Problem Set 1
Precision Care Medicine(BME 580.480/680)

September 4, 2019

1. Suppose that the number of patients arriving to the ER obeys a Poisson arrival process with arrival rate of 5 patients per hour. What is the probability that at least 1 patient (1 or 2 or 3, etc.....) arrive in 3 hours? Please show your work.

2. Suppose that the number of patients arriving to the ER obeys a Poisson arrival process with arrival rate of 3 patients per 30 minutes. What is the probability that two patients will arrive between 9AM-9:30AM and 3 patients will arrive between 10AM-10:15AM? Please show your work.

3. Suppose y is a Gaussian random variable with mean $\mu = 3$ and variance $\sigma^2 = 1$. Let $z = ay + b$. What is the probability density function for z conditioned on y?

4. Suppose y is a Gaussian random variable with mean $\mu = 0$ and variance $\sigma^2 = 1$. Let $z = 3y$. Write an expression in the form of an integral to compute $Pr(z > 1)$. No need to solve the integral.

5. Suppose two independent fair coins are tossed in the air. If either coin lands on tails, then that coin is tossed again until it lands on heads. Let y_i be the number of tosses of coin i for $i = 1, 2$. What is the probability that the number of total tosses equals 10, i.e., $Pr(y_1 + y_2 = 10)$? Please show your work.

6. Suppose you toss a coin 50 times. The coin has probability of heads equal to p (unknown) and you observe heads for the first 25 tosses and tails for the last 25 tosses.

   (a) Derive the data likelihood function as a function of p

   (b) Compute the maximum likelihood estimate for p. Show all your work.

7. Suppose you toss a coin 100 times. The coin has probability of heads equal to p (unknown) and you observe heads for 20 tosses and tails for 80 tosses.

   (a) Derive the data likelihood function as a function of p

   (b) Compute the maximum likelihood estimate for p. Show all your work.

8. Suppose two independent fair coins are tossed twice. For each coin, if it lands on tails, then you receive 5, else you pay 10. Let r equal the amount of money you have left (can be positive or negative) after your two tosses of the two coins. Calculate your expected earnings. Show your work.

9. Suppose that the time in days until ICU discharge for a patient population recovering from cardiac arrest follows a density $f(x) = \lambda e^{-x/3}$ for $x > 0$

   (a) What value of λ would makes this density a valid pdf?

   (b) What is the probability that a patient stays in the ICU for longer than a week ?

10. Suppose you play the coin-tossing game with your friend again like in problem 8. However, this time, there is only one coin and the coin is biased with the probability of head is 0.3. To compensate for the biased coin, your friend says that she will give you 6 dollars if the coin lands on head and you give her 4 dollars if it comes up tails. There will be 100 rounds in total. What is your expected total earnings and variance of your total earnings? Show your work.

11. Suppose that the systolic blood pressures (SBP) of Hopkins students are normally distributed with mean 110 (mmHg) and variance 16. Suppose that 20 students are randomly selected for a health exam. Suppose that the only criterion for hypertension in a young adult is defined as SBP $>$130 mmHg. Calculate the probability that 3 students or more are hypertensive?

12. Suppose that heart rates obtained from ICU patient data is normally distributed with a mean of 86 beats/second and standard deviation of 49 beats/second. Knowing that this includes heart rates that are physiologically impossible, you decide that only the heart rates between 20 beats/second and 220 beats/second are valid.

    a) What proportion of the data will be within the valid range?

    Suppose that you select a random patient and find that during their ICU stay, there were 1000 heart rate measurements.

    b) What is the probability that 90 percent of the data points are within the valid range.

13. Suppose that on average 1 in-hospital cardiac arrests occur per day. What is the probability that the hospital experiences:

    a) no in-hospital cardiac arrest for an entire week.

    b) 5 in-hospital cardiac arrest in 3 days.

14. Derive the maximum likelihood estimates for mean and standard deviation for a normal distribution N(μ, σ^2) if you have k independent observations $x_1, x_2, ..., x_k$.

15. A probability density function for a random variable is given by:

$$f(x) = \begin{cases} k(x^2 + x) & \text{if } 0 \leq \text{x} \leq 1, \\ 0 & \text{else} \end{cases}$$

a) Find the value k that makes f(x) a probability density function.

b) Find the cumulative density function of f(x)?

c) Use the CDF to find Pr(0.25 $\leq$ x $\leq$ 0.75).

d) Find the mean, variance, and standard deviation of x.