Problem 5

Derive the order of the error with respect to the sin and cos approximations,

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

Approximation: $\sin\left(\frac{1}{N}\right) \cong \frac{1}{N}$ $\cos\left(\frac{1}{N}\right) \cong 1 - \frac{1}{2N^2}$

$\Rightarrow$ errorsin $\left(\frac{1}{N}\right)$

$$\sin\left(\frac{1}{N}\right) = \frac{1}{N} - \frac{\frac{1}{N^3}}{6} + \frac{\frac{1}{N^5}}{5!} - \frac{\frac{1}{N^7}}{7!} + \ldots$$

$$\underbrace{\sin\left(\frac{1}{N}\right) - \frac{1}{N}}_{\text{error}} = -\frac{1}{6N^3} + \frac{1}{5!N^5} - \frac{1}{7!N^7} + \ldots$$

$$\boxed{\text{errorsin}\left(\frac{1}{N}\right) = O(N^3)}$$

$\Rightarrow$ errorcos $\left(\frac{1}{N}\right)$

$$\cos\left(\frac{1}{N}\right) = 1 - \frac{\frac{1}{N^2}}{2} + \frac{\frac{1}{N^4}}{4!} - \frac{\frac{1}{N^6}}{6!} + \ldots$$

$$\underbrace{\cos\left(\frac{1}{N}\right) - \left[1 - \frac{1}{2N^2}\right]}_{\text{error}} = \frac{1}{4!N^4} - \frac{1}{6!N^6}$$

$$\boxed{\text{errorcos}\left(\frac{1}{N}\right) = O(N^4)}$$