FUNCTIONAL DPENDENCIES AND CANDIDATE KEYS

(Practice Questions)

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Ex-1

Let a Relation R have attributes {a1,a2,a3} and a1 is the candidate key. Then how many super keys are possible?

Here, any superset of a1 is the super key.
Super keys are = {a1, a1 a2, a1 a3, a1 a2 a3}

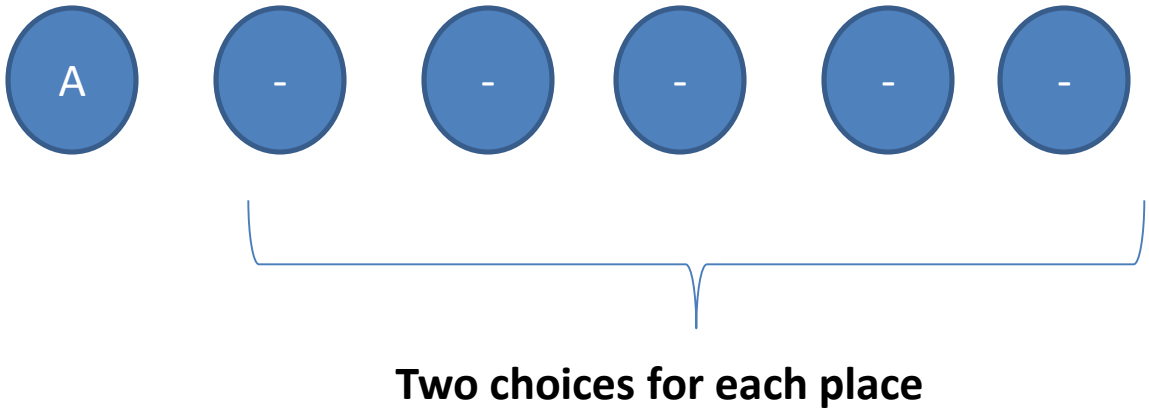
Thus we see that 4 Super keys are possible in this case.

In general, if we have ‘N’ attributes with one candidate key then the number of possible superkeys is $2^{(N-1)}$.

Ex-2

- R(A,B,C,D,E,F) where A is c candidate key, how many super keys can be there?

  $2^5 = 32$



Finding Candidate Keys

- Determine all essential attributes of the given relation.
- Essential attributes are those attributes which are not present on RHS of any functional dependency.
- Essential attributes are always a part of every candidate key.
- This is because they can not be determined by other attributes.

Example: Let R(A, B, C, D, E, F) be a relation scheme with the following functional dependencies-

- $A \rightarrow B$
- $C \rightarrow D$
- $D \rightarrow E$

Here, the attributes which are not present on RHS of any functional dependency are A, C and F.

So, essential attributes are- **A, C and F**.

Ex-3

Let R = (A, B, C, D, E) be a relation scheme with the following dependencies- AB → C, C → D, B → E

Determine the candidate keys and write number of super keys.

Attributes on RHS {C,D,E}. Thus essential attributes are {A,B}

we find the closure of AB.

$\{ AB \}^+$

= { A , B }

= { A , B , C } (Using AB → C)

= { A , B , C , D } (Using C → D)

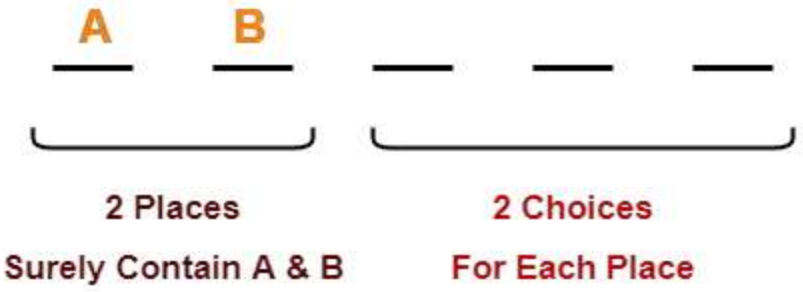
= { A , B , C , D , E } (Using B → E)

Thus AB can determine all the attributes of the given relation.

Total Number of Super Keys

There are total 5 attributes in the given relation of which-

- There are 2 essential attributes:A and B.
- Remaining 3 attributes are non-essential attributes.
- Essential attributes will be definitely present in every key.
- Non-essential attributes may or may not be taken in every super key.



So, number of super keys possible = 2 x 2 x 2 = 8.
Thus, total number of super keys possible = 8.

Example-4

Consider the Universal relation R={ABCDEFGHIJ} and the set of FDs, F= {AB → C, A → DE, B → F, F → GH, D → IJ}.

What is the key of R?

Attributes on RHS {C,D,E,F,G,H,I,J }. Thus essential attributes are {A,B}

we find the closure of AB. $\{ AB \}^+$

= { A , B }

= { A , B , C } (Using AB → C)

= { A , B , C , D ,E} (Using A→ DE)

= { A , B , C , D , E, F } (Using B → F)

= { A , B , C , D , E, F ,G,H} (Using F → GH)

= { A , B , C , D , E, F ,G,H,I,J} (Using D → IJ)

Thus AB can determine all the attributes of the given relation.

Ex-5

- Consider the relation R=(A B C D) and set of FD { A → B, B →C, C→A}
- Find candidate keys

{ABCD} is super key …as A determines B and C, we can eliminate B and C.

Thus AD is candidate key

Also, CD is Candidate key as C determines A

Also, BD is candidate key as b determines C.

Thus, AD, CD, BD are candidate keys.

Another approach

Essential attribute is D (as it is never appearing on RHS of any FD)

Possible candidate keys are AD, BD, CD

Compute their clousre to know if they are candidate key or not.

$\{AD\}^+ = \{A,D, B,C\}$

$\{BD\}^+ = \{B,D, C,A\}$

$\{CD\}^+ = \{C,D, A,B\}$

Thus, AD, CD, BD are candidate keys.

Ex-6

- Consider the relation R=(A B C D E) and set of FD { A → C, C →BD, D→A}
- Find ALL candidate keys

E is missing on RHS thus E must be part of CK.

(ABCDE) is super key

(AE) IS CANDIDATE KEY

$(DE)^+$ ={A,C,B,D,E}. Thus DE is also candidate key

$(CE)^+$ = {C,E,B,D,A} .. CE is Candidate key

Thus, AE,DE,CE are candidate keys

Ex-7

Consider the relation R=(A B C D) and set of FD { BC → A, AD →B, CD→B, AC → D}

Find ALL candidate keys

Attributes on RHS of FDs are{ A,B,D}

C is missing on RHS thus C must be part of CK.

Possible Candidate keys are AC,BC,CD.

- As BC→ A, (BCD) is super key also CD→ B thus CD is super key. C alone or D alone not a key thus **CD** is candidate key.
- As CD→ B, (ACD) is super key also AC→ D thus AC is super key. C alone or A alone not a key thus **AC** is candidate key.
- Can BC be candidate key? No

Ex-8

Consider the relation scheme R(E, F, G, H, I, J, K, L, M, N) and the set of functional dependencies-

{ E, F } → G , F → { I , J },{ E, H } → { K, L }, K → M, L → N

What is the key for R?

1. { E, F }
2. { E, F, H }
3. { E, F, H, K, L }
4. { E }

Also, determine the total number of candidate keys and super keys.

- Attributes on RHS{ G,I,J,K,L,M,N}
- Essential attribute {E,F,H} thus option 1 and 4 are wrong
- We calculate closure of $\{E,F,H\}^{+}$ ={E,F,H,G,I,J,K,L,M,N} thus it is candidate key

Total Number of Super Keys

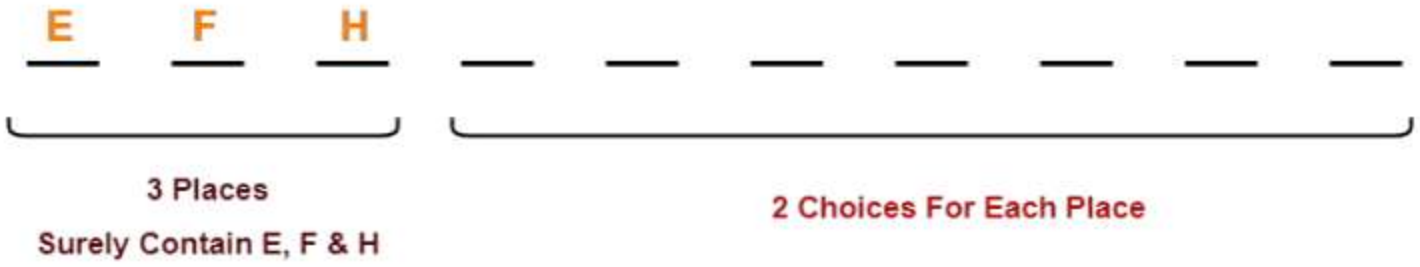
There are total 10 attributes in the given relation of which-

There are 3 essential attributes- E, F and H.

Remaining 7 attributes are non-essential attributes.

Essential attributes will be definitely present in every key.

Non-essential attributes may or may not be taken in every super key.



So, number of super keys possible = 2 x 2 x 2 x 2 x 2 x 2 x 2 = 128.

Thus, total number of super keys possible = 128.