12) If the system of equations $x+2y-3z=2$, $2x+\lambda y+5z=5$, $14x+3y+\mu z=33$ has infinitely many solutions, then $\lambda+\mu$ is equal to: [JEE MAIN 2025]

Solution:

$$D=\begin{vmatrix} 1 & 2 & -3 \\ 2 & \lambda & 5 \\ 14 & 3 & \mu \end{vmatrix}=0, \lambda\mu+42\lambda-4\mu+107=0$$

$D_1=2\lambda\mu+99\lambda-10\mu+255$

$D_2=13-\mu$

$D_3=5\lambda+5$

$D_2=0 \Rightarrow \mu=13$ $D_3=0 \Rightarrow \lambda=-1$

check verify for these values $D \& D_2=0$

(13) For some a, b, let

$$f(x)=\begin{vmatrix} a+\frac{\sin x}{x} & 1 & b \\ a & 1+\frac{\sin x}{x} & b \\ a & 1 & b+\frac{\sin x}{x} \end{vmatrix}, \quad x \neq 0,$$

$\lim_{x \to 0} f(x)=\lambda+\mu a+vb$. Then $(\lambda+\mu+v)^2$ is equal to: [JEE MAIN 2025]

Solution:

$$\lim_{x \to 0} f(x)=\begin{vmatrix} a+1 & 1 & b \\ a & 1+1 & b \\ a & 1 & b+1 \end{vmatrix}$$

$=(a+1)(2(b+1)-b)+1(ab-a(b+1))+ba$

$=(a+1)(b+2)-a+ab$

$=b+a+2=\lambda+\mu a+vb$

$\lambda=2, \mu=1, v=1 \Rightarrow(\lambda+\mu+v)^2=16$

(14) Let M denote the set of all real matrices of order 3×3 and let $S=\{-3,-2,-1,1,2\}$. Let

$S_1=\left\{A=[a_{ij}] \in M: A=A^T \text{ and } a_{ij} \in S, \forall i, j\right\}$

$S_2=\left\{A=[a_{ij}] \in M: A=-A^T \text{ and } a_{ij} \in S, \forall i, j\right\}$

$S_3=$

$\{A=[a_{ij}] \in M: a_{11}+a_{22}+a_{33}=0 \text{ and } a_{ij} \in S, \forall i, j\}$

If $n(S_1 \cup S_2 \cup S_3)=125\alpha$, then α equals. [JEE MAIN 2025]

Solution:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

No. of elements in $S_1: A=A^T \Rightarrow 5^3 \times 5^3$

No. of elements in $A=-A^T \Rightarrow 0$

since no. zero in 5

No. of elements in $S_3 \Rightarrow$

$$\left.\begin{array}{c} a_{11}+a_{22}+a_{33}=0 \Rightarrow(1,2,-3) \Rightarrow 31 \\ \text{or} \\ (1,1,-2) \Rightarrow 3 \\ \text{or} \\ (-1,-1,2) \Rightarrow 3 \end{array}\right\} \Rightarrow 12 \times 5^6$$

$n(S_1 \cap S_3)=12 \times 5^3$

$n(S_1 \cup S_2 \cup S_3)=5^6(1+12)-12 \times 5^3$

$\Rightarrow 5^3 \times\left[13 \times 5^3-12\right]=125\alpha$

$\alpha=1613$

(15) Let $A=\begin{bmatrix} \frac{1}{\sqrt{2}} & -2 \\ 0 & 1 \end{bmatrix}$ and $P=\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \theta>0$. If $B=PAP^T, C=P^TB^{10}P$ and the sum of the diagonal elements of C is $\frac{m}{n}$, where $\gcd(m, n)=1$, then $m+n$ is : [JEE MAIN 2025]

Solution:

$$P=\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$\because P^TP=I$

$B=PAPT$

Pre multiply by P^T (Given)

$P^TB=P^TPAP^T=AP^T$

Now post multiply by P

$P^TBP=AP^TP=A$

So $A^2=\underbrace{P^TBPP^TBP}_{I}$

$A^2=P^TB^2P$

Similarly $A^{10}=P^TB^{10}P=C$

$A=\begin{bmatrix} \frac{1}{\sqrt{2}} & -2 \\ 0 & 1 \end{bmatrix}$ (Given)

$\Rightarrow A^2=\begin{bmatrix} \frac{1}{2} & -\sqrt{2}-2 \\ 0 & 1 \end{bmatrix}$

Similarly check A^3 and so on since $C=A^{10}$

$\Rightarrow$ Sum of diagonal elements of C is $\left(\frac{1}{\sqrt{2}}\right)^{10}+1$

$=\frac{1}{32}+1=\frac{33}{32}=\frac{m}{n}$

$\operatorname{g\,cd}(m, n)=1$ (Given)

$\Rightarrow m+n=65$

(16) Let M and m respectively be the maximum and the minimum values of

$$f(x)=\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 4x \\ \sin^2 x & 1+\cos^2 x & 4\sin 4x \\ \sin^2 x & \cos^2 x & 1+4\sin 4x \end{vmatrix}, x \in R \text{ Then}$$

M^4-m^4 is equal to :____ [JEE MAIN 2025]

Solution:

$$\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 4x \\ \sin^2 x & 1+\cos^2 x & 4\sin 4x \\ \sin^2 x & \cos^2 x & 1+4\sin 4x \end{vmatrix}, x \in R$$

$R_2 \rightarrow R_2-R_1 \& R_3 \rightarrow R_3 \rightarrow R_1$

$$f(x)\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 4x \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

Expand about R_1, use get

$f(x)=2+4\sin 4x$

$\therefore M=$ max value of $f(x)=6$

$M=$ min value of $f(x)=-2$

$\therefore M^4-M^4=1280$

(17) Let $A=[a_{ij}]=\begin{bmatrix} \log_5 128 & \log_4 5 \\ \log_5 8 & \log_4 25 \end{bmatrix}$. If A_{ij} is the cofactor of $a_{ij}, C_{ij}=\sum_{k=1}^{2} a_{ik}A_{jk}, 1 \leq i, j \leq 2$, and $C=[C_{ij}]$, then $8|C|$ is equal to : [JEE MAIN 2025]

Solution:

$|A|=\frac{11}{2}$

$C_{11}=\sum_{k=1}^{2} a_{1k} \cdot A_{1k}=a_{11}A_{11}+a_{12}A_{12}=|A|=\frac{11}{2}$

$C_{12}=\sum_{k=1}^{2} a_{1k} \cdot A_{2k}=0$

$C_{21}=\sum_{k=1}^{2} a_{2k} \cdot A_{1k}=0$

$C_{22}=\sum_{k=1}^{2} a_{2k}\cdot A_{2k}=|A|=\frac{11}{2}$
$C=\left[\begin{array}{cc} 11/2 & 0 \\ 0 & 11/2 \end{array}\right]$
$|C|=\frac{121}{4}$
$8|C|=242$

(18) Let $S=\left\{m\in Z: A^{m^2}+A^m=3I-A^{-6}\right\}$, where $A=\left[\begin{array}{cc} 2 & -1 \\ 1 & 0 \end{array}\right]$. Then $n(S)$ is equal to [JEE MAIN 2025]

Solution:
$A=\left[\begin{array}{cc} 2 & -1 \\ 1 & 0 \end{array}\right]$
$A^2=\left[\begin{array}{cc} 3 & -2 \\ 2 & -1 \end{array}\right], A^3=\left[\begin{array}{cc} 4 & -3 \\ 3 & -2 \end{array}\right], A^4=\left[\begin{array}{cc} 5 & -4 \\ 4 & -3 \end{array}\right]$
and so on
$A^6=\left[\begin{array}{cc} 7 & -6 \\ 6 & -5 \end{array}\right]$
$A^m=\left[\begin{array}{cc} m+1 & -m \\ m & -m-1 \end{array}\right]$
$A^{m^2}=\left[\begin{array}{cc} m^2+1 & -m^2 \\ m^2 & -\left(m^2-1\right) \end{array}\right]$
$A^{m^2}+A^m=3I-A^{-6}$
$\left[\begin{array}{cc} m+1 & -m^2 \\ m^2 & -\left(m^2-1\right) \end{array}\right]+\left[\begin{array}{cc} m+1 & -m \\ m & -(m-1) \end{array}\right]$
$=3\left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right]-\left[\begin{array}{cc} -5 & 6 \\ -6 & 7 \end{array}\right]$
$=\left[\begin{array}{cc} 8 & -6 \\ 6 & -4 \end{array}\right]$
$=m^2+1+m+1=8$
$=m^2+m-6=0\Rightarrow m=-3,2$
$n(s)=2$

(19) Let $\alpha,\beta(\alpha\neq\beta)$ be the values of m, for which the equations $x+y+z=1; x+2y+4z=m$ and $x+4y+10z=m^2$ have infinitely many solutions. Then the value of $\sum_{n=1}^{10}\left(n^\alpha+n^\beta\right)$ is equal to : [JEE MAIN 2025]

Solution:
$\Delta=\left|\begin{array}{ccc} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{array}\right|=1(20-16)-1(10-4)+1(4-2)$
$=4-6+2=0$
For infinite solutions
$\Delta_x=\Delta_y=\Delta_z=0$
$m^2-3x+2=0$
$m=1,2$
$\alpha=1,\beta=2$
$\therefore \sum_{n=1}^{10}\left(n^\alpha+n^\beta\right)=\sum_{n=1}^{10} n^1+\sum_{n=1}^{10} n^2$
$=\frac{10(11)}{2}+\frac{10(11)(21)}{6}$
$=55+385$
$=440$

(20) Let $A=[a_{ij}]$ be a matrix of order 3×3, with $a_{ij}=(\sqrt{2})^{i+j}$. If the sum of all the elements in the third row of A^2 is $\alpha+\beta\sqrt{2},\alpha,\beta\in Z$, then $\alpha+\beta$ is equal to [JEE MAIN 2025]

Solution:
$A=\left[\begin{array}{ccc} (\sqrt{2})^2 & (\sqrt{2})^3 & (\sqrt{2})^4 \\ (\sqrt{2})^3 & (\sqrt{2})^4 & (\sqrt{2})^5 \\ (\sqrt{2})^4 & (\sqrt{2})^5 & (\sqrt{2})^6 \end{array}\right]$
$A=\left[\begin{array}{ccc} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{array}\right]$
$A^2=2^2\left[\begin{array}{ccc} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 2 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 4 \end{array}\right]\left[\begin{array}{ccc} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 2 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 4 \end{array}\right]$
$=4\left[\begin{array}{ccc} - & - & - \\ - & - & - \\ (2+4+8) & (2\sqrt{2}+4\sqrt{2}+8\sqrt{2}) & (4+8+16) \end{array}\right]$
Sum of elements of 3rd row $=4(14+14\sqrt{2}+28)$
$=4(42+14\sqrt{2})$
$=168+56\sqrt{2}$
$\alpha+\beta\sqrt{2}$
$\therefore \alpha\alpha+\beta=168+56=224$

(21) Let integers $a,b\in[-3,3]$ be such that $a+b\neq 0$. Then the number of all possible ordered pairs (a,b), for which $\left|\frac{z-a}{z+b}\right|=1$ and $\left|\begin{array}{ccc} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{array}\right|=1, z\in C$, where ω and ω^2 are the roots of $x^2+x+1=0$, is equal to________ [JEE MAIN 2025]

Solution:
$a,b\in I,-3\leq a,b\leq 3,a+b\neq 0$
$|z-a|=|z+b|$
$\left|\begin{array}{ccc} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{array}\right|=1$
$\Rightarrow\left|\begin{array}{ccc} z & z & z \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{array}\right|=1$
$\Rightarrow z\left|\begin{array}{ccc} 1 & 1 & 1 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{array}\right|=1$
$\Rightarrow z\left|\begin{array}{ccc} 1 & 0 & 0 \\ \omega & z+\omega^2-\omega & 1-\omega \\ \omega^2 & 1-\omega^2 & z+\omega-\omega^2 \end{array}\right|=1$
$\Rightarrow z^3=1$
$\Rightarrow z=\omega,\omega^2,1$
Now
$|1-a|=|1+b|$
$\Rightarrow 10$ pairs

(22) If the system of linear equations $3x+y+\beta z=3$, $2x+\alpha y-z=-3$, $x+2y+z=4$ has infinitely many solutions, then the value of $22\beta-9\alpha$ is : [JEE MAIN 2025]

Solution:
$\Delta=\left|\begin{array}{ccc} 3 & 1 & \beta \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{array}\right|=0$
$3\alpha+4\beta-\alpha\beta+3=0$
$\Delta_3=\left|\begin{array}{ccc} 3 & 1 & 3 \\ 2 & \alpha & -3 \\ 1 & 2 & 4 \end{array}\right|=0$
$9\alpha+19=0$
$\alpha=\frac{-19}{9},\beta=\frac{6}{11}$
$\Rightarrow 22\beta-9\alpha=31$

(23) Let $a\in R$ and A be a matrix of order 3×3 such that det$(A)=-4$ and $A+I=\left[\begin{array}{ccc} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{array}\right]$, where I is the identity matrix of order 3×3. If det$((a+1)\,$adj$((a-1)A))$ is $2^m 3^n, m,n\in\{0,1,2,\ldots\ldots 20\}$, then $m+n$ is equal to : [JEE MAIN 2025]

Solution:
$A=\left[\begin{array}{ccc} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{array}\right]-I=\left[\begin{array}{ccc} 0 & a & 1 \\ 2 & 0 & 0 \\ a & 1 & 1 \end{array}\right]$
$|A|=-4\Rightarrow 2-2a=-4\Rightarrow a=3$

$|(a+1) \text{ adj } (a-1)A| = |4 \text{ adj } 3A|$
$= 4^3 |\text{ adj } 3A|$
$= 4^3 \times |3A|^{3-1} = 64|3A|^2$
$= 64 \times (3^3)^2 |A|^2$
$= 2^6 \times 3^6 \times 16$
$2^m \times 3^n = 2^{10} \times 3^6$
$\therefore m = 10, n = 6$
$\Rightarrow m + n = 16$

(24) If the system of equation $2x + \lambda y + 3z = 5$, $3x + 2y - z = 7$, $4x + 5y + \mu z = 9$ has infinitely many solutions, then $(\lambda^2 + \mu^2)$ is equal to : [JEE MAIN 2025]

Solution:
$\Delta = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \mu \end{vmatrix} = 0$
$\Rightarrow 2(2\mu + 5) + \lambda(-4 - 3\mu) + 3(7) = 0$
$\Rightarrow 4\mu - 3\lambda\mu - 4\lambda + 31 = 0 \ldots\ldots(1)$
$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 5 \\ 3 & 2 & 7 \\ 4 & 5 & 9 \end{vmatrix} = 0$
$\Rightarrow 2(-17) + \lambda(1) + 5(7) = 0$
$\Rightarrow \lambda = -1$
from equation (1)
$4\mu + 3\mu + 4 + 31 = 0 \Rightarrow \mu = -5$
$\therefore \lambda^2 + \mu^2 = 26$

(25) Let A be a 3×3 real matrix such that $A^2(A - 2I) - 4(A - I) = O$, where I and O are the identity and null matrices, respectively. If $A^5 = \alpha A^2 + \beta A + \gamma I$, where α, β and γ are real constants, then $\alpha + \beta + \gamma$ is equal to: [JEE MAIN 2025]

Solution:
$A^3 - 2A^2 - 4A + 4I = 0$
$A^3 = 2A^2 + 4A - 4I$
$A^4 = 2A^3 + 4A^2 - 4A$
$= 2(2A^2 + 4A - 4I) + 4A^2 - 4A$
$A^4 = 8A^2 + 4A - 8I$
$A^5 = 8A^3 + 4A^2 - 8A$
$= 8(2A^2 + 4A - 4I) + 4A^2 - 8A$
$A^5 = 20A^2 + 24A - 32I$
$\therefore \alpha = 20, \beta = 24, \gamma = -32$
$\therefore \alpha + \beta + \gamma = 12$

(26) Let A be a matrix of order 3×3 and $|A| = 5$. If $|2 \text{ adj}(3A \text{ adj}(2A))| = 2^\alpha \cdot 3^\beta \cdot 5^\gamma \alpha, \beta, \gamma \in N$ then $\alpha + \beta + \gamma$ is equal to [JEE MAIN 2025]

Solution:
$|2 \text{ adj}(3A \text{ adj}(2A))|$
$2^3 \cdot |3A \text{ adj}(2A)|^2$
$2^3 \cdot (3^3)^2 \cdot |A|^2 \cdot |adj(2A)|^2$
$2^3 \cdot 3^6 \cdot |A|^2 \cdot (|2A|^2)^2$
$2^3 \cdot 3^6 \cdot |A|^2 \left[(2^3)^2 \cdot |A|^2\right]^2$
$2^3 \cdot 3^6 \cdot |A|^2 \cdot 2^{12} \cdot |A|^4$
$2^{15} \cdot 3^6 \cdot |A|^6$
$2^{15} \cdot 3^6 \cdot 5^6 = 2^\alpha \cdot 3^\beta \cdot 5^\gamma$
$\alpha = 15, \beta = 6, \gamma = 6$
$\alpha + \beta + \gamma = 27$

(27) Let I be the identity matrix of order 3×3 and for the matrix $A = \begin{bmatrix} \lambda & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -1 & 2 \end{bmatrix}$, $|A| = -1$. Let B be the inverse of the matrix adj $(A \text{ adj } (A^2))$. Then $|(\lambda B + 1)|$ is equal to ______. [JEE MAIN 2025]

Solution:
$|A| = \begin{vmatrix} \lambda & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -1 & 2 \end{vmatrix} = -1$
$\lambda(16) - 2(-34) + 3(-39) = -1$
$16\lambda = 48 \Rightarrow \lambda = 3$
$B^{-1} = \text{adj}(A \cdot \text{adj}(A^2))$
Let $C = A. \text{adj}(A^2)$
$AC = A^2 \text{ adj}(A^2) = |A|^2 \cdot I = I \Rightarrow C = A^{-1}$
Now $B^{-1} = \text{adj}(A^{-1}) = B = \text{adj}(A)$
Now $\lambda B + I \Rightarrow 3B + I$
Let $P = 3B + I$
$P = 3 \text{ adj}(A) + I$
$AP = 3 \text{ Aadj}(A) + A$
$AP = 3|A| \cdot I + A$
$AP = A - 3I$
$|AP| = |A - 3I|$
$|A| \cdot |P| = \begin{vmatrix} 0 & 2 & 3 \\ 4 & 2 & 6 \\ 7 & -1 & -1 \end{vmatrix} = 38$
$|P| = -38$

(28) Let $A = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$. If for some $\theta \in (0, \pi)$, $A^2 = A^T$, then the sum of the diagonal elements of the matrix $(A + I)^3 + (A - I)^3 - 6A$ is equal to ______. [JEE MAIN 2025]

Solution:
$\because A$ is orthogonal matrix
$\therefore A^T = A^{-1}$
$\Rightarrow A^2 = A^{-1}$
$\Rightarrow A^3 = I$
let $B = (A + I)^3 + (A - I)^3 - 6A$
$= 2(A^3 + 3A) - 6A$
$= 2A^3$
$B = 2I = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
Now sum of diagonal elements $= 2 + 2 + 2 = 6$

(29) Let the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ satisfy $A^n = A^{n-2} + A^2 - I$ for $n \geq 3$. Then the sum of all the elements of A^{50} is :- [JEE MAIN 2025]

Solution:
$A^{50} = A^{48} + A^2 - I$
$= A^{46} + 2(A^2 - I)$
$= A^{44} + 3(A^2 - I)$
$= A^2 + 24(A^2 - I)$
$= 25A^2 - 24I$
$= 25 \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} - 24 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
$= \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$
Sum $= 53$

(30) Let A be a 3×3 matrix such that $|\text{adj}(\text{adj}(\text{adj } A))| = 81$. If

$S=\left\{n \in Z : (|\operatorname{adj}(\operatorname{adj} A)|)^{\frac{(n-1)^2}{2}}=|A|^{\left(3 n^2-5 n-4\right)}\right\}$, then $\sum_{n \in S}\left|A^{\left(n^2+n\right)}\right|$ is equal to [JEE MAIN 2025]

Solution:

$|\operatorname{adj}(\operatorname{adj})(\operatorname{adj} A)|=81$
$\Rightarrow|\operatorname{adj} A|^4=81$
$\Rightarrow|\operatorname{adj} A|=3$
$\Rightarrow|A|^2=3$
$\Rightarrow|A|=\sqrt{3}$
$\left(|A|^4\right)^{\frac{(n-1)^2}{2}}=|A|^{3 n^2-5 n-4}$
$\Rightarrow 2(n-1)^2=3 n^2-5 n-4$
$\Rightarrow 2 n^2-4 n+2=3 n^2-5 n-4$
$\Rightarrow n^2-n-6=0$
$\Rightarrow(n-3)(n+2)=0$
$\Rightarrow n=3,-2$
$\sum_{n \in s}\left|A^{n^2+n}\right|$
$=\left|A^2\right|+\left|A^{12}\right|$
$=3+36=3+729=732$

(31) Let the system of equations $x+5 y-z=1$, $4 x+3 y-3 z=7$, $24 x+y+\lambda z=\mu$, $\lambda, \mu \in R$, have infinitely many solutions. Then the number of the solutions of this system, If x, y, z are integers and satisfy $7 \leq x+y+z \leq 77$, is [JEE MAIN 2025]

Solution:

For infinitely many solution
$\Delta=0$
$\left|\begin{array}{ccc}1 & 5 & -1 \\ 4 & 3 & -3 \\ 24 & 1 & \lambda\end{array}\right|=0$
$\Rightarrow 1(3 \lambda+3)-5(4 \lambda+72)-1(4-72)=0$
$\Rightarrow-17 \lambda+3-4 \times 72-4=0$
$\Rightarrow 17 \lambda=-289$
$\Rightarrow \lambda=-17$
$\Delta_1=0$
$\Rightarrow\left|\begin{array}{ccc}1 & 5 & -1 \\ 7 & 3 & -3 \\ \mu & 1 & -17\end{array}\right|=0$
$\Rightarrow 1(-51+3)-5(-119+3 \mu)-1(7-3 \mu)=0$
$\Rightarrow-48+595-15 \mu-7+3 \mu=0$
$\Rightarrow 12 \mu=540$
$\mu=45$
$x+5 y-z=1$
$4 x+3 y-3 z=7$
$24 x+y-17 z=45$
Let $z=1$
$x+5 y=1+\lambda] \times 4$
$4 x+3 y=7+3 \lambda$
$\underline{4 x+20 y=\underline{4}+\underline{4} \lambda}$
$-17 y=3-\lambda$
$y=\frac{\lambda-3}{17}, x=1+\lambda-\frac{5 \lambda-15}{17}$
$=\frac{32\_12 \lambda}{17}$
$7 \leq \frac{\lambda-3}{17}+\frac{32+12 \lambda}{17}+\lambda \leq 77$
$7 \leq \frac{30 \lambda+29}{17} \leq 77$
$3 \leq \lambda \leq 42$
$\lambda=3,20,37$

(32) Let α be a solution of $x^2+x+1=0$, and for some a and b in R, $\left[\begin{array}{lll}4 & a & b\end{array}\right]\left[\begin{array}{ccc}1 & 16 & 13 \\ -1 & -1 & 2 \\ -2 & -14 & -8\end{array}\right]=\left[\begin{array}{lll}0 & 0 & 0\end{array}\right]$. If $\frac{4}{\alpha^4}$ $+\frac{m}{\alpha^a}+\frac{n}{\alpha^b}=3$, then $m+n$ is equal to [JEE MAIN 2025]

Solution:

$x^2+x+1=0$
α is root
$\therefore \alpha^2+\alpha+1=0$
$\Rightarrow \alpha=\omega$ as ω^2 [cube root of unity]
also
$\left[\begin{array}{lll}4-a-2 b & 64-a-14 b & 52+2 a-8 b\end{array}\right]$
$=\left[\begin{array}{lll}0 & 0 & 0\end{array}\right]$
$\therefore a+2 b=4$
$a+14 b=64$
$\Rightarrow 12 b=60 \Rightarrow b=5$
$\Rightarrow a=-6$
$\therefore \frac{4}{\alpha^4}+\frac{m}{\alpha^{-6}}+\frac{n}{\alpha^5}=3$
$\Rightarrow \frac{4}{\omega}+\frac{m}{1}+\frac{n}{\omega^2}=3$
$\Rightarrow 4 \omega^2+m+n \omega=3$
$\Rightarrow 4\left(-\frac{1}{2}-\frac{\sqrt{3}}{2} i\right)+m+n\left(-\frac{1}{2}+\frac{\sqrt{3}}{2} i\right)=3$
$\therefore-2+m-\frac{n}{2}=3$
$\frac{-4 \sqrt{3}}{2}+\frac{n \sqrt{3}}{2}=0$
$\therefore n=4$
$m=7$
$\therefore m+n=11$

(33) Let $A=\left[\begin{array}{ccc}2 & 2+p & 2+p+q \\ 4 & 6+2 p & 8+3 p+2 q \\ 6 & 12+3 p & 20+6 p+3 q\end{array}\right]$ If $\operatorname{det}(\operatorname{adj}(\operatorname{adj}(3 A)))=2^m \cdot 3^n, m, n \in N$, then $m+n$ is equal to [JEE MAIN 2025]

Solution:

$|A|=\left|\begin{array}{ccc}2 & 2+p & 2+p+q \\ 4 & 6+2 p & 8+3 p+2 q \\ 6 & 12+3 p & 20+6 p+3 q\end{array}\right|$
$C_3 \rightarrow C_3-C_2-C_1 \times \frac{q}{2}$
Then $C_3 \rightarrow C_2-C_1 x\left(1+\frac{p}{2}\right)$
$\Rightarrow|A|=\left|\begin{array}{ccc}2 & 0 & 0 \\ 4 & 2 & 2+p \\ 6 & 6 & 8+3 p\end{array}\right|$
$\Rightarrow|A|=2(16+6 p-12-6 p)=8=2^3$
$|\operatorname{adj}(\operatorname{adj}(3 A))|=|3 A|^{(3-1)^2}=|3 A|^4$
$=\left(3^3|A|\right)^4=\left(3^3 \times 2^3\right)^4=2^{12} \times 3^{12}$
$\Rightarrow m+n=24$

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2025 Matrices Determinants PYQs

(Solutions)

Paper Set : 1
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Time : 1H:20M

Mathematics - Section A (MCQ)

(1) જો A, B અને $\left(\text{adj}\left(A^{-1}\right)+\text{adj}\left(B^{-1}\right)\right)$ એ સમાન કક્ષાના શૂન્યતર શ્રેણિક છે તો $A\left(\text{adj}\left(A^{-1}\right)+\text{adj}\left(B^{-1}\right)\right)^{-1} B$ નો વ્યસ્ત શ્રેણિક મેળવો. [JEE MAIN 2025]

(A) $AB^{-1}+A^{-1}B$
(B) $\text{adj}\left(B^{-1}\right)+\text{adj}\left(A^{-1}\right)$
(C) $\frac{1}{|AB|}(\text{adj}(B)+\text{adj}(A))$
(D) $\frac{AB^{-1}}{|A|}+\frac{BA^{-1}}{|B|}$

Solution:(Correct Answer:C)

$\left[A\left(\text{adj}\left(A^{-1}\right)+\text{adj}\left(B^{-1}\right)\right)^{-1}\cdot B\right]^{-1}$
$B^{-1}\cdot\left(\text{adj}\left(A^{-1}\right)+\text{adj}\left(B^{-1}\right)\right)\cdot A^{-1}$
$B^{-1}\,\text{adj}\left(A^{-1}\right)A^{-1}+B^{-1}\left(\text{adj}\left(B^{-1}\right)\right)\cdot A^{-1}$
$B^{-1}\left|A^{-1}\right|I+\left|B^{-1}\right|IA^{-1}$
$\frac{B^{-1}}{|A|}+\frac{A^{-1}}{|B|}$
$\Rightarrow\frac{\text{adj }B}{|B||A|}+\frac{\text{adj }A}{|A||B|}$
$=\frac{1}{|A||B|}(\text{adjB}+\text{adj }A)$

(2) સમીકરણોની સંહતિ $x+y+z=6$, $x+2y+5z=9$, $x+5y+\lambda z=\mu$ ને એકપણ ઉકેલ નો હોય જો . . .. [JEE MAIN 2025]

(A) $\lambda=17, \mu\neq 18$
(B) $\lambda\neq 17, \mu\neq 18$
(C) $\lambda=15, \mu\neq 17$
(D) $\lambda=17, \mu=18$

Solution:(Correct Answer:A)

$$D=\begin{vmatrix}1&1&1\\1&2&5\\1&5&\lambda\end{vmatrix}=0$$
$\lambda=17$
$$D_z=\begin{vmatrix}1&1&6\\1&2&9\\1&5&\mu\end{vmatrix}\neq 0$$
$\mu\neq 18$

(3) અહી $A=[a_{ij}]$ એ 3×3 શ્રેણિક છે કે જેથી $A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}$ અને $A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$, તો a_{23} ની કિંમત મેળવો. [JEE MAIN 2025]

(A) -1
(B) 0
(C) 2
(D) 1

Solution:(Correct Answer:A)

Let $A=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}$

$A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}\Rightarrow\begin{bmatrix}a_{12}\\a_{22}\\a_{32}\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}\Rightarrow\begin{matrix}a_{22}=0; a_{12}=0\\a_{32}=1\end{matrix}$

$A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}\Rightarrow\begin{matrix}4a_{11}+a_{12}+3a_{13}=0\\4a_{21}+a_{22}+3a_{25}=1\Rightarrow 4a_{21}+3a_{25}\\4a_{31}+a_{32}+3a_{33}=0\end{matrix}$

$A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}\Rightarrow\begin{matrix}2a_{11}+a_{12}+2a_{13}=1\\2a_{21}+a_{22}+2a_{23}=0\Rightarrow a_{21}+a_{23}=\\2a_{31}+a_{32}+2a_{33}=0\end{matrix}$

$-4a_{23}+3a_{23}=1\Rightarrow a_{23}=-1$

(4) અહી $A=\begin{bmatrix}\alpha&-1\\6&\beta\end{bmatrix}$, $\alpha>0$, આપેલ છે કે જેથી $\det(A)=0$ અને $\alpha+\beta=1$ છે. જો I એ 2×2 એકમ શ્રેણિક હોય તો શ્રેણિક $(I+A)^8=$ [JEE MAIN 2025]

(A) $\begin{bmatrix}4&-1\\6&-1\end{bmatrix}$
(B) $\begin{bmatrix}257&-64\\514&-127\end{bmatrix}$
(C) $\begin{bmatrix}1025&-511\\2024&-1024\end{bmatrix}$
(D) $\begin{bmatrix}766&-255\\1530&-509\end{bmatrix}$

Solution:(Correct Answer:D)

$|A|=0$
$\alpha\beta+6=0$
$\alpha\beta=-6$
$\alpha+\beta=1$
$\Rightarrow\alpha=3,\beta=-2$
$A=\begin{bmatrix}3&-1\\6&-2\end{bmatrix}$
$A^2=\begin{bmatrix}3&-1\\6&-2\end{bmatrix}\begin{bmatrix}3&-1\\6&-2\end{bmatrix}=\begin{bmatrix}3&-1\\6&-2\end{bmatrix}$
$\therefore A^2=A$
$A=A^2=A^3=A^4=A^5$
$(I+A)^8$
$=I+{}^8C_1A^7+{}^8C_2A^6+\ldots.+{}^8C_8A^8$
$=I+A\left({}^8C_1+{}^8C_2+\ldots..+{}^8C_8\right)$
$=I+A\left(2^8-1\right)$
$=\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}765&-255\\1530&-510\end{bmatrix}$
$=\begin{bmatrix}766&-255\\1530&-509\end{bmatrix}$

(5) અહી સમીકરણ સંહતિ : $2x+3y+5z=9$, $7x+3y-2z=8$, $12x+3y-(4+\lambda)z=16-\mu$ ને અનંત ઉકેલ ધરાવે છે. તો વર્તુળ કે જેનું કેન્દ્ર (λ,μ) અને જે રેખા $4x=3y$ ને સ્પર્શે છે તેની ત્રિજ્યા મેળવો. [JEE MAIN 2025]

(A) $\frac{17}{5}$
(B) $\frac{7}{5}$
(C) 7
(D) $\frac{21}{5}$

Solution:(Correct Answer:B)

$$\begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 12 & 3 & -(\lambda+4) \end{vmatrix}=0$$
$\Rightarrow 12(-21)-3(-39)-(\lambda+4)(-15)=0$
$\Rightarrow -252+117+15(1+4)=0$
$\Rightarrow 15\lambda+177-252=0$
$\Rightarrow 15\lambda-75=0 \Rightarrow \lambda=5$
$$\begin{vmatrix} 9 & 3 & 5 \\ 8 & 3 & -2 \\ 16-\mu & 3 & -9 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 1 & 0 & 7 \\ \mu-8 & 0 & 7 \\ 16-\mu & 3 & -9 \end{vmatrix}=0$$
$\Rightarrow 7-7(\mu-8)=0 \Rightarrow 1-(\mu-8)=0 \Rightarrow \mu=9$
$\Rightarrow$ centre of circle $(5,9)$
radius $=$ length of $\perp$ from centre $(5,9)=\left|\frac{20-27}{5}\right|=\frac{7}{5}$

Mathematics - Section B (NUMERIC)

(6) અહી A 3 કક્ષાવાળો ચોરસ શ્રેણિક છે કે જેથી $\det(A)=-2$ અને $\det(3\,\text{adj}(-6\,\text{adj}(3A)))=2^{m+n}\cdot 3^{mn}$, $m>n$ હોય તો $4m+2n$ ની કિંમત મેળવો. [JEE MAIN 2025]

Solution:
$|A|=-2$
$\det(3\,\text{adj}(-6\,\text{adj}(3A)))$
$=3^3\det(\text{adj}(-\text{adj}(3A)))$
$=3^3(-6)^6(\det(3A))^4$
$=3^{21}\times 2^{10}$
$m+n=10$
$mn=21$
$m=7; n=3$

(7) એક 3×3 શ્રેણિક M માટે ધારોકે (M) એ M ના તમામ વિકર્ણી ઘટકોનો સરવાળો દર્શાવે છે. ધારોકે A એવો 3×3 શ્રેણિક છે કે જેથી $|A|=\frac{1}{2}$ તથા trace $(A)=3$. જો $B=\text{adj}(\text{adj}(2A))$ હોય, તો $|B|+\text{trace}(B)$ નું મૂલ્ય = _______ [JEE MAIN 2025]

Solution:
$|A|=\frac{1}{2}, \text{trace}(A)=3, B=\text{adj}(\text{adj}(2A))=|2A|^{2-2}(2A)$
$n=3, B=|2A|(2A)=2^3\cdot|A|(2A)=8A$
$|B|=|8A|=8^3\cdot|A|=2^8=256$
$\text{trace}(B)=8\,\text{trace}(A)=24$
$|B|+\text{trace}(B)=280$

(8) જો સુરેખ સમીકરણોની સંહતિ $x+y+2z=6$, $2x+3y+az=a+1$, $-x-3y+bz=2b$ જ્યાં $a,b\in R$, ને અસંખ્ય ઉકેલો હોય, તો $7a+3b=$ ______ [JEE MAIN 2025]

Solution:
$$\Delta=\begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & a \\ -1 & -3 & b \end{vmatrix}=0$$
$\Rightarrow 2a+b-6=0$
$$\Delta_1=\begin{vmatrix} 1 & 1 & 6 \\ 2 & 3 & a+1 \\ -1 & -3 & 2b \end{vmatrix}=0$$
$\Rightarrow a+b-8=0$
Solving $(1)+(2)$
$a=-2, b=10$
$\Rightarrow 7a+3b=16$

(9) જો સમીકરણની સંહતિ $(\lambda-1)x+(\lambda-4)y+\lambda z=5$, $\lambda x+(\lambda-1)y+(\lambda-4)z=7$, $(\lambda+1)x+(\lambda+2)y-(\lambda+2)z=9$ ને અનંત ઉકેલો હોય તો $\lambda^2+\lambda$ ની કિંમત મેળવો. [JEE MAIN 2025]

Solution:
$(\lambda-1)x+(\lambda-4)y+\lambda z=5$
$\lambda x+(\lambda-1)y+(\lambda-4)z=7$
$(\lambda+1)x+(\lambda+2)y-(\lambda+2)z=9$
For infinitely many solutions
$$D=\begin{vmatrix} \lambda-1 & \lambda-4 & \lambda \\ \lambda & \lambda-1 & \lambda-4 \\ \lambda+1 & \lambda+2 & -(\lambda+2) \end{vmatrix}=0$$
$(\lambda-3)(2\lambda+1)=0$
$$D_x=\begin{vmatrix} 5 & \lambda-4 & \lambda \\ 7 & \lambda-1 & \lambda-4 \\ 9 & \lambda+2 & -(\lambda+2) \end{vmatrix}=0$$
$2(3-\lambda)(23-2\lambda)=0$
$\lambda=3$
$\therefore \lambda^2+\lambda=9+3=12$

(10) જો સમીકરણ સંહિતા $2x-y+z=4$, $5x+\lambda y+3z=12, 100x-47y+\mu z=212$ ને અસંખ્ય ઉકેલો હોય તો $\mu-2\lambda=$ .... [JEE MAIN 2025]

Solution:
$$\Delta=0 \Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ 5 & \lambda & 3 \\ 100 & -47 & \mu \end{vmatrix}=0$$
$2(\lambda\mu+141)+(5\mu-300)-235-100\lambda=0\ldots$
$$\Delta_3=0 \Rightarrow \begin{vmatrix} 2 & -1 & 4 \\ 5 & \lambda & 12 \\ 100 & -47 & 212 \end{vmatrix}=0$$
$6\lambda=-12 \Rightarrow \lambda=-2$
Put $\lambda=2$ in.......(1)
$2(-2\mu+141)+5\mu-300-235+200=0$
$\mu=53$
$\therefore 57$

(11) ધારો કે A એવો એક 3×3 શ્રેણિક છે કે જેથી પ્રત્યેક શૂન્યેત્તર 3×1 શ્રેણિકો $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ માટે $X^TAX=0$ થાય. જો $A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}, A\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$ અને $\det(\text{adj}(2(A+I)))=2^\alpha 3^\beta 5^\gamma, \alpha,\beta,\gamma\in N$ હોય, તો $\alpha^2+\beta^2+\gamma^2=$ _______ [JEE MAIN 2025]

Solution:
$X^TAX=0$
$$(xyz)\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=0$$
$$(xyz)\begin{pmatrix} a_1x+a_2y+a_3z \\ b_1x+b_2y+b_3z \\ c_1x+c_2y+c_3z \end{pmatrix}=0$$
$x(a_1x+a_2y+a_3z)+y(b_1x+b_2y+b_3z)$
$+z(c_1x+c_2y+c_3z)=0$
$a_1=0, b_2=0 c_3=0$
$a_2+b_1=0, a_3+c_1=0, b_3=c_2=0$
$A=$ skew symm matrix
$$A=\begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix};\quad A=\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}=\begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$$
$$\Rightarrow A=\begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}=\begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$$
$x+y=1$
$-x+z=4$
$y+z=5$

$$\begin{pmatrix}0 & x & y\\ -x & 0 & z\\ -y & -z & 0\end{pmatrix}\begin{pmatrix}1\\ 2\\ 1\end{pmatrix}=\begin{pmatrix}1\\ 4\\ -8\end{pmatrix}$$
$2x+y=0x=-1$
$-x+z=4y=2$
$-y-2z=-8z=3$
$$A=\begin{pmatrix}0 & -1 & 2\\ 1 & 0 & 3\\ -2 & -3 & 0\end{pmatrix}$$
$$2(A+I)=\begin{pmatrix}2 & -2 & 4\\ 2 & 2 & 6\\ -2 & -6 & 2\end{pmatrix}$$
$2(A+I)=120\Rightarrow \det|\operatorname{adi}(2(A+I))|$
$=120^2=2^6\cdot 3^2\cdot 5^2$
$\alpha=6,\beta=2,\gamma=2$

(12) જો સમીકરણ સંહિત $x+2y-3z=2, 2x+\lambda y+5z=5,$ $14x+3y+\mu z=33$ ને અસંખ્ય ઉકેલો હોય, તો $\lambda+\mu$ ______ [JEE MAIN 2025]

Solution:
$$D=\begin{vmatrix}1 & 2 & -3\\ 2 & \lambda & 5\\ 14 & 3 & \mu\end{vmatrix}=0, \lambda\mu+42\lambda-4\mu+107=0$$
$D_1=2\lambda\mu+99\lambda-10\mu+255$
$D_2=13-\mu$
$D_3=5\lambda+5$
$D_2=0\Rightarrow\mu=13\ D_3=0\Rightarrow\lambda=-1$
check verify for these values $D \& D_2=0$

(13) કોઈક $a, b,$ માટે ધારો કે
$$f(x)=\begin{vmatrix}a+\frac{\sin x}{x} & 1 & b\\ a & 1+\frac{\sin x}{x} & b\\ a & 1 & b+\frac{\sin x}{x}\end{vmatrix}, \quad x\neq 0,$$
$\lim_{x\to 0} f(x)=\lambda+\mu a+vb.$ તો $(\lambda+\mu+v)^2$ __________. [JEE MAIN 2025]

Solution:
$$\lim_{x\to 0} f(x)=\begin{vmatrix}a+1 & 1 & b\\ a & 1+1 & b\\ a & 1 & b+1\end{vmatrix}$$
$=(a+1)(2(b+1)-b)+1(ab-a(b+1))+ba$
$=(a+1)(b+2)-a+ab$
$=b+a+2=\lambda+\mu a+vb$
$\lambda=2,\mu=1,v=1\Rightarrow(\lambda+\mu+v)^2=16$

(14) ધારો કે M એ કક્ષા 3×3 વાળા તમામ વાસ્તવિક શ્રેણિકોનો ગણ દર્શાવ છે તથા $S=\{-3,-2,-1,1,2\}$. ધારો કે
$S_1=\{A=[a_{ij}]\in M: A=A^T$ and $a_{ij}\in S,\forall i,j\}$
$S_2=\{A=[a_{ij}]\in M: A=-A^T$ and $a_{ij}\in S,\forall i,j\}$
$S_3=$
$\{A=[a_{ij}]\in M: a_{11}+a_{22}+a_{33}=0$ and $a_{ij}\in S,\forall i,j\}$
જો $n(S_1\cup S_2\cup S_3)=125\alpha$, હોય તો $\alpha=$_____. [JEE MAIN 2025]

Solution:
$$\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$$
No. of elements in $S_1: A=A^T\Rightarrow 5^3\times 5^3$
No. of elements in $A=-A^T\Rightarrow 0$
since no. zero in 5
No. of elements in $S_3\Rightarrow$
$$\left.\begin{array}{c}a_{11}+a_{22}+a_{33}=0\Rightarrow(1,2,-3)\Rightarrow 31\\ \text{or}\\ (1,1,-2)\Rightarrow 3\\ \text{or}\\ (-1,-1,2)\Rightarrow 3\end{array}\right\}\Rightarrow 12\times 5^6$$

$n(S_1\cap S_3)=12\times 5^3$
$n(S_1\cup S_2\cup S_3)=5^6(1+12)-12\times 5^3$
$\Rightarrow 5^3\times[13\times 5^3-12]=125\alpha$
$\alpha=1613$

(15) ધારો કે $A=\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}$ અને $P=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}, \theta>0.$
જો $B=PAP^T, C=P^TB^{10}P$ અને C ના વિકીર્ણ ઘટકોનો સરવાળો $\frac{m}{n}$, હોય, જ્યાં ગુ.સા.અ. $(m,n)=1$, તો $m+n=$________ [JEE MAIN 2025]

Solution:
$$P=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$
$\because P^TP=I$
$B=PAPT$
Pre multiply by P^T (Given)
$P^TB=P^TPAP^T=AP^T$
Now post multiply by P
$P^TBP=AP^TP=A$
So $A^2=\underbrace{P^TBPP^TBP}_{I}$
$A^2=P^TB^2P$
Similarly $A^{10}=P^TB^{10}P=C$
$A=\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}$ (Given)
$\Rightarrow A^2=\begin{bmatrix}\frac{1}{2} & -\sqrt{2}-2\\ 0 & 1\end{bmatrix}$
Similarly check A^3 and so on since $C=A^{10}$
$\Rightarrow$ Sum of diagonal elements of C is $\left(\frac{1}{\sqrt{2}}\right)^{10}+1$
$=\frac{1}{32}+1=\frac{33}{32}=\frac{m}{n}$
g $\operatorname{cd}(m,n)=1$ (Given)
$\Rightarrow m+n=65$

(16) જો M અને m એ અનુક્રમે
$$f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 4x\\ \sin^2 x & 1+\cos^2 x & 4\sin 4x\\ \sin^2 x & \cos^2 x & 1+4\sin 4x\end{vmatrix}, x\in R \text{ ની}$$
મહતમ અને ન્યૂનતમ કિમતો હોય તો M^4-m^4 ની કિમંત મેળવો. [JEE MAIN 2025]

Solution:
$$\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 4x\\ \sin^2 x & 1+\cos^2 x & 4\sin 4x\\ \sin^2 x & \cos^2 x & 1+4\sin 4x\end{vmatrix}, x\in R$$
$R_2\to R_2-R_1 \& R_3\to R_3\to R_1$
$$f(x)\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 4x\\ -1 & 1 & 0\\ -1 & 0 & 1\end{vmatrix}$$
Expand about R_1, use get
$f(x)=2+4\sin 4x$
$\therefore M=$ max value of $f(x)=6$
$M=$ min value of $f(x)=-2$
$\therefore M^4-M^4=1280$

(17) અહી $A=[a_{ij}]=\begin{bmatrix}\log_5 128 & \log_4 5\\ \log_5 8 & \log_4 25\end{bmatrix}$ છે. જો A_{ij} એ a_{ij} નો સહઅવયજ શ્રેણિક છે. જો $C_{ij}=\sum_{k=1}^{2} a_{ik}A_{jk}, 1\leq i,j\leq 2$ અને $C=[C_{ij}]$ આપેલ હોય તો $8|C|$ ની કિમંત મેળવો. [JEE MAIN 2025]

Solution:
$|A|=\frac{11}{2}$
$C_{11}=\sum_{k=1}^{2} a_{1k}\cdot A_{1k}=a_{11}A_{11}+a_{12}A_{12}=|A|=\frac{11}{2}$
$C_{12}=\sum_{k=1}^{2} a_{1k}\cdot A_{2k}=0$
$C_{21}=\sum_{k=1}^{2} a_{2k}\cdot A_{1k}=0$
$C_{22}=\sum_{k=1}^{2} a_{2k}\cdot A_{2k}=|A|=\frac{11}{2}$

$C=\begin{bmatrix} 11/2 & 0 \\ 0 & 11/2 \end{bmatrix}$
$|C|=\frac{121}{4}$
$8|C|=242$

(18) અહી $S=\left\{m\in Z: A^{m^2}+A^m=3I-A^{-6}\right\}$ કે જ્યાં $A=\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$ હોય તો $n(S)$ ની કિંમત મેળવો. [JEE MAIN 2025]

Solution:
$A=\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$
$A^2=\begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}, A^3=\begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}, A^4=\begin{bmatrix} 5 & -4 \\ 4 & -3 \end{bmatrix}$
and so on
$A^6=\begin{bmatrix} 7 & -6 \\ 6 & -5 \end{bmatrix}$
$A^m=\begin{bmatrix} m+1 & -m \\ m & -m-1 \end{bmatrix}$
$A^{m^2}=\begin{bmatrix} m^2+1 & -m^2 \\ m^2 & -\left(m^2-1\right) \end{bmatrix}$
$A^{m^2}+A^m=3I-A^{-6}$
$\begin{bmatrix} m+1 & -m^2 \\ m^2 & -\left(m^2-1\right) \end{bmatrix}+\begin{bmatrix} m+1 & -m \\ m & -(m-1) \end{bmatrix}$
$=3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}-\begin{bmatrix} -5 & 6 \\ -6 & 7 \end{bmatrix}$
$=\begin{bmatrix} 8 & -6 \\ 6 & -4 \end{bmatrix}$
$=m^2+1+m+1=8$
$=m^2+m-6=0 \Rightarrow m=-3,2$
$n(s)=2$

(19) ધારોકે, $\alpha, \beta(\alpha \neq \beta)$ એ m ની એવી કિંમતો છે કે જેના માટે સમીકરણો $x+y+z=1; x+2y+4z=m$ અને $x+4y+10z=m^2$ ને અસંખ્ય ઉકેલો હોય તો $\sum_{n=1}^{10}\left(n^\alpha+n^\beta\right)$ નું મૂલ્ય______ છે. [JEE MAIN 2025]

Solution:
$\Delta=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix}=1(20-16)-1(10-4)+1(4-2)$
$=4-6+2=0$
For infinite solutions
$\Delta_x=\Delta_y=\Delta_z=0$
$m^2-3x+2=0$
$m=1,2$
$\alpha=1, \beta=2$
$\therefore \sum_{n=1}^{10}\left(n^\alpha+n^\beta\right)=\sum_{n=1}^{10} n^1+\sum_{n=1}^{10} n^2$
$=\frac{10(11)}{2}+\frac{10(11)(21)}{6}$
$=55+385$
$=440$

(20) ધારો કે $A=(a_{ij})$ એ કક્ષા 3×3 નો એક શ્રેણિક છે, જ્યાં $a_{ij}=(\sqrt{2})^{i+j}$ છે. જો A^2 ની ત્રીજી હારના તમામ ઘટકોનો સરવાળો $\alpha+\beta\sqrt{2}, \alpha+\beta \in Z$ હોય તો $\alpha+\beta=$ ________. [JEE MAIN 2025]

Solution:
$A=\begin{bmatrix} (\sqrt{2})^2 & (\sqrt{2})^3 & (\sqrt{2})^4 \\ (\sqrt{2})^3 & (\sqrt{2})^4 & (\sqrt{2})^5 \\ (\sqrt{2})^4 & (\sqrt{2})^5 & (\sqrt{2})^6 \end{bmatrix}$
$A=\begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix}$
$A^2=2^2\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 2 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 4 \end{bmatrix}\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 2 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 4 \end{bmatrix}$
$=4\begin{bmatrix} - & - & - \\ - & - & - \\ (2+4+8) & (2\sqrt{2}+4\sqrt{2}+8\sqrt{2}) & (4+8+16) \end{bmatrix}$
Sum of elements of 3rd row $=4(14+14\sqrt{2}+28)$
$=4(42+14\sqrt{2})$
$=168+56\sqrt{2}$
$\alpha+\beta\sqrt{2}$
$\therefore \alpha\alpha+\beta=168+56=224$

(21) ધારો કે જો પૂર્ણાંકો $a, b \in[-3,3]$ એવાં છે કે જેથી $a+b\neq 0$. તો શક્ય તમામ એવી જોડ (a,b) ની સંખ્યા શોધો, કે જેના માટે $\left|\frac{z-a}{z+b}\right|=1$ અને $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=1, z\in C$, જ્યાં ω અને ω^2 એ $x^2+x+1=0$, નાં બીજ છે. [JEE MAIN 2025]

Solution:
$a, b \in I, -3\leq a, b\leq 3, a+b\neq 0$
$|z-a|=|z+b|$
$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=1$
$\Rightarrow\begin{vmatrix} z & z & z \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=1$
$\Rightarrow z\begin{vmatrix} 1 & 1 & 1 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=1$
$\Rightarrow z\begin{vmatrix} 1 & 0 & 0 \\ \omega & z+\omega^2-\omega & 1-\omega \\ \omega^2 & 1-\omega^2 & z+\omega-\omega^2 \end{vmatrix}=1$
$\Rightarrow z^3=1$
$\Rightarrow z=\omega, \omega^2, 1$
Now
$|1-a|=|1+b|$
$\Rightarrow$ 10 pairs

(22) જો સુરેખ સંહતિઓ $3x+y+\beta z=3$, $2x+\alpha y-z=-3$, $x+2y+z=4$ ને અનંત ઉકેલો હોય તો $22\beta-9\alpha$ ની કિંમત મેળવો. [JEE MAIN 2025]

Solution:
$\Delta=\begin{vmatrix} 3 & 1 & \beta \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix}=0$
$3\alpha+4\beta-\alpha\beta+3=0$
$\Delta_3=\begin{vmatrix} 3 & 1 & 3 \\ 2 & \alpha & -3 \\ 1 & 2 & 4 \end{vmatrix}=0$
$9\alpha+19=0$
$\alpha=\frac{-19}{9}, \beta=\frac{6}{11}$
$\Rightarrow 22\beta-9\alpha=31$

(23) અહી $a\in R$ અને શ્રેણિક A એ 3×3 કક્ષાનો છે કે જેથી $\det(A)=-4$ અને $A+I=\begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$, કે જ્યાં I એ 3×3 કક્ષાનો એકમ શ્રેણિક છે. જો $\det((a+1)\operatorname{adj}((a-1)A))$ એ $2^m3^n, m, n\in\{0,1,2,\ldots\ldots 20\}$, હોય તો $m+n$ ની કિંમત મેળવો. [JEE MAIN 2025]

Solution:
$A=\begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}-I=\begin{bmatrix} 0 & a & 1 \\ 2 & 0 & 0 \\ a & 1 & 1 \end{bmatrix}$
$|A|=-4\Rightarrow 2-2a=-4\Rightarrow a=3$

$|(a+1)\operatorname{adj}(a-1)A| = |4\operatorname{adj}3A|$
$= 4^3|\operatorname{adj}3A|$
$= 4^3 \times |3A|^{3-1} = 64|3A|^2$
$= 64 \times (3^3)^2|A|^2$
$= 2^6 \times 3^6 \times 16$
$2^m \times 3^n = 2^{10} \times 3^6$
$\therefore m = 10, n = 6$
$\Rightarrow m+n = 16$

(24) જો સમીકરણ સંહતીઓ $2x+\lambda y+3z=5, 3x+2y-z=7,$ $4x+5y+\mu z=9$ ને અનંત ઉકેલ હોય તો $(\lambda^2+\mu^2)$ ની કિંમત મેળવો. [JEE MAIN 2025]

Solution:

$\Delta = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \mu \end{vmatrix} = 0$
$\Rightarrow 2(2\mu+5)+\lambda(-4-3\mu)+3(7)=0$
$\Rightarrow 4\mu-3\lambda\mu-4\lambda+31=0\ldots\ldots(1)$
$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 5 \\ 3 & 2 & 7 \\ 4 & 5 & 9 \end{vmatrix} = 0$
$\Rightarrow 2(-17)+\lambda(1)+5(7)=0$
$\Rightarrow \lambda = -1$
from equation (1)
$4\mu+3\mu+4+31=0 \Rightarrow \mu=-5$
$\therefore \lambda^2+\mu^2 = 26$

(25) અહી A એ 3×3 કક્ષાનો વાસ્તવિક શ્રેણિક છે કે જેથી $A^2(A-2I)-4(A-I)=O$ છે જ્યાં I અને O અનુક્રમે એકમ અને શૂન્ય શ્રેણિક છે . જો $A^5=\alpha A^2+\beta A+\gamma I$ કે જ્યાં α,β અને γ એ વાસ્તવિક અચળાંક છે તો $\alpha+\beta+\gamma$ ની કિંમત મેળવો. [JEE MAIN 2025]

Solution:

$A^3-2A^2-4A+4I=0$
$A^3=2A^2+4A-4I$
$A^4=2A^3+4A^2-4A$
$=2(2A^2+4A-4I)+4A^2-4A$
$A^4=8A^2+4A-8I$
$A^5=8A^3+4A^2-8A$
$=8(2A^2+4A-4I)+4A^2-8A$
$A^5=20A^2+24A-32I$
$\therefore \alpha=20, \beta=24, \gamma=-32$
$\therefore \alpha+\beta+\gamma=12$

(26) અહી A એ 3×3 કક્ષાનો શ્રેણી છે અને $|A|=5$ છે. જો $|2\operatorname{adj}(3A\operatorname{adj}(2A))| = 2^\alpha\cdot3^\beta\cdot5^\gamma \alpha,\beta,\gamma\in N$ હોય તો $\alpha+\beta+\gamma$ મેળવો. [JEE MAIN 2025]

Solution:

$|2\operatorname{adj}(3A\operatorname{adj}(2A))|$
$2^3\cdot|3A\operatorname{adj}(2A)|^2$
$2^3\cdot(3^3)^2\cdot|A|^2\cdot|adj(2A)|^2$
$2^3\cdot3^6\cdot|A|^2\cdot(|2A|^2)^2$
$2^3\cdot3^6\cdot|A|^2\left[(2^3)^2\cdot|A|^2\right]^2$
$2^3\cdot3^6\cdot|A|^2\cdot2^{12}\cdot|A|^4$
$2^{15}\cdot3^6\cdot|A|^6$
$2^{15}\cdot3^6\cdot5^6=2^\alpha\cdot3^\beta\cdot5^\gamma$
$\alpha=15, \beta=6, \gamma=6$
$\alpha+\beta+\gamma=27$

(27) અહી I એ 3×3 કક્ષાનો એકમ શ્રેણિક છે અને શ્રેણિક $A=\begin{bmatrix} \lambda & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -1 & 2 \end{bmatrix}, |A|=-1$ આપેલ છે . જો B એ શ્રેણિક $\operatorname{adj}(A\operatorname{adj}(A^2))$ વ્યસ્ત શ્રેણિક હોય તો $|(\lambda B+1)|$ ની કિંમત મેળવો. [JEE MAIN 2025]

Solution:

$|A| = \begin{vmatrix} \lambda & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -1 & 2 \end{vmatrix} = -1$
$\lambda(16)-2(-34)+3(-39)=-1$
$16\lambda=48 \Rightarrow \lambda=3$
$B^{-1}=\operatorname{adj}(A\cdot\operatorname{adj}(A^2))$
Let $C=A.\operatorname{adj}(A^2)$
$AC=A^2\operatorname{adj}(A^2)=|A|^2\cdot I=I \Rightarrow C=A^{-1}$
Now $B^{-1}=\operatorname{adj}(A^{-1})=B=\operatorname{adj}(A)$
Now $\lambda B+I \Rightarrow 3B+I$
Let $P=3B+I$
$P=3\operatorname{adj}(A)+I$
$AP=3\operatorname{Aadj}(A)+A$
$AP=3|A|\cdot I+A$
$AP=A-3I$
$|AP|=|A-3I|$
$|A|\cdot|P| = \begin{vmatrix} 0 & 2 & 3 \\ 4 & 2 & 6 \\ 7 & -1 & -1 \end{vmatrix} = 38$
$|P|=-38$

(28) ધારો કે $A=\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$. કોઇક $\theta\in(0,\pi)$ માટ, જો $A^2=A^T$ હોય, તો શ્રેણિક $(A+I)^3+(A-I)^3-6A$ ના વિકીર્ણ ઘટકીનો સરવાળો ________ છે. [JEE MAIN 2025]

Solution:

$\because A$ is orthogonal matrix
$\therefore A^T=A^{-1}$
$\Rightarrow A^2=A^{-1}$
$\Rightarrow A^3=I$
let $B=(A+I)^3+(A-I)^3-6A$
$=2(A^3+3A)-6A$
$=2A^3$
$B=2I=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
Now sum of diagonal elements $=2+2+2=6$

(29) અહી શ્રેણિક $A=\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ એ $n\geq3$ માટે $A^n=A^{n-2}+A^2-I$ નું સમાધાન કરે છે તો A^{50} ના બધાજ ઘટકનો સરવાળો મેળવો. [JEE MAIN 2025]

Solution:

$A^{50}=A^{48}+A^2-I$
$=A^{46}+2(A^2-I)$
$=A^{44}+3(A^2-I)$
$=A^2+24(A^2-I)$
$=25A^2-24I$
$=25\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}-24\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
$=\begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$
Sum $=53$

(30) અહી A એ 3×3 કક્ષાનો શ્રેણિક છે કે જેથી $|\operatorname{adj}(\operatorname{adj}(\operatorname{adj}A))|=81$ છે. જો $S=\left\{n\in Z:(|\operatorname{adj}(\operatorname{adj}A)|)^{\frac{(n-1)^2}{2}}=|A|^{(3n^2-5n-4)}\right\}$ હોય તો $\sum_{n\in S}\left|A^{(n^2+n)}\right|$ ની કિંમત મેળવો. [JEE MAIN 2025]

Solution:

$|\text{adj}(\text{adj})(\text{adj } A)| = 81$
$\Rightarrow |\text{adj } A|^4 = 81$
$\Rightarrow |\text{adj } A| = 3$
$\Rightarrow |A|^2 = 3$
$\Rightarrow |A| = \sqrt{3}$
$\left(|A|^4\right)^{\frac{(n-1)^2}{2}} = |A|^{3n^2-5n-4}$
$\Rightarrow 2(n-1)^2 = 3n^2 - 5n - 4$
$\Rightarrow 2n^2 - 4n + 2 = 3n^2 - 5n - 4$
$\Rightarrow n^2 - n - 6 = 0$
$\Rightarrow (n-3)(n+2) = 0$
$\Rightarrow n = 3, -2$
$\sum_{n\in s} \left|A^{n^2+n}\right|$
$= \left|A^2\right| + \left|A^{12}\right|$
$= 3 + 36 = 3 + 729 = 732$

(31) સમીકરણ સહંતિ $x + 5y - z = 1$, $4x + 3y - 3z = 7$, $24x + y + \lambda z = \mu$, $\lambda, \mu \in R$ ના ઉકેલની સંખ્યા અનંત હોય તો આ સમીકરણોની સંહતિના ઉકેલ ની સંખ્યા મેળવો કે જેમાં x, y, z એ પૂર્ણાંક હોય અને $7 \leq x + y + z \leq 77$ નું સમાધાન કરતું હોય . [JEE MAIN 2025]

Solution:

For infinitely many solution
$\Delta = 0$
$\begin{vmatrix} 1 & 5 & -1 \\ 4 & 3 & -3 \\ 24 & 1 & \lambda \end{vmatrix} = 0$
$\Rightarrow 1(3\lambda + 3) - 5(4\lambda + 72) - 1(4 - 72) = 0$
$\Rightarrow -17\lambda + 3 - 4 \times 72 - 4 = 0$
$\Rightarrow 17\lambda = -289$
$\Rightarrow \lambda = -17$
$\Delta_1 = 0$
$\Rightarrow \begin{vmatrix} 1 & 5 & -1 \\ 7 & 3 & -3 \\ \mu & 1 & -17 \end{vmatrix} = 0$
$\Rightarrow 1(-51 + 3) - 5(-119 + 3\mu) - 1(7 - 3\mu) = 0$
$\Rightarrow -48 + 595 - 15\mu - 7 + 3\mu = 0$
$\Rightarrow 12\mu = 540$
$\mu = 45$
$x + 5y - z = 1$
$4x + 3y - 3z = 7$
$24x + y - 17z = 45$
Let $z = 1$
$x + 5y = 1 + \lambda] \times 4$
$4x + 3y = 7 + 3\lambda$
$4x + 20y = \underline{4} + \underline{4}\lambda$
$-17y = 3 - \lambda$
$y = \frac{\lambda - 3}{17}, x = 1 + \lambda - \frac{5\lambda - 15}{17}$
$= \frac{32 - 12\lambda}{17}$
$7 \leq \frac{\lambda - 3}{17} + \frac{32 + 12\lambda}{17} + \lambda \leq 77$
$7 \leq \frac{30\lambda + 29}{17} \leq 77$
$3 \leq \lambda \leq 42$
$\lambda = 3, 20, 37$

(32) ધારો કે α એ $x^2 + x + 1 = 0$ નું બીજ છે,તથા a અને b એ R, $\begin{bmatrix} 4 & a & b \end{bmatrix} \begin{bmatrix} 1 & 16 & 13 \\ -1 & -1 & 2 \\ -2 & -14 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ નું સમાધાન કરે છે. જો $\frac{4}{\alpha^4} + \frac{m}{\alpha^a} + \frac{n}{\alpha^b} = 3$, હોય, તો $m + n =$_____ [JEE MAIN 2025]

Solution:

$x^2 + x + 1 = 0$
α is root
$\therefore \alpha^2 + \alpha + 1 = 0$
$\Rightarrow \alpha = \omega$ as ω^2 [cube root of unity]
also
$\begin{bmatrix} 4 - a - 2b & 64 - a - 14b & 52 + 2a - 8b \end{bmatrix}$
$= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$
$\therefore a + 2b = 4$
$a + 14b = 64$
$\Rightarrow 12b = 60 \Rightarrow b = 5$
$\Rightarrow a = -6$
$\therefore \frac{4}{\alpha^4} + \frac{m}{\alpha^{-6}} + \frac{n}{\alpha^5} = 3$
$\Rightarrow \frac{4}{\omega} + \frac{m}{1} + \frac{n}{\omega^2} = 3$
$\Rightarrow 4\omega^2 + m + n\omega = 3$
$\Rightarrow 4\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) + m + n\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 3$
$\therefore -2 + m - \frac{n}{2} = 3$
$\frac{-4\sqrt{3}}{2} + \frac{n\sqrt{3}}{2} = 0$
$\therefore n = 4$
$m = 7$
$\therefore m + n = 11$

(33) ધારો કે $A = \begin{bmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{bmatrix}$ જો $\det(\text{adj}(\text{adj}(3A))) = 2^m \cdot 3^n, m, n \in N$, હોય તો $m + n =$ _______ [JEE MAIN 2025]

Solution:

$|A| = \begin{vmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{vmatrix}$
$C_3 \to C_3 - C_2 - C_1 \times \frac{q}{2}$
Then $C_3 \to C_2 - C_1 x\left(1 + \frac{p}{2}\right)$
$\Rightarrow |A| = \begin{vmatrix} 2 & 0 & 0 \\ 4 & 2 & 2+p \\ 6 & 6 & 8+3p \end{vmatrix}$
$\Rightarrow |A| = 2(16 + 6p - 12 - 6p) = 8 = 2^3$
$|\text{adj}(\text{adj}(3A))| = |3A|^{(3-1)^2} = |3A|^4$
$= \left(3^3|A|\right)^4 = \left(3^3 \times 2^3\right)^4 = 2^{12} \times 3^{12}$
$\Rightarrow m + n = 24$