

Grammar

A grammar G is defined as quadruple

$$G = (V, T, P, S)$$

where V is a finite set of objects called variables or nonterminal symbols

T is a finite set of objects called Terminal symbols

$S \in V$ is the start symbol (non terminal)

P is a finite set of productions.

Let $G = (V, T, P, S)$ be a grammar. Then the set Language: $L(G) = \{w \in T^* : S \xRightarrow{*} w\}$ is the language set of terminals generated by G .
 ($\xRightarrow{}$ means in many steps)*

Example: Consider the grammar $G = (\{S\}, \{a, b\}, S, P)$ with

P given by $S \rightarrow aSb \mid \lambda$

$$L(G) = \{\lambda, ab, aabb, aaabbb, \dots\}$$

$$L(G) = \{a^n b^n : n \geq 0\}$$

$$\begin{aligned} S &\rightarrow aSb \rightarrow \\ aSb &\rightarrow aabb \end{aligned}$$

Q Consider the Grammar $G = (\{A, S\}, \{a, b\}, S, P)$ with P consisting of the production.

(i) $S \rightarrow aAb$

$A \rightarrow aAb \mid \lambda$

(ii) $S \rightarrow aA$

$A \rightarrow bS$

(iii) $S \rightarrow \lambda$

$S \rightarrow Aa$

$A \rightarrow B$

$B \rightarrow Aa$

Find language generated by Grammar

Ans i), $S \rightarrow aAb \rightarrow ab$

$\{ab, aabb, aaabbb, \dots\}$ $S \rightarrow aAb$
 $aAb \rightarrow ab$

$L(h_1) = \{ab, a^2b^2, a^3b^3, \dots\}$

$L(h_1) = \{a^n b^n : n \geq 1\}$

(ii) $S \rightarrow aA$

$A \rightarrow bS$

$S \rightarrow \lambda$

$L(h_2) = \{\lambda, ab, abab, \dots\}$

$= \{\lambda, ab, (ab)^2, (ab)^3, \dots\}$

$L(h_2) = \{(ab)^n : n \geq 0\}$

(iii) $S \rightarrow Aa$

$A \rightarrow B$

$B \rightarrow Aa$

So $L(h_3) = \{\} \text{ or } \phi$

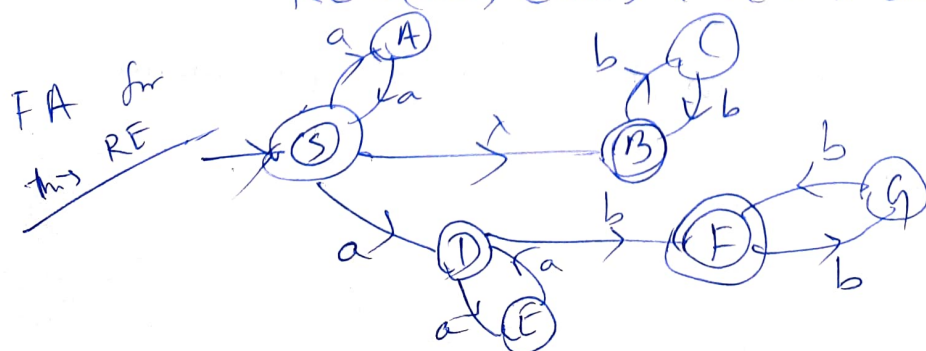
null set.

There is no termination.

~~Next step~~
 Find a Regular Grammar for a language

$L = \{a^n b^m : n+m \text{ is even}\}$

RE = $(aa)^* (bb)^* + a(aa)^* b(bb)^*$



Regular grammar

Right linear grammar

$$S \rightarrow \lambda / aA / B / aD$$

$$A \rightarrow aS$$

$$B \rightarrow \lambda / bC$$

$$C \rightarrow bB$$

$$D \rightarrow aE / bF$$

$$E \rightarrow aD$$

$$F \rightarrow \lambda / bG$$

$$G \rightarrow bF$$

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Q Find right linear and left linear grammar for the language accepted by the following nfa.



RLG for the language L

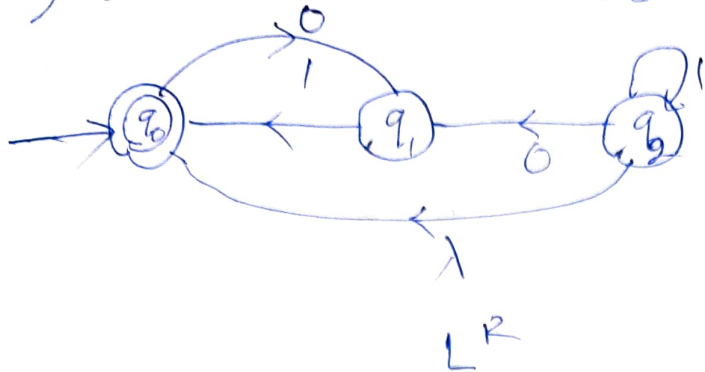
$$q_0 \rightarrow \lambda / 1q_1 / 1q_2$$

$$q_1 \rightarrow 0q_0 / 0q_2$$

$$q_2 \rightarrow 1q_2$$

This is the right linear grammar for the NFA.

I have to take the reverse of NFA to get left linear grammar.



Right Linear grammar for the language L^R

$$q_0 \rightarrow \lambda / 0q_1$$

$$q_1 \rightarrow 1q_0$$

$$q_2 \rightarrow 0q_1 / 1q_2 / q_0$$

Left Linear grammar for L

$$q_0 \rightarrow \lambda / q_1 0$$

$$q_1 \rightarrow q_0 1$$

$$q_2 \rightarrow q_1 0 / q_2 1 / q_0$$

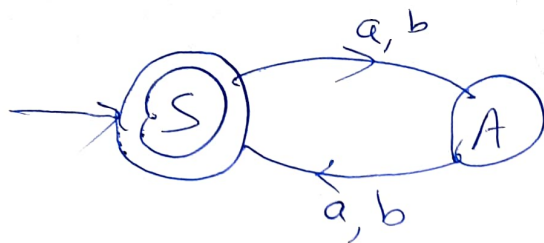
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Regular Grammar

Find a regular grammar that generates the language

$$L = \{w \in \{a, b\}^* : \sigma_a(w) + 3\sigma_b(w) \text{ is even}\}$$

$$L = \{\lambda, aa, bb, ab, ba, aaaa, aaab, aabb, \dots\}$$



$$S \rightarrow \lambda | aA | bA$$

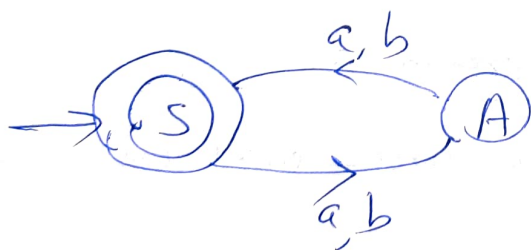
$$A \rightarrow aS | bS$$

This is the regular grammar for the regular language.

This is the right linear grammar

If I want to get the left linear grammar for the given FA then

I will simply reverse the FA.



Right Linear grammar

$$S \rightarrow \lambda | aA | bA$$

$$A \rightarrow aS | bS$$

left linear grammar will be

$$S \rightarrow \lambda | Aa | Ab$$

$$A \rightarrow Sa | Sb$$

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Left Linear grammar and Right Linear grammar

A grammar $G = (V, T, P, S)$ is said to be left linear grammar if it is of the form

$$A \rightarrow Bu | \lambda$$

and it is said to be right linear if it is of the form

$$A \rightarrow \lambda B | u$$

where $A, B \in V$ and $u \in T^*$

Note

A grammar is said to be regular if it is either left linear or right linear.

EX

$$S \rightarrow aA$$

$$A \rightarrow bA | \lambda$$

This is right linear grammar

It is a regular grammar as it is right linear

EX

$$S \rightarrow Ba$$

$$B \rightarrow ab$$

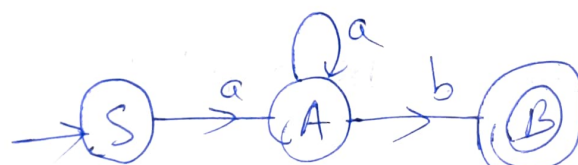
This is left linear grammar.

It is also a regular grammar as it is left linear.

Ex 3 $S \rightarrow aA$ right linear
 $A \rightarrow bB$ right linear
 $B \rightarrow Cd$ left linear
 $C \rightarrow e$

The grammar is neither left linear or right linear. This is not a regular grammar.

EX $S \rightarrow aA$
 $A \rightarrow aA|b$



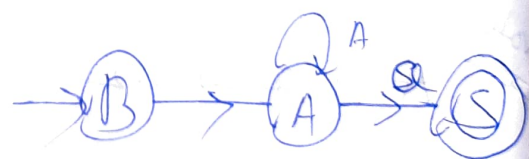
RE $aa^*b = a^+b$

$S \rightarrow Aa$
 $A \rightarrow Aa|b$

R.E baa^*
 $= ba^+$

$FA \rightarrow RL G \rightarrow Rev FA$
 $(L) \quad (L) \quad (L)$

$\rightarrow RL G (Rev FA) \rightarrow LL G$
 $(L^R) \quad (L)$



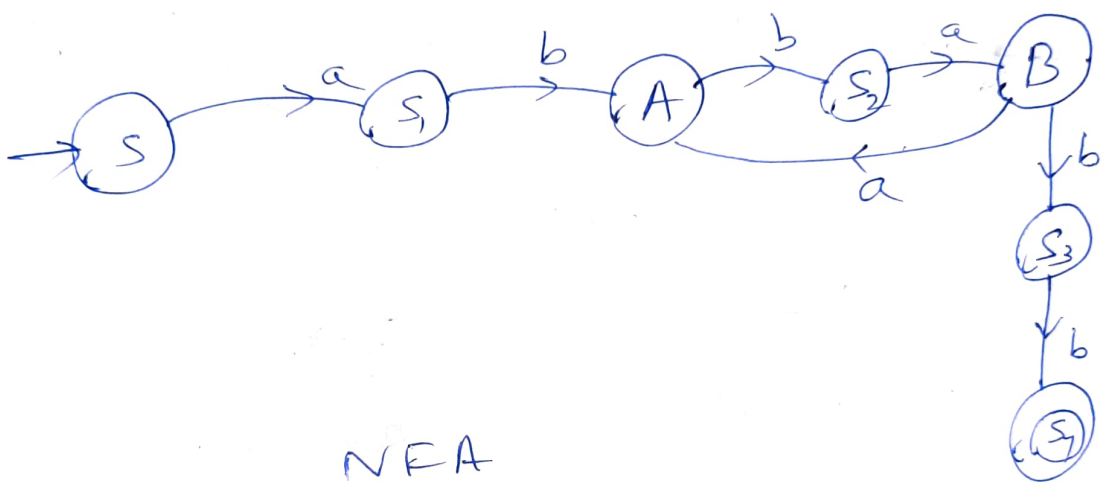
$B \rightarrow bA$ $B \rightarrow Ab$
 $A \rightarrow aA|a$ $A \rightarrow Aa$

Q Construct a DFA that accepts the language generated by the grammar.

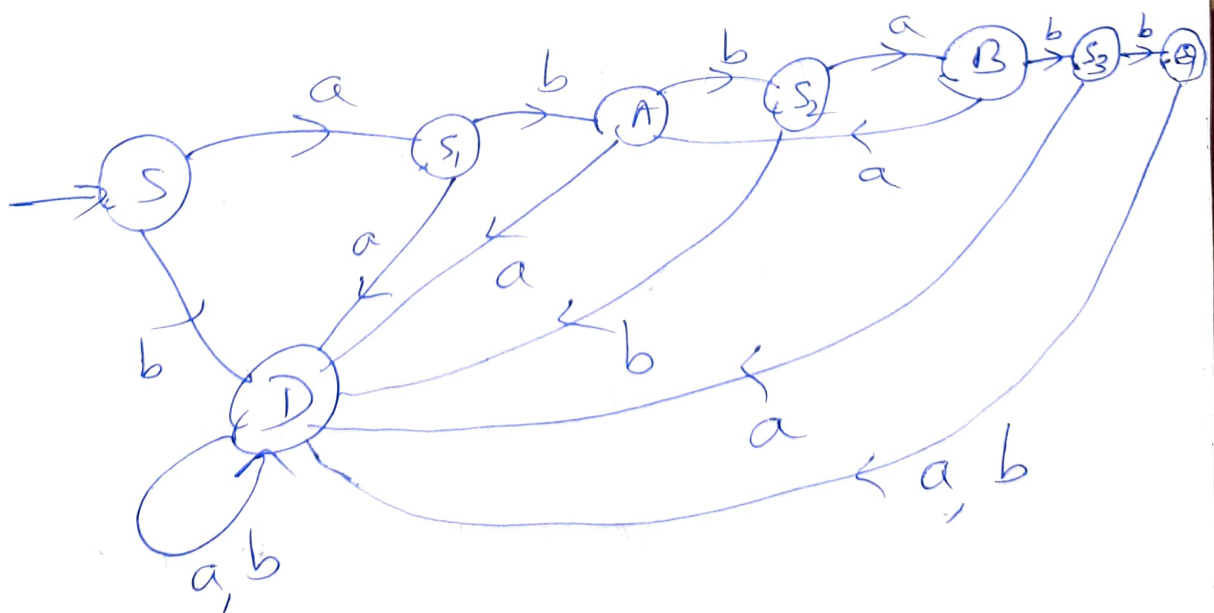
$S \rightarrow abA$

$A \rightarrow baB$

$B \rightarrow aA \mid bb$

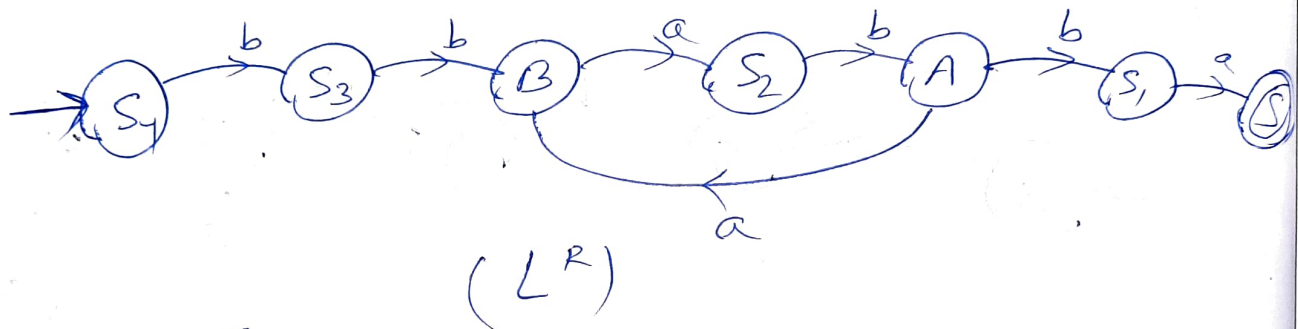
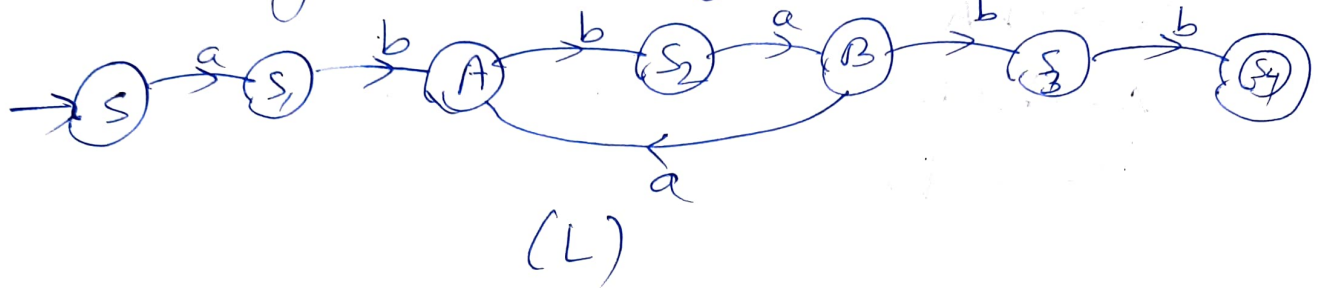


NFA



DFA for this right linear grammar

Find the left linear grammar equivalent to the right linear grammar.



$$S_4 \rightarrow bS_3$$

$$S_3 \rightarrow bB$$

$$B \rightarrow aS_2$$

$$S_2 \rightarrow bA$$

$$A \rightarrow bS_1 | aB$$

$$S_1 \rightarrow aS$$

$$S \rightarrow \lambda$$

$$S_4 \rightarrow bbB$$

$$B \rightarrow abA$$

$$A \rightarrow aB | baS$$

$$S \rightarrow \lambda$$

Both are correct

If I reverse this right linear grammar then

$$S \rightarrow Bbb$$

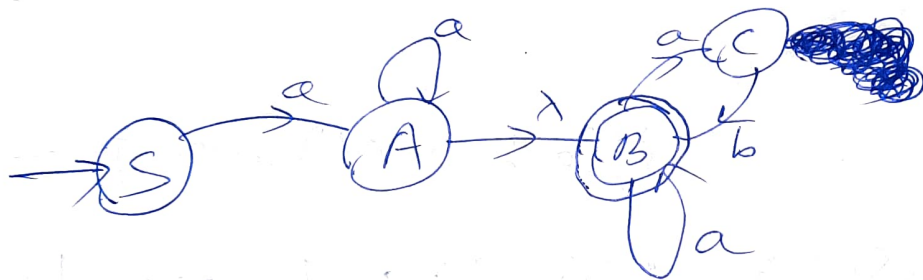
$$B \rightarrow \cancel{Aba} A ba$$

$$A \rightarrow Ba | Sab$$

$$S \rightarrow \lambda$$

This is the left linear grammar.

lect-89 Find a regular grammar that generates the language $L(aa^*(abta)^*)$



Right linear grammar or Regular grammar for $L(aa^*(abta)^*)$ is

$$S \rightarrow aA$$

$$A \rightarrow aA | B$$

$$B \rightarrow \cancel{aB} | aB | aC$$

$$C \rightarrow bB$$