Multi-focus image fusion using Singular Value Decomposition in DCT domain

Mostafa Amin-Naji, Pardis Ranjbar-Noiey, and Ali Aghagolzadeh
Faculty of Electrical and Computer Engineering, Babol Noshirvani University of Technology, Babol, Iran.
Mostafa.Amin.Naji@gmail.com, Pardis.Ranjbar@gmail.com, and aghagol@nit.ac.ir

Abstract—**Multi-focus image fusion is a process that fuses several images from a scene with different focal lengths into a whole image in which all areas are focused on. Image fusion methods in the Discrete Cosine Transform (DCT) domain are efficient due to their low time and energy consumption, and low complexity. This is especially true when fusing images are compressed in JPEG format in Visual Sensor Networks (VSN). In this paper, a low complexity multi-focus image fusion in DCT domain is presented which increases the output image quality. Our proposed method makes it suitable for real-time applications because of its implementation in DCT domain. On the other hand, it is stable in noisy conditions. The proposed method uses the singular values of Singular Value Decomposition (SVD) of 8×8 input blocks in DCT domain. The geometric mean of the 5 largest singular values (out of 8 singular values) is computed as a criterion of focused block detection. The blocks which have the highest geometric mean value among other corresponding blocks is selected as the focused block. These blocks are then used for constructing the output image. This method can be utilized both in DCT domain and in spatial domain. Various experiments and comparisons between the proposed method and the previous methods in noisy and noiseless conditions have been presented, which confirm the increase in image quality and stability in noisy images.**

Keywords—*multi-focus; image fusion; discrete cosine transform; singular value decomposition; visual sensor networks*

I. INTRODUCTION

The process of image fusion consists of combining multiple images to obtain a single image that is more precise than the original images. An aim of the image fusion is to decrease the information quantity, while simultaneously making it more interpretable for both human and machine perception. In multi-focus image fusion case, the single output fused image is constructed from the multiple input images which each of them is focused on a part of scene and the other parts are blurred [1, 2]. Technically speaking, due to the limited depth of focus in optical lenses of CMOS/CCD cameras, recording an image which has distinct components is challenging [3]. In order to create a vivid image, every part of image must remain in the focal length. However, this is usually not practical and only a fraction of the image, which is remaining in the focal length could be explicit, leaving the rest obscure [4, 5]. In Visual Sensor Networks (VSN) there is an opportunity to capture more images from a scene by multi cameras with different depths of focus [5-8].

Various multi-focus image fusion researches have been done in the spatial domain [9-15]. These methods are time and energy consuming, making them unsuitable for real-time applications. Multi-scale transforms include gradient, Laplacian, and morphological pyramid transform [16-18], Discrete Wavelet Transform (DWT) [19], Shift-Invariant Wavelet Transform (SIDWT) [20] and Discrete Cosine Harmonic Wavelet Transform (DCHWT) [21]. These create complexity in computation, making it not only unsuitable considering time consumption, but also inappropriate because of the ringing artifacts. Ringing artifacts at the edge of the images decrease the quality of the output image. Recently, several methods have been proposed for fusing images by the properties and attributes of Singular Value Decomposition (SVD) such as Multi-resolution Singular Value Decomposition (MSVD) and Higher Order Singular Value Decomposition (HOSVD) [22-26]. Naidu introduced Multi-resolution Singular Value Decomposition (MSVD) fusion of images [26]. The output of MSVD is four sub-bands consisting of Y.LL (low-frequency details) as approximation and Y.LH, Y.HL, T.HH as high-frequency details. The fusion scheme of this method is averaging of low-frequency, selecting the maximum absolute value of high-frequency sub-band of source images and, finally, applying IMSVD. This method has undesirable side effects such as blurring and ringing artifacts over the edges of the output image.

Feature extraction is a key procedure in image fusion which is usually employed to reduce the complexity of calculation, time and energy consumption. Therefore, scientists presented significant ways of implementing image fusion algorithms in Discrete Cosine Transform (DCT) domain for real-time applications; In other words, image fusion methods in DCT domain for images which are compressed in JPEG format take less time and are more efficient [5, 27, 28]. After dividing input images into smaller blocks (e.g. 8×8), DCT based methods utilize appropriate criteria to select focused blocks. Subsequently, the output fused image is merged using these selected blocks. Tang introduces DCT+Average and DCT+Contrast methods for multi-focus image fusion [27]. In DCT+Average, the output image is computed by taking the mean of all DCT coefficients. Likewise, DCT+Contrast chooses the maximum AC value of DCT representation block coefficients for creating the output image. Throughout this method, the undesirable side effects such as blurring and blocking artifact have been noticed in the fused images. Haghighat et al in the DCT+Variance method have calculated the variance for every block in DCT domain [5]. Thus, the block with the maximum variance in DCT domain, has been chosen as the focused block. In DCT+Ac_Max method, the block which

978-1-5386-4405-8/17/$31.00 ©2017 IEEE

has more quantity of the maximum value of AC coefficients in DCT domain has been chosen as the block for the output image [28]. Considering Spatial Frequency criterion, DCT+SF has been introduced by Cao et al [29]. In this method, Spatial Frequency has been calculated in DCT domain, while the block with the maximum SF value has been chosen for the output image. In [30], the focus criterion Sum-Modified-Laplacian (SML) is calculated in DCT domain. So the blocks with higher SML values are selected for the output image. On the other hand, the Consistency Verification (CV) process is considered in the main algorithm of DCT+SML. Whereas CV is a post-processing step and it should not be considered completely with DCT+SML method because this is DCT+SML+CV. So there is a lot of unsuitable block selection in DCT+SML. Due to their selective criteria, these methods (DCT+Variance, DCT+Ac-max, DCT+SF, and DCT+SML) create abundant errors during suitable block selection. Hence, these methods face quality loss and blocking artifacts. Amin-Naji and Aghagolzadeh introduced new methods for multi-focus image fusion in DCT domain [2, 31, 32]. In DCT+Corr method, the images are passed through a low-pass filter to make artificial blurred images. Therefore the correlation coefficient between source blocks and the artificially blurred blocks used as a focus criterion. In DCT+Sharp method, the blocks are passed through the unsharp filter in order to make bigger distance of variances' values. In addition, two arbitrary thresholds have been considered to identify blocks suspicious of close variance values. Thus, this increases the image quality and decreases blocking artifacts in the output images. Although having minor errors throughout selecting suitable blocks, these methods (DCT+Corr and DCT+Sharp), have had great improvements in comparison to the previous DCT methods. Furthermore, every DCT domain method mentioned has sensitivity towards noise. Consequently, in noisy multi-focus images, they do not have good performance.

Just as mentioned previously, due to their high time and energy consumption, spatial-frequency and DWT based methods were not suitable for JPEG images in real-time applications. Nevertheless, with the development of DCT based methods, the execution time for multi-focus image fusion algorithm for JPEG images in VSN has been decreased impressively. In this manuscript, a method has been presented which not only has increased the quality of the fused images, but has also decreased the execution time of the DCT algorithms. This feature makes our method suitable for real-time applications. The proposed method utilizes singular value decomposition. This proposed method is applicable in DCT and spatial domains, while having superior stability in noisy conditions compared to other methods.

This paper is organized as follows: in Section 2, DCT & SVD and their properties have been introduced. In Section 3, the proposed multi-focus image fusion method's algorithm has been presented. In Section 4, experiments and comparisons with previous methods have been presented and the paper is concluded.

II. Preliminary Mathematics

A. Discrete Cosine Transform using vector processing

Two-dimensional DCT transform of N×N blocks of the image a(m,n) are represented as (1) [33].

$$A= C.a.C^T \ , \tag{1}$$

where the orthogonal matrix C consists of coefficients of the discrete cosine transform and superscripts ^{T} stands for the transpose operator. On the other hand, A represents the DCT coefficients for image's matrix of a. Furthermore C, is known as a Hermitian matrix, i.e.:

$$C^{-1} = C^T \ . \tag{2}$$

The DCT inverse of A is therefore defined as (3).

$$a = C^T.A.C \ . \tag{3}$$

B. Singular Value Decomposition (SVD)

In this paper, Singular Value Decomposition (SVD) is considered, which converts an M×N matrix a into three separate matrices, multiplied by each other. Matrix a can be decomposed via the singular value decomposition as:

$$a = U\Sigma V^T \tag{4}$$

where U is an M×M orthogonal matrix and Eigenvector of aa^t, while Σ is an M×N matrix consisted of singular diagonal entities and finally V^T is an N×N orthogonal matrix and Eigenvector of $a^t a$.

However, an alternative way of showing a in rank 1 matrices is:

$$a = \sigma_1 u_1 V_1^T + \sigma_2 u_2 V_2^T + \cdots + \sigma_n u_n V_n^T \ . \tag{5}$$

In this paper n is set to 8, due to the experiments executed on 8×8 blocks. The singular values σ_i are ordered as:

$$\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \cdots \geq \sigma_8. \tag{6}$$

The eigenvalues of the matrix aa^T can be obtained by the following equation:

$$\det(\lambda I - aa^T)=0 \ , \tag{7}$$

which the eigenvalues are $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_8$.

Therefore:

$$\Sigma = \begin{bmatrix} \sqrt{\lambda_1} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{\lambda_2} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\lambda_3} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \sqrt{\lambda_8} \end{bmatrix}_{8\times 8} . \tag{8}$$

III. Proposed Method

The general structure of the proposed methods is depicted in Fig.1. In order to simplify the description of the proposed method, a fusion of two multi-focus images is considered. However, the proposed algorithm can easily be extended to more than two images. In addition, it is assumed that multi-focus images are already registered. In multi-focus images, areas which are in the focal length and are clear in one image, may be vague in another. This focused area has more contrast and clearness compared to the defocused area. Furthermore, it is expected that these focused areas have larger singular values σ_i; therefore the largest value of σ_i is selected as a criterion of measurement which represents the amount of focus is applied there. The proposed method starts by dividing the input images into 8×8 blocks similar to the encoding of JPEG image. Each block is transferred to the DCT domain by (1). By computing Singular Value Decomposition (SVD) in DCT domain, focused blocks in DCT domain are recognized.

A. Singular Value Decomposition in DCT domain

The previous discussions are applicable for the spatial domain. By transforming the discussion into DCT domain and comparing the SVD matrix, it is concluded that matrix Σ in (4) has the exact data in both spatial and DCT domains; so

$$b_{8\times 8} = U\Sigma_{8\times 8}V^T \quad \text{(Spatial domain)}, \tag{9}$$

$$B_{8\times 8} = U'\Sigma'_{8\times 8}V'^T \quad \text{(DCT domain)}. \tag{10}$$

All parameters are different; however, $\sum$and $\sum'$ are identical. In order to prove this case:

Remark:

Defining:

$$b = U\sum V^T \tag{11}$$

$$B = U'\sum' V'^T \tag{12}$$

where:

$$\mathrm{B} = \mathrm{C\,b\,C^T} \tag{13}$$

$$U' = C\,U\,C^T \tag{14}$$

$$\sum' = C\sum C^T \tag{15}$$

$$V'^T = C\,V^T C^T\ , \tag{16}$$

Proof:

then:

$$\mathrm{B} = \mathrm{CbC^T} = U'\sum' V'^T = CUC^T C\sum C^T\, CV^T C^T \tag{17}$$

where $\mathrm{C^TC} = \mathrm{I}$, then

$$\mathrm{B} = C\,U\sum V^T\, C^T. \tag{18}$$

It can be observed that $CU = U'$ and $V^TC^T = V'^T$, hence:

$$B = U'\sum V'^{\mathrm{T}} \tag{19}$$

whereas it was known that

$$B = U'\sum' V'^{\mathrm{T}}. \tag{20}$$

Therefore

$$\sum' = \sum. \blacksquare \tag{21}$$

In the equations shown above, it has been proven that $\sum'\ (DCT\ domain)$ and $\sum\ (spatial\ domain)$ matrices are exactly equal. To put it in other words, eigenvalues of bb^T and BB^T are the same.

B. Image fusion Based on SVD in DCT domain

By computing SVD and eigenvalues of BB^T for 8×8 blocks in DCT, singular values σ_i ($\sigma_1, \dots, \sigma_8$) are extracted for each block. The lower indices are for low frequencies and the higher indices are for high frequencies. Singular values σ_6 , σ_7 , and σ_8 have smaller values, near to zero, which block noise has great impact on changing their values. Therefore, in order to prevent selecting unsuitable blocks, it is better to use $\sigma_1, \dots, \sigma_5$ only. Geometric mean of 5 largest singular values is chosen as a criterion for the focused blocks:

$$\mathrm{SVD}^5_{\mathrm{DCT}} = \sqrt[5]{\sigma_1\sigma_2\sigma_3\sigma_4\,\sigma_5}\ . \tag{22}$$

Consequently, the block with the most value of $\mathrm{SVD}^5_{\mathrm{DCT}}$ is selected as the focused block. The output fused image is constructed by the decision map (M) of selected suitable focused blocks. The decision map of two multi-focused images (imA and imB) is as follows:

$$\mathrm{M(i,j)} = \begin{cases} 1 & \text{if}\quad \mathrm{SVD^5_{DCT}(imA)} > \mathrm{SVD^5_{DCT}(imB)} \\ -1 & \text{if}\quad \text{else} \end{cases} \tag{23}$$

where i and j = 1, 2 … $\frac{\text{size of the input image}}{8}$.

Accordingly for the $\mathrm{(i,j)^{th}}$ block of the output image, if $\mathrm{M(i,j)} = 1$, imA is used and if $\mathrm{M(i,j)} = -1$, imB is used. This stage of the proposed method is called DCT+SVD.

Consistency Verification (CV) which has been presented in [18], is a common step that is being used for post-processing in multi-focus image fusion algorithm to improve the output fused image quality. In this case, by considering an area of the decision map which the middle block is from imA while the majority blocks are from imB, the middle block must be from imB. This is possible by applying the majority filter (e.g 3×3 averaging filter) as follows:

$$\mathrm{W(i,j)} = \frac{1}{9}\sum_{\mathrm{k}=-1}^{+1}\sum_{\mathrm{l}=-1}^{+1}\mathrm{M(i+k,j+l)} \tag{24}$$

For W(i,j) > 0, the selected block for the output image is selected from imA and for W(i,j) < 0, the selected block for the output image is selected from imB. This stage has been named as DCT+SVD+CV.

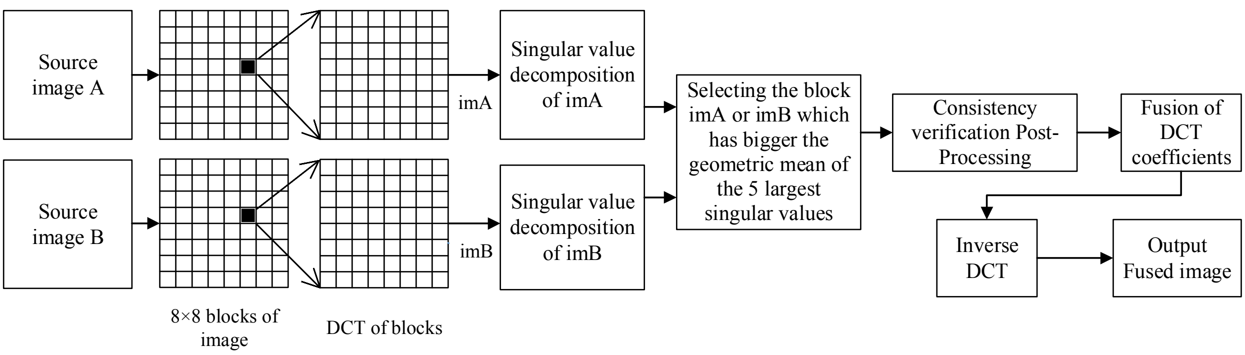

Figure 1. The general structure of the proposed method.

IV. Experimental Results

By comparing the previous methods, the simulation results of the proposed method have been stated in this section. Comparing the results of the proposed method with the previous and state of the art methods is done by both visual interpretation and image fusion quality metrics. The previous methods which are based on multi-scale transform and DCT domain are discussed and compared with the proposed method in this paper. The most significant multi-scale transform methods are DWT [19], SIDWT [20] and DCHWT [21], while the most prominent DCT domain transform methods are DCT+Average [27], DCT+Contrast [27], DCT+Variance [5], DCT+AC-Max [28], DCT+SF [29], DCT+SML [30], DCT+Sharp [31], and DCT+Corr [32].

A. Simulation Conditions

Simulations were performed using MATLAB® 2016b software. The Wavelet-based methods, such as DWT with DBSS(2.2), and SIDWT with Haar basis, were used and the input image decomposed into three levels. Simulation of these techniques were done by "Image Fusion Toolbox" [34]. In simulation of DCHWT, the algorithm mentioned in [35] was used. The simulation MATLAB code of DCT+Variance was taken from an online database [36]. The simulation of the other methods based on DCT-comparison such as DCT+Average [27], DCT+Contrast [27] DCT+AC_Max [28], DCT+SF [29], DCT+SML [30], DCT+Corr [31], and DCT+Sharp [32] were done by out of shelves codes or written by the authors for the best conditions of simulation. For all DCT based methods which were used in experiments, an averaging mask with the size of 3×3 was used for the majority filter for CV.

In this section in order to compare the proposed method with the previous ones, the algorithm has been executed on several images. In the first class of images, the ground-truth images are available; therefore the comparison of the outputs of the algorithms with their ground-truth image could be feasible. Seven gray-level 512×512 images, shown in Fig. 2, have been selected for these experiments. These standard images have been extracted from database [37]. By using two disk averaging filters, with the radius of 5 and 9, the multi-focus condition has been created artificially as follows. First, the filter has been applied to the right half section of the image, then to the left half section. In this way, 14 pairs of multi-focus images have been created from seven images of Fig. 2. The second class of images, are those that have been recorded with a different focus depth in camera than in real life. To experiment on these images, "Diver" multi-focus image has been used.

In order to compare the output of the proposed algorithm with the results of the previous ones, quantitative criteria are used to analyze the quality of the output images. To measure the quality of the output image for referenced multi-focus image fusion, SSIM [38] and MSE [38] metrics are used. For noise measurement, PSNR [38] is used. The higher amounts of SSIM and PSNR along with the lower amount of MSE present the better quality of the fused output images. The quality of the output images of the fusion of non-referenced multi-focus images is measured by the following metrics: $Q^{AB/F}$, $L^{AB/F}$, and $N^{AB/F}$ [39, 40], whereas $Q^{AB/F}$ is the total information transferred from the source images to the fused image, $L^{AB/F}$ is the total loss of information and $N^{AB/F}$ is the noise or artifacts added in the fused image due to fusion process. In addition, two other criteria, Spatial Frequency (SF) [12] and Feature Mutual Information (FMI) [41] are used; edge information extraction ability is the base for FMI. The higher amounts of $Q^{AB/F}$ and FMI along with the lower amounts of $L^{AB/F}$ and $N^{AB/F}$ present the better quality of the fused output images.

Figure 2. Standard source images employed in simulations.

B. Results Discussion

In the first part of the experiments, the proposed method and the previous methods were executed on 14 pairs of multi-focus images, which have been created artificially by the images in Fig. 2. The quality of the fused output images is evaluated by SSIM and MSE metrics. The average and standard deviation values of 14 fused multi-focus images are given in Table I. Considering the results from Table I, it can be concluded that the results of the proposed method (DCT+SVD) are the best in terms of quality among the previous methods, which has an SSIM_Average=0.9953 and an MSE_Average=1.6127. Another factor that has made the proposed method more preferable is due to the fact that the standard deviation of the SSIM and MSE metrics for the 14 fused images for the proposed method is more suitable (less) than those of the previous methods; these values are SSIM_Std=0.0070 and MSE_Std=2.1876. In addition, by adding the Consistency Verification (CV) as a post-processing step, the results given in Table I show the best output quality for the proposed method (DCT+SVD+CV).

At the second stage of experimentation, the algorithms were executed on non-referenced multi-focus image "Diver", which were recorded by the variation of the focus depth in real life. The "Diver" multi-focus image, which is focused in the front and in the back, is shown in Fig. 3(a) and (b), respectively. For visual comparison, the fused output images are shown in Fig. 3 (c)-(o). The fused output image of the proposed method (DCT+SVD) in Fig. 3(o) has the highest quality. As for the quantitative comparison, the values of $Q^{AB/F}$, $L^{AB/F}$, $N^{AB/F}$, FMI, and SF are given in Table II. Obviously, the proposed method (DCT+SVD) has the maximum amount of $Q^{AB/F}$ & FMI & SF, and the minimum amount of $L^{AB/F}$ & $N^{AB/F}$.

At the final stage, the stability and accountability of the proposed method and the other methods based on DCT, were studied considering noise. For this purpose, "Pirater" multi-focus images were fused with 4 types of noises. The Gaussian noise with (μ=0 & σ=0.001) and (μ=0 & σ=0.005) and also the "salt & pepper" noise with the density of D=0.02 & D=0.04 were considered. The PSNR amount in the proposed method (DCT+SVD+CV) and the other DCT based methods with CV post-processing are given in Table III. It is obvious that the proposed method has the best results considering different noises. In order to prove the case by visual interpretation, the noisy multi-focus images "Pirater" using "salt & pepper" with the noise density of D=0.02 and fusion results of the other DCT based methods are shown in Fig. 4. The difference images between noisy source images and the output fused images of the methods are also depicted in Fig. 4 (k)-(q). It is visible that the difference image for the proposed method has the least discrepancy with a PSNR=22.3032.

V. CONCLUSION

In this paper, a new multi-focus image fusion method using singular value decomposition in DCT domain was presented. Additionally, it was proven that singular values matrix ($\sum$) in spatial domain and ($\sum'$) in DCT domain are exactly the same. Throughout this method, the properties of Singular Value Decomposition (SVD) of input 8×8 blocks in DCT domain were used. The geometric mean of the 5 largest singular values is computed. The block which has the superior amount of the geometric mean among the other blocks, is chosen for the focused block. This block is then used for constructing the output image. Not only does the proposed method execute in DCT domain, but it also works in the spatial domain as well. Due to easy implementation in DCT domain, the proposed method is suitable for real-time applications. In order to analyze and compare other methods and the proposed method, numerous experiments were made on multi-focused JPEG images. The proposed method shows the best output image quality, regarding quantitative and qualitative criteria compared to the other methods. Moreover, this method has the most PSNR value and quality in noisy condition of multi-focus images among the other methods.

TABLE I. THE AVERAGE AND STANDARD DEVIATION OF SSIM AND MSE VALUES OF THE METHODS IN 14 PAIRS OF MULTI-FOCUS IMAGES WHICH ARE GENERATED ARTIFICIALLY FROM FIG.2.

Methods	**Average (Std) of SSIM**	**Average (Std) of MSE**
DCT+Average [27]	0.9187 (0.0232)	64.2293 (32.8139)
DCT+Contrast [27]	0.9645 (0.0161)	23.1248 (12.2736)
DWT [19]	0.9786 (0.0088)	17.1877 (9.8871)
SIDWT [20]	0.9808 (0.0095)	13.1387 (7.0087)
MSVD [26]	0.9333 (0.0217)	57.2680 (29.4494)
DCHWT [21]	0.9900 (0.0058)	4.9004 (3.4440)
DCT+Variance [5]	0.9713 (0.0167)	17.5789 (12.1764)
DCT+AC-Max [28]	0.9905 (0.0145)	4.7244 (7.3791)
DCT+SF [29]	0.9888 (0.0115)	6.0810 (6.4798)
DCT+Corr [31]	0.9929 (0.0059)	7.9954 (6.5302)
DCT+Sharp [32]	0.9858 (0.0137)	7.7916 (8.6286)
DCT+SML [30]	0.9803 (0.0322)	11.2477 (22.2358)
DCT+SVD (our proposed)	**0.9953 (0.0070)**	**1.6127 (2.1876)**
DCT+Variance+CV [5]	0.9909 (0.0099)	9.0675(11.2167)
DCT+AC-Max+CV [28]	0.9968 (0.0064)	1.5753 (2.9696)
DCT+SF+CV [29]	0.9964 (0.0063)	2.4377 (4.1082)
DCT+Corr+CV [31]	0.9964 (0.0051)	3.5226 (2.6533)
DCT+Sharp+CV [32]	0.9949 (0.0073)	2.4762 (4.2742)
DCT+SML+CV [30]	0.9953 (0.0111)	2.7315 (6.8176)
DCT+SVD+CV (our proposed)	**0.9979 (0.0051)**	**0.4374 (0.9286)**

TABLE II. THE $Q^{AB/F}$, $L^{AB/F}$, $N^{AB/F}$, FMI, AND SF COMPARISON OF THE VARIOUS MULTI-FOCUS IMAGE FUSION METHODS FOR "DIVER" NON-REFERENCED IMAGES.

Methods	Qabf	Labf	Nabf	FMI	SF
DCT+Average [27]	0.6247	0.3742	0.0022	0.8677	7.010
DCT+Contrast [27]	0.6873	0.2156	0.3018	0.8656	14.007
DWT [19]	0.7103	0.1933	0.3115	0.8705	14.141
SIDWT [20]	0.7386	0.2274	0.0954	0.8701	13.367
MSVD [26]	0.6207	0.3645	0.0311	0.8606	11.848
DCHWT [21]	0.7280	0.2511	0.0541	0.8714	13.173
DCT+Variance [5]	0.7413	0.2534	0.0113	0.8725	14.044
DCT+AC-Max [28]	0.7300	0.2612	0.0188	0.8732	14.029
DCT+SF [29]	0.7428	0.2532	0.0087	0.8737	14.148
DCT+Corr [31]	0.7478	0.2485	0.0080	0.8741	14.108
DCT+Sharp [32]	0.7454	0.2507	0.0085	0.8736	14.142
DCT+SML [30]	0.7352	0.2578	0.0152	0.8735	14.105
DCT+SVD (our proposed)	**0.7501**	**0.2462**	**0.0079**	**0.8742**	**14.161**

TABLE III. THE PSNR COMPARISON OF NOISY MULTI-FOCUS IMAGE FUSION METHODS FOR "PIRATER" IMAGES.

Methods	Salt & Pepper D=0.02	Salt & Pepper D=0.04	Gaussian μ=0 & σ=0.001	Gaussian μ=0 & σ=0.005
DCT+Variance+CV [5]	21.6101	18.7496	29.4928	22.7639
DCT+AC_Max+CV [28]	21.1156	18.5202	29.7557	22.6949
DCT+SF+CV [29]	21.1887	18.4093	29.8117	22.7239
DCT+Corr+CV [31]	21.2125	18.2987	29.2501	22.0268
DCT+Sharp+CV [32]	21.9789	18.9379	29.7477	22.6098
DCT+SML+CV [30]	21.2488	18.3706	26.8710	21.6382
DCT+SVD+CV (our proposed)	**22.3032**	**19.1128**	**29.8761**	**22.8456**

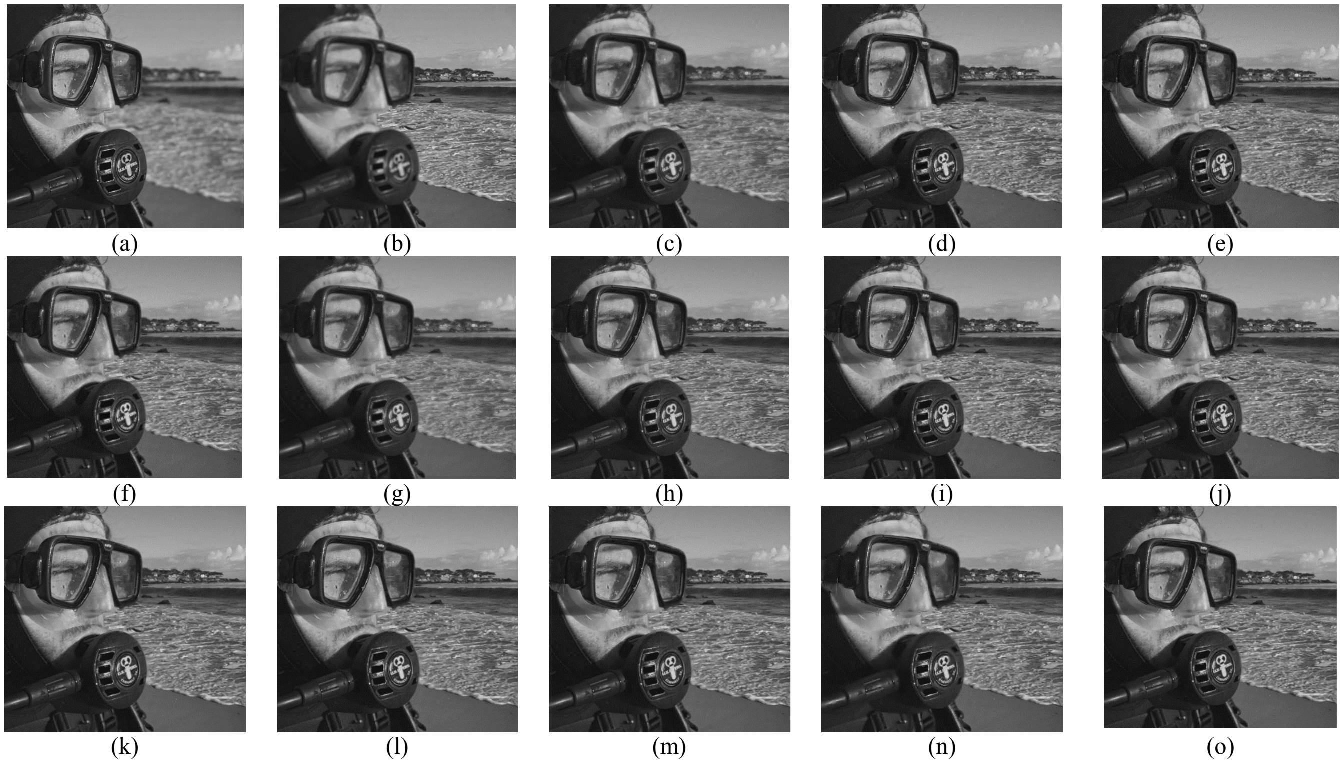

Figure 3. Real Multi-focus images “Diver” and the fusion result of methods. (a) The first source image with focus on left. (b) The second source image with focus on right. (c) DCT+Average. (d) DCT+Contrast. (e) DWT. (f) SIDWT. (g) MSVD. (h) DCHWT. (i) DCT+Variance. (j) DCT+AC_Max. (k) DCT+SF. (l) DCT+Corr. (m) DCT+Sharp. (n) DCT+SML. **(o) DCT+SVD (our proposed method).**

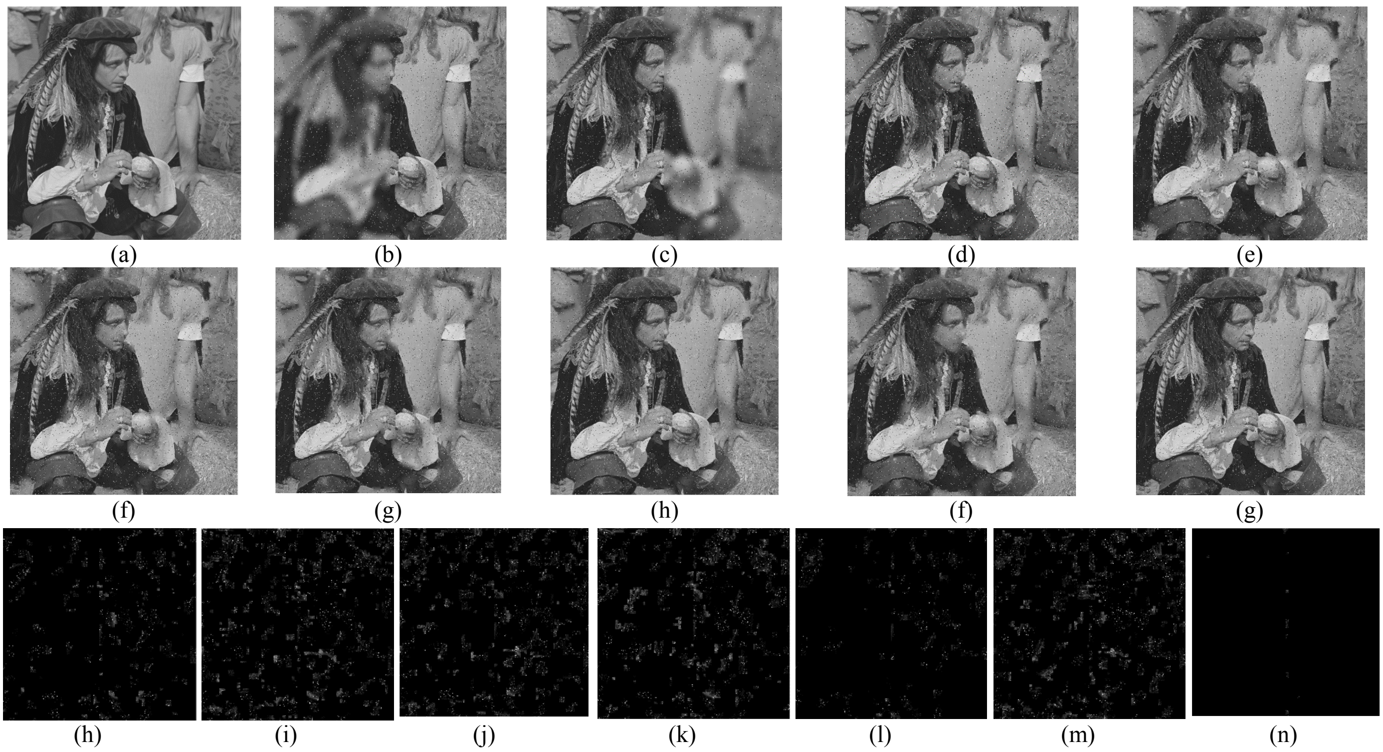

Figure 4. The noisy multi-focus image fusion of “Pirater” using “salt & pepper” with noise density of D=0.02 and the fusion results of different methods. (a) Ground-Truth image. (b) The first noisy image with focus on the right half. (c) The second noisy image with focus on the left half. (d) DCT+Variance+CV. (e) DCT+AC_Max+CV. (f) DCT+SF+CV. (g) DCT+Corr+CV. (h) DCT+Sharp+CV. (i) DCT+SML+CV. **(j) DCT+SVD+CV (our proposed).** (k), (l), (m), (n), (o), (p), and (q) are the difference images between the noisy source images and (d), (e), (f), (g), (h), (i) and (j), respectively.

REFERENCES

[1] M. Amin-Naji and A. Aghagolzadeh. "Multi-focus image fusion in DCT domain using variance and energy of Laplacian and correlation coefficient for visual sensor networks." *Journal of AI and Data Mining*, in press.

[2] M. Nejati, S. Samavi, N. Karimi, S.M.R. Soroushmehr, S. Shirani, I. Rosta, and K. Najarian. "Surface area-based focus criterion for multi-focus image fusion." Information Fusion, vol. 36, July, pp. 284–295, 2017.

[3] W. Wu, X. Yang, Y. Pang, J. Peng, and G. Jeon. "A multifocus image fusion method by using hidden Markov model." *Optics Communications*, vol. 287, January, pp.63-72, 2013.

[4] M. Nejati, S. Shadrokh and S. Shirani. "Multi-focus image fusion using dictionary-based sparse representation." *Information Fusion*, vol. 25, September, pp.72-84, 2015.

[5] M. Haghighat, A. Aghagolzadeh and H. Seyedarabi, "Multi-focus image fusion for visual sensor networks in DCT domain," *Computers & Electrical Engineering*, vol. 37, no. 5, pp. 789-797, 2011.

[6] S. Soro, & W. Heinzelman. "A Survey of Visual Sensor Networks." *Advances in Multimedia*, vol. 2009, 21 pages, 2009.

[7] M. Amin-Naji, A. Aghagolzadeh. "Multi-focus image fusion using VOL and EOL in DCT domain." In *2016 1st International Conference on New Research Achievements in Electrical and Computer Engineering*, pp. 728-733, 2016.

[8] M. B. A. Haghighat, A. Aghagolzadeh, and H. Seyedarabi, "Real-time fusion of multi-focus images for visual sensor networks," in *2010 6th Iranian Machine Vision and Image Processing (MVIP)*, Isfahan, pp. 1-6, 2010.

[9] W. Huang and Z. Jing, "Evaluation of focus measures in multi-focus image fusion," *Pattern Recognition Letters*, vol. 28, no. 4, pp. 493-500, 2007.

[10] S. Pertuz, D. Puig, and M. A. Garcia, "Analysis of focus measure operators for shape-from-focus," *Pattern Recognition*, vol. 46, no. 5, pp. 1415-1432, 2013.

[11] H. Zhao, Q. Li, and H. Feng. "Multi-focus color image fusion in the HSI space using the sum-modified-Laplacian and a coarse edge map." *Image and Vision Computing*, vol. 26, no.9, pp. 1285-1295, 2008.

[12] S. Li, and B. Yang. "Multifocus image fusion using region segmentation and spatial frequency." *Image and Vision Computing*, vol. 26, no.7, pp.971-979, 2008.

[13] S. Mahajan, and A. Singh. "A Comparative Analysis of Different Image Fusion Techniques." *IPASJ International Journal of Computer Science (IIJCS)*, vol. 2, no. 1, 2014.

[14] H.A. Eltoukhy and S. Kavusi. "Computationally efficient algorithm for multifocus image reconstruction." *Proceedings SPIE Electronic Imaging*, vol. 5017, pp. 332–341, 2003.

[15] S. Li, J.T. Kwok, and Y. Wang. "Combination of images with diverse focuses using the spatial frequency," *Information fusion*, vol. 2, no. 3, pp. 169-176, 2001.

[16] V.S. Petrovic and C.S. Xydeas. "Gradient-based multiresolution image fusion." *IEEE Transactions Image Processing*. vol. 13, no. 2, pp. 228-237, 2004.

[17] P.J. Burt, and E.H. Adelson. "The Laplacian pyramid as a compact image code." *IEEE Transactions Communications*. vol. 31, no. 4, pp. 532-540, 1983.

[18] I. De and B. Chanda. "A simple and efficient algorithm for multifocus image fusion using morphological wavelets." *Signal Processing*. vol. 86, no. 5, pp. 924-936, 2006.

[19] H. Li, B. Manjunath and S. Mitra, "Multisensor Image Fusion Using the Wavelet Transform," *Graphical Models and Image Processing*, vol. 57, no. 3, pp. 235-245, 1995.

[20] O. Rockinger, "Image sequence fusion using a shift-invariant wavelet transform," in *Proceedings of IEEE International Conference on Image Processing*, vol. 3, Santa Barbara, pp. 288-291, 1997.

[21] B.K.S. Kumar. "Multifocus and multispectral image fusion based on pixel significance using discrete cosine harmonic wavelet transform." *Signal, Image and Video Processing*, vol. 7, no. 6, pp.1125-1143, 2013.

[22] J. Liang, Y. He, D. Liu, and X. Zeng. "Image fusion using higher order singular value decomposition." *IEEE Transactions on Image Processing,* vol. 21, no. 5, pp. 2898-2909, 2012.

[23] C. Zhang, X. Luo, Z. Zhang, R. Gao, and X. Wu. "Multi-Focus Image Fusion Method Using Higher Order Singular Value Decomposition and Fuzzy Reasoning." *Journal of Algorithms & Computational Technology*, vol.9, no.3, pp. 303-321, 2015.

[24] S. Gautam, and M. Kumar. "An Effective Image Fusion Technique based on Multiresolution Singular Value Decomposition." *INFOCOMP Journal of Computer Science*, vol. 14, no.2, pp.31-43, 2016.

[25] C. Zhang, X. Luo, Z. Zhang, R. Gao, and X. Wu. "Multi-Focus Image Fusion Method Using Higher Order Singular Value Decomposition and Fuzzy Reasoning." *Journal of Algorithms & Computational Technology*, vol. 9, no.3, pp. 303-321, 2015.

[26] V.P.S. Naidu, "Image fusion technique using multi-resolution singular value decomposition," *Defence Science Journal*, vol. 61, no. 5, pp. 479-484, 2011.

[27] J. Tang, "A contrast based image fusion technique in the DCT domain," *Digital Signal Processing*, vol. 14, no. 3, pp. 218-226, 2004.

[28] Y. Phamila and R. Amutha, "*Discrete Cosine Transform based fusion of multi-focus images for visual sensor networks*," Signal Processing, vol. 95, February, pp. 161-170, 2014.

[29] L. Cao, L. Jin, H. Tao, G. Li, Z. Zhuang and Y. Zhang, "*Multi-Focus Image Fusion Based on Spatial Frequency in Discrete Cosine Transform Domain*," IEEE Signal Processing Letters, vol. 22, no. 2, pp. 220-224, 2015.

[30] M. Abdollahzadeh, T. Malekzadeh, and H. Seyedarabi. "Multi-focus image fusion for visual sensor networks." In *2016 24th Iranian Conference on Electrical Engineering (ICEE)*, pp. 1673-1677, 2016.

[31] M.A.Naji and A. Aghagolzadeh, "Multi-focus image fusion in DCT domain based on correlation coefficient," in *2015 2nd International Conference on Knowledge-Based Engineering and Innovation (KBEI)*, pp. 632-639, 2015.

[32] M.A.Naji and A. Aghagolzadeh, "A new multi-focus image fusion technique based on variance in DCT domain," in *2015 2nd International Conference on Knowledge-Based Engineering and Innovation (KBEI)*, pp. 478-484, 2015.

[33] M. Amin-Naji, A. Aghagolzadeh. "Block DCT filtering using vector processing." In 2016 *1st International Conference on New Research Achievements in Electrical and Computer Engineering*, 2016.

[34] O. Rockinger, image fusion toolbox, 1999. Available: http://www.metapix.de/toolbox.htm

[35] B.K.S. Kumar, Multifocus and multispectral image fusion based on pixel significance using dchwt, 2013, Available: https://www.mathworks.com/matlabcentral/fileexchange/43051-multifocus-and-multispectral-image-fusion-based-on-pixel-significance-using-dchwt?focused=3793307&tab=function

[36] M.B.A. Haghighat, Multi-Focus Image Fusion in DCT Domain, 2015, Available: https://github.com/mhaghighat/dctVarFusion

[37] Gonzalez, R.C., Woods, R.E., Eddins, S.L. Image processing place, Available: http://www.imageprocessingplace.com/root_files_V3/image_databases.htm

[38] Z. Wang and A. Bovik. "Mean squared error: Love it or leave it? A new look at Signal Fidelity Measures." *IEEE Signal Processing Magazine*, vol. 26, no. 1, pp. 98-117, 2009.

[39] C. Xydeas, and V. Petrovic. "Objective image fusion performance measure." *Electronics Letters*, vol. 36, no.4, pp. 308-309, 2009.

[40] V. Petrovic and C. Xydeas. "Objective image fusion performance characterization." in *Tenth IEEE International Conference on Computer Vision (ICCV)*, pp. 1866-1871, 2005.

[41] M.B.A. Haghighat, A. Aghagolzadeh, and H. Seyedarabi. "A non-reference image fusion metric based on mutual information of image features." *Computers & Electrical Engineering*, vol. 37, no. 5, pp. 744-756, 2011.