Rank of a Matrix

Method:- Echelon form.

⇒ Only row operation are allowed.

⇒ Reduce matrix to upper triangular matrix.

Defn- Upper triangular matrix:- A square matrix in which all the elements below the principle diagonal is zero.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 8 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$ Diagonal

⇒ $\rho(A)$ = no of non-zero rows.

i.e, Rank of matrix

Eg:-

$$A = \begin{array}{c} \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \end{array} \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

Soln:- $R_2 + 2R_1$

$R_3 - R_1$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2 - 3R_4$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2 \Leftrightarrow R_4$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -2 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{non-zero rows.} \\ \\ \\ \end{matrix}$$

$\therefore \rho(A) = 2$ (no of non zero rows)