

## Problem 1 Analysis

Show  $O()$  by mathematical analysis:

### Linear Search

```
for (int i = 0; i < n; i++) {  
    if (val == a[i])  
        return i;  
}
```

```
return -1;
```

$$\Rightarrow O_b + \sum_{i=0}^{n-1} (O_i + Pos) \quad \text{Let } O_i + Pos = O_i's$$

$$\Rightarrow O_b + \sum_{i=0}^{n-1} O_i's$$

$$= \sum_{i=x}^y 1 = (y-x)+1$$

$$\Rightarrow O_b + O_i's (n-x-0+1)$$

$$\Rightarrow O_i's (n) + O_b \Rightarrow \boxed{c'N + C} \quad \text{behaves like an order } N \text{ function}$$

$\uparrow$   $\uparrow$   
 $c'$   $c$

$\therefore$  Linear Search is  $O(N)$

### Binary Search

```
int low = 0;
```

```
int high = 0;
```

```
do {
```

```
    int middle = (high + low) / 2;
```

```
    if (val == a[middle]) return middle;
```

```

else if (val > a[middle]) low = middle + 1;
else high = middle - 1;
} while (low <= high)

```

return -1;

$$\Rightarrow O_b + \sum_{i=0}^{\log_2(n)} (O_i + P O_s)$$

$$\Rightarrow O_b + \sum_{i=0}^{\log_2(n)} O_i$$

$$\Rightarrow O_b + O_i \sum_{i=0}^{\log_2(n)} 1$$

$$\Rightarrow O_b + O_i (1 + \log_2(n))$$

$$\Rightarrow \underbrace{O_b + O_i}_c + \underbrace{O_i}_{c'} \log_2(n)$$

$$\Rightarrow c + c' \log_2(n) = f(n)$$

$$\Rightarrow \boxed{f(n) = O(\log(N))}$$

Array size = n

After one;

$$\text{size} = \frac{n}{2}$$

After two;

$$\text{size} = \frac{n}{4}$$

After k iterations

$$1 = \frac{N}{2^k}$$

$$2^k = N$$

$$k = \log_2(N)$$