EXERCISE SET 1.2

1. Which of the following sets are equal?

$A = \{a, b, c, d\}$ $\quad B = \{d, e, a, c\}$

$C = \{d, b, a, c\}$ $\quad D = \{a, a, d, e, c, e\}$

2. Write in words how to read each of the following out loud.
   a. $\{x \in \mathbf{R}^+ \mid 0 < x < 1\}$
   b. $\{x \in \mathbf{R} \mid x \leq 0 \text{ or } x \geq 1\}$
   c. $\{n \in \mathbf{Z} \mid n \text{ is a factor of } 6\}$
   d. $\{n \in \mathbf{Z}^+ \mid n \text{ is a factor of } 6\}$

3. a. Is $4 = \{4\}$?
   b. How many elements are in the set $\{3, 4, 3, 5\}$?
   c. How many elements are in the set $\{1, \{1\}, \{1, \{1\}\}\}$?

4. a. Is $2 \in \{2\}$?
   b. How many elements are in the set $\{2, 2, 2, 2\}$?
   c. How many elements are in the set $\{0, \{0\}\}$?
   d. Is $\{0\} \in \{\{0\}, \{1\}\}$?
   e. Is $0 \in \{\{0\}, \{1\}\}$?

H 5. Which of the following sets are equal?

$A = \{0, 1, 2\}$

$B = \{x \in \mathbf{R} \mid -1 \leq x < 3\}$

$C = \{x \in \mathbf{R} \mid -1 < x < 3\}$

$D = \{x \in \mathbf{Z} \mid -1 < x < 3\}$

$E = \{x \in \mathbf{Z}^+ \mid -1 < x < 3\}$

H 6. For each integer n, let $T_n = \{n, n^2\}$. How many elements are in each of T_2, T_{-3}, T_1, and T_0? Justify your answers.

7. Use the set-roster notation to indicate the elements in each of the following sets.
   a. $S = \{n \in \mathbf{Z} \mid n = (-1)^k$, for some integer $k\}$.
   b. $T = \{m \in \mathbf{Z} \mid m = 1 + (-1)^i$, for some integer $i\}$.
   c. $U = \{r \in \mathbf{Z} \mid 2 \leq r \leq -2\}$
   d. $V = \{s \in \mathbf{Z} \mid s > 2 \text{ or } s < 3\}$
   e. $W = \{t \in \mathbf{Z} \mid 1 < t < -3\}$
   f. $X = \{u \in \mathbf{Z} \mid u \leq 4 \text{ or } u \geq 1\}$

8. Let $A = \{c, d, f, g\}$, $B = \{f, j\}$, and $C = \{d, g\}$. Answer each of the following questions. Give reasons for your answers.
   a. Is $B \subseteq A$?
   b. Is $C \subseteq A$?
   c. Is $C \subseteq C$?
   d. Is C a proper subset of A?

9. a. Is $3 \in \{1, 2, 3\}$?
   b. Is $1 \subseteq \{1\}$?
   c. Is $\{2\} \in \{1, 2\}$?
   d. Is $\{3\} \in \{1, \{2\}, \{3\}\}$?
   e. Is $1 \in \{1\}$?
   f. Is $\{2\} \subseteq \{1, \{2\}, \{3\}\}$?
   g. Is $\{1\} \subseteq \{1, 2\}$?
   h. Is $1 \in \{\{1\}, 2\}$?
   i. Is $\{1\} \subseteq \{1, \{2\}\}$?
   j. Is $\{1\} \subseteq \{1\}$?

10. a. Is $((-2)^2, -2^2) = (-2^2, (-2)^2)$?
    b. Is $(5, -5) = (-5, 5)$?
    c. Is $(8 - 9, \sqrt[3]{-1}) = (-1, -1)$?
    d. Is $\left(\frac{-2}{-4}, (-2)^3\right) = \left(\frac{3}{6}, -8\right)$?

11. Let $A = \{w, x, y, z\}$ and $B = \{a, b\}$. Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set.
    a. $A \times B$
    b. $B \times A$
    c. $A \times A$
    d. $B \times B$

EXERCISE SET 1.3

1. Let $A = \{2, 3, 4\}$ and $B = \{6, 8, 10\}$ and define a relation R from A to B as follows: For every $(x, y) \in A \times B$,

$$(x, y) \in R \quad \text{means that} \quad \frac{y}{x} \text{ is an integer.}$$

   a. Is $4\,R\,6$? Is $4\,R\,8$? Is $(3, 8) \in R$? Is $(2, 10) \in R$?
   b. Write R as a set of ordered pairs.
   c. Write the domain and co-domain of R.
   d. Draw an arrow diagram for R.

2. Let $C = D = \{-3, -2, -1, 1, 2, 3\}$ and define a relation S from C to D as follows: For every $(x, y) \in C \times D$,

$$(x, y) \in S \quad \text{means that} \quad \tfrac{1}{x} - \tfrac{1}{y} \text{ is an integer.}$$

   a. Is $2\,S\,2$? Is $-1\,S\,-1$? Is $(3, 3) \in S$? Is $(3, -3) \in S$?
   b. Write S as a set of ordered pairs.
   c. Write the domain and co-domain of S.
   d. Draw an arrow diagram for S.

3. Let $E = \{1, 2, 3\}$ and $F = \{-2, -1, 0\}$ and define a relation T from E to F as follows: For every $(x, y) \in E \times F$,

$$(x, y) \in T \quad \text{means that} \quad \frac{x - y}{3} \text{ is an integer.}$$

   a. Is $3\,T\,0$? Is $1\,T\,(-1)$? Is $(2, -1) \in T$? Is $(3, -2) \in T$?
   b. Write T as a set of ordered pairs.
   c. Write the domain and co-domain of T.
   d. Draw an arrow diagram for T.

4. Let $G = \{-2, 0, 2\}$ and $H = \{4, 6, 8\}$ and define a relation V from G to H as follows: For every $(x, y) \in G \times H$,

$$(x, y) \in V \quad \text{means that} \quad \frac{x - y}{4} \text{ is an integer.}$$

   a. Is $2\,V\,6$? Is $(-2)\,V\,(8)$? Is $(0, 6) \in V$? Is $(2, 4) \in V$?
   b. Write V as a set of ordered pairs.
   c. Write the domain and co-domain of V.
   d. Draw an arrow diagram for V.

5. Define a relation S from $\mathbf{R}$ to $\mathbf{R}$ as follows: For every $(x, y) \in \mathbf{R} \times \mathbf{R}$,

$$(x, y) \in S \quad \text{means that} \quad x \geq y.$$

   a. Is $(2, 1) \in S$? Is $(2, 2) \in S$? Is $2\,S\,3$? Is $(-1)\,S\,(-2)$?
   b. Draw the graph of S in the Cartesian plane.

6. Define a relation R from $\mathbf{R}$ to $\mathbf{R}$ as follows: For every $(x, y) \in \mathbf{R} \times \mathbf{R}$,

$$(x, y) \in R \quad \text{means that} \quad y = x^2.$$

   a. Is $(2, 4) \in R$? Is $(4, 2) \in R$? Is $(-3)\,R\,9$? Is $9\,R\,(-3)$?
   b. Draw the graph of R in the Cartesian plane.

7. Let $A = \{4, 5, 6\}$ and $B = \{5, 6, 7\}$ and define relations R, S, and T from A to B as follows: For every $(x, y) \in A \times B$:

$$(x, y) \in R \quad \text{means that} \quad x \geq y.$$
$$(x, y) \in S \quad \text{means that} \quad \frac{x - y}{2} \text{ is an integer.}$$
$$T = \{(4, 7), (6, 5), (6, 7)\}.$$

   a. Draw arrow diagrams for R, S, and T.
   b. Indicate whether any of the relations R, S, and T are functions.

8. Let $A = \{2, 4\}$ and $B = \{1, 3, 5\}$ and define relations U, V, and W from A to B as follows: For every $(x, y) \in A \times B$:

$$(x, y) \in U \quad \text{means that} \quad x > 2.$$
$$(x, y) \in V \quad \text{means that} \quad y - 1 = \tfrac{x}{2}.$$
$$W = \{(2, 5), (4, 1), (2, 3)\}.$$

   a. Draw arrow diagrams for U, V, and W.
   b. Indicate whether any of the relations U, V, and W are functions.

EXERCISE SET 2.1

In each of 1–4 represent the common form of each argument using letters to stand for component sentences, and fill in the blanks so that the argument in part (b) has the same logical form as the argument in part (a).

1. a. If all integers are rational, then the number 1 is rational.
All integers are rational.
Therefore, the number 1 is rational.

b. If all algebraic expressions can be written in prefix notation, then __________________.

____________________________.

Therefore, $(a+2b)(a^2-b)$ can be written in prefix notation.

2. a. If all computer programs contain errors, then this program contains an error.
This program does not contain an error.
Therefore, it is not the case that all computer programs contain errors.

b. If ________, then ________.
2 is not odd.
Therefore, it is not the case that all prime numbers are odd.

3. a. This number is even or this number is odd.
This number is not even.
Therefore, this number is odd.

b. ________ or logic is confusing.
My mind is not shot.
Therefore, ________.

4. a. If the program syntax is faulty, then the computer will generate an error message.
If the computer generates an error message, then the program will not run.
Therefore, if the program syntax is faulty, then the program will not run.

b. If this simple graph ________, then it is complete.
If this graph ________, then any two of its vertices can be joined by a path.
Therefore, if this simple graph has 4 vertices and 6 edges, then ________.

5. Indicate which of the following sentences are statements.

a. 1,024 is the smallest four-digit number that is a perfect square.

b. She is a mathematics major.

c. $128 = 2^6$

d. $x = 2^6$

Write the statements in 6–9 in symbolic form using the symbols $\sim$, $\vee$, and $\wedge$ and the indicated letters to represent component statements.

6. Let s = "stocks are increasing" and i = "interest rates are steady."

a. Stocks are increasing but interest rates are steady.

b. Neither are stocks increasing nor are interest rates steady.

7. Juan is a math major but not a computer science major. (m = "Juan is a math major," c = "Juan is a computer science major")

8. Let h = "John is healthy," w = "John is wealthy," and s = "John is wise."

a. John is healthy and wealthy but not wise.

b. John is not wealthy but he is healthy and wise.

c. John is neither healthy, wealthy, nor wise.

d. John is neither wealthy nor wise, but he is healthy.

e. John is wealthy, but he is not both healthy and wise.

9. Let p = "$x > 5$," q = "$x = 5$," and r = "$10 > x$."

a. $x \geq 5$

b. $10 > x > 5$

c. $10 > x \geq 5$

10. Let p be the statement "DATAENDFLAG is off," q the statement "ERROR equals 0," and r the statement "SUM is less than 1,000." Express the following sentences in symbolic notation.

a. DATAENDFLAG is off, ERROR equals 0, and SUM is less than 1,000.

b. DATAENDFLAG is off but ERROR is not equal to 0.

c. DATAENDFLAG is off; however, ERROR is not 0 or SUM is greater than or equal to 1,000.

d. DATAENDFLAG is on and ERROR equals 0 but SUM is greater than or equal to 1,000.

e. Either DATAENDFLAG is on or it is the case that both ERROR equals 0 and SUM is less than 1,000.

11. In the following sentence, is the word *or* used in its inclusive or exclusive sense? A team wins the playoffs if it wins two games in a row or a total of three games.

Write truth tables for the statement forms in 12–15.

12. $\sim p \wedge q$

13. $\sim(p \wedge q) \vee (p \vee q)$

14. $p \wedge (q \wedge r)$

15. $p \wedge (\sim q \vee r)$

Determine whether the statement forms in 16–24 are logically equivalent. In each case, construct a truth table and include a sentence justifying your answer. Your sentence should show that you understand the meaning of logical equivalence.

16. $p \vee (p \wedge q)$ and p

17. $\sim(p \wedge q)$ and $\sim p \wedge \sim q$

18. $p \vee \mathbf{t}$ and $\mathbf{t}$

19. $p \wedge \mathbf{t}$ and p

20. $p \wedge \mathbf{c}$ and $p \vee \mathbf{c}$

21. $(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$

22. $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$

23. $(p \wedge q) \vee r$ and $p \wedge (q \vee r)$

24. $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$

Rewrite the statements in 1–4 in if-then form.

1. This loop will repeat exactly N times if it does not contain a **stop** or a **go to.**

2. I am on time for work if I catch the 8:05 bus.

3. Freeze or I'll shoot.

4. Fix my ceiling or I won't pay my rent.

Construct truth tables for the statement forms in 5–11.

5. $\sim p \vee q \rightarrow \sim q$

6. $(p \vee q) \vee (\sim p \wedge q) \rightarrow q$

7. $p \wedge \sim q \rightarrow r$

8. $\sim p \vee q \rightarrow r$

9. $p \wedge \sim r \leftrightarrow q \vee r$

10. $(p \rightarrow r) \leftrightarrow (q \rightarrow r)$

11. $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$

12. Use the logical equivalence established in Example 2.2.3, $p \vee q \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$, to rewrite the following statement. (Assume that x represents a fixed real number.)

    If $x > 2$ or $x < -2$, then $x^2 > 4$.

13. Use truth tables to verify the following logical equivalences. Include a few words of explanation with your answers.
    a. $p \rightarrow q \equiv \sim p \vee q$
    b. $\sim(p \rightarrow q) \equiv p \wedge \sim q$.

H 14. a. Show that the following statement forms are all logically equivalent:

    $p \rightarrow q \vee r, \quad p \wedge \sim q \rightarrow r, \quad \text{and} \quad p \wedge \sim r \rightarrow q$

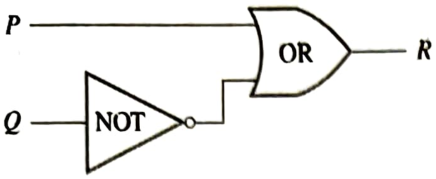
    b. Use the logical equivalences establisheo in part (a) to rewrite the following sentence in two different ways. (Assume that n represents a fixed integer.)

    If n is prime, then n is odd or n is 2.

EXERCISE SET 2.4

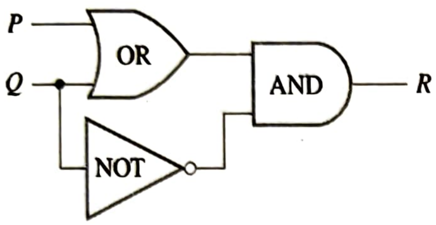
Give the output signals for the circuits in 1–4 if the input signals are as indicated.

1.



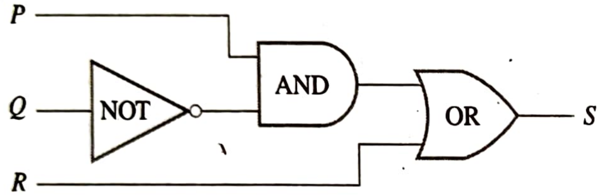
input signals: $P = 1$ and $Q = 1$

2.



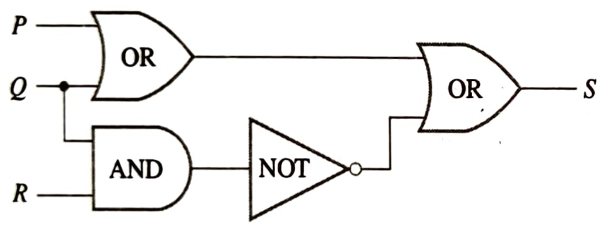
input signals: $P = 1$ and $Q = 0$

3.



input signals: $P = 1$, $Q = 0$, $R = 0$

4.



input signals: $P = 0$, $Q = 0$, $R = 0$

In 5–8, write an input/output table for the circuit in the referenced exercise.

5. Exercise 1 **6.** Exercise 2

7. Exercise 3 **8.** Exercise 4

In 9–12, find the Boolean expression that corresponds to the circuit in the referenced exercise.

9. Exercise 1 **10.** Exercise 2

11. Exercise 3 **12.** Exercise 4

Construct circuits for the Boolean expressions in 13–17.

13. $\sim P \vee Q$ **14.** $\sim(P \vee Q)$

15. $P \vee (\sim P \wedge \sim Q)$ **16.** $(P \wedge Q) \vee \sim R$

17. $(P \wedge \sim Q) \vee (\sim P \wedge R)$

For each of the tables in 18–21, construct (a) a Boolean expression having the given table as its truth table and (b) a circuit having the given table as its input/output table.

18.

P	Q	R	S
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

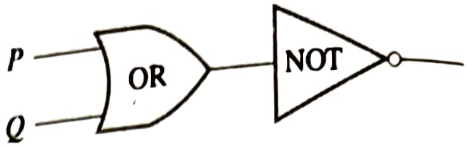
19.

P	Q	R	S
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	0
0	0	0	0

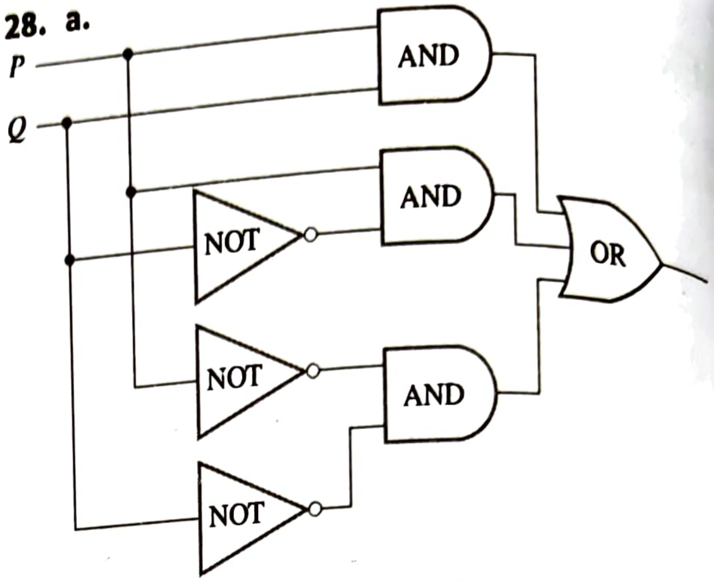
20.

P	Q	R	S
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

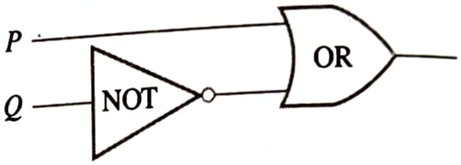
b.



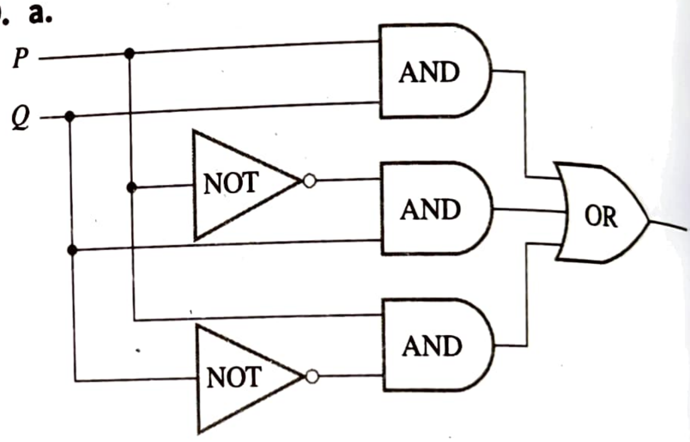
28. a.



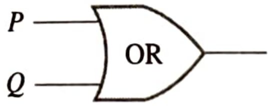
b.



29. a.



b.



For the circuits corresponding to the Boolean expressions in each of 30 and 31 there is an equivalent circuit with at most two logic gates. Find such a circuit.

30. $(P \wedge Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$

31. $(\sim P \wedge \sim Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q)$

32. The Boolean expression for the circuit in Example 2.4.5 is

$$(P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R)$$

(a disjunctive normal form). Find a circuit with at most three logic gates that is equivalent to this circuit.