Important questions

Que 1 Recurrence of Merge Sort.

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ 2T\left(\frac{n}{2}\right)+n & \text{if } n>1 \end{cases}$$

Sol By Master Method

$a=2,\ b=2,\ f(n)=n$

$n^{\log_b a} = n^{\log_2 2} = n^1 = n$

By case 2 of Master Method

$f(n) = \Theta(n^{\log_b a})$

then solⁿ to the recurrence is

$T(n) = \Theta(n^{\log_b a} \log n)$

$= O(n \log n)$ Ans

Que 2 Compute the value of K from the Given recurrence.

$\Rightarrow T(n) = KT(n-1) \qquad n>0$

$T_0 = 1$

$T_4 = 10000$

$T_1 = K\,T(1-1)$

$K\,\underbrace{T(0)}_{1}$

$\boxed{T_1 = K}$ **

$T_2 = K\,T(2-1)$

$K\,T(1)$ [from **]

$\Rightarrow K \cdot K = K^2$

$T_3 = K\,T(3-1)$

$\Rightarrow K \cdot T(2)$

$\Rightarrow K \cdot K^2 = K^3$

$T_4 = K\,T(4-1)$

$\Rightarrow K\,T(3)$

$\Rightarrow K \cdot K^3 = K^4 = 10000$

$\Rightarrow 10^4$

$\boxed{\text{So, the value of } K \text{ is } 10}$

Que 3 $A = \Theta(n \log n)$

$\boxed{A} \rightarrow$ Solves a problem of size 32 in 5 seconds.

G How much time it will take to solve a problem of size 128.

$\Rightarrow A = \Theta(n \log n)$

$\hookrightarrow 32 \log 32$

$\Rightarrow 32 \log 2^5$

$\Rightarrow 32 \times 5 \underbrace{\log_2 2}_{1}$

$\Rightarrow 160$ seconds.

In 1 second $= \frac{160}{5} = 32$

In ___ seconds 128 size $= 128 \log 128$

$128 \log_2 2^7$

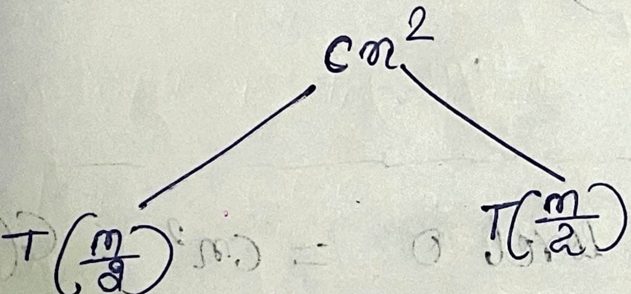
128×7

$\frac{\cancel{128 \times 7}^{\,4}}{\cancel{32}} = 4 \times 7 = \boxed{28 \text{ seconds}}$ Ans

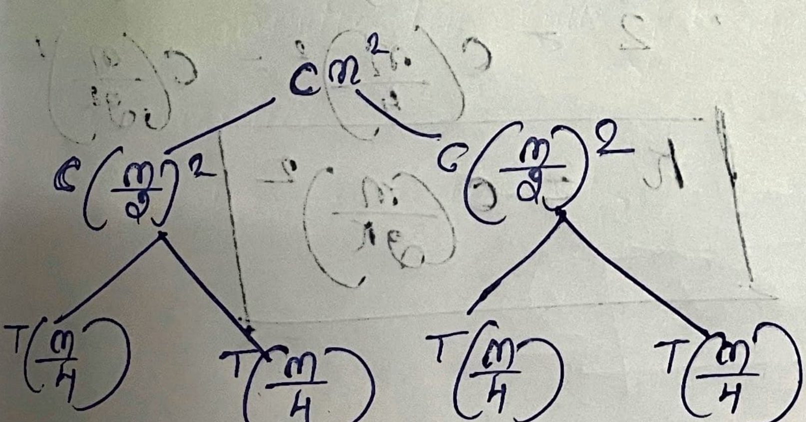
Que $T(n) = 2T\left(\frac{n}{2}\right) + O(n^2)$. Solve the given recurrence using recursion Tree.

$\Rightarrow T(n) = 2T\left(\frac{n}{2}\right) + O(n^2)$

$= 2T\left(\frac{n}{2}\right) + cn^2$



(a)



(b)

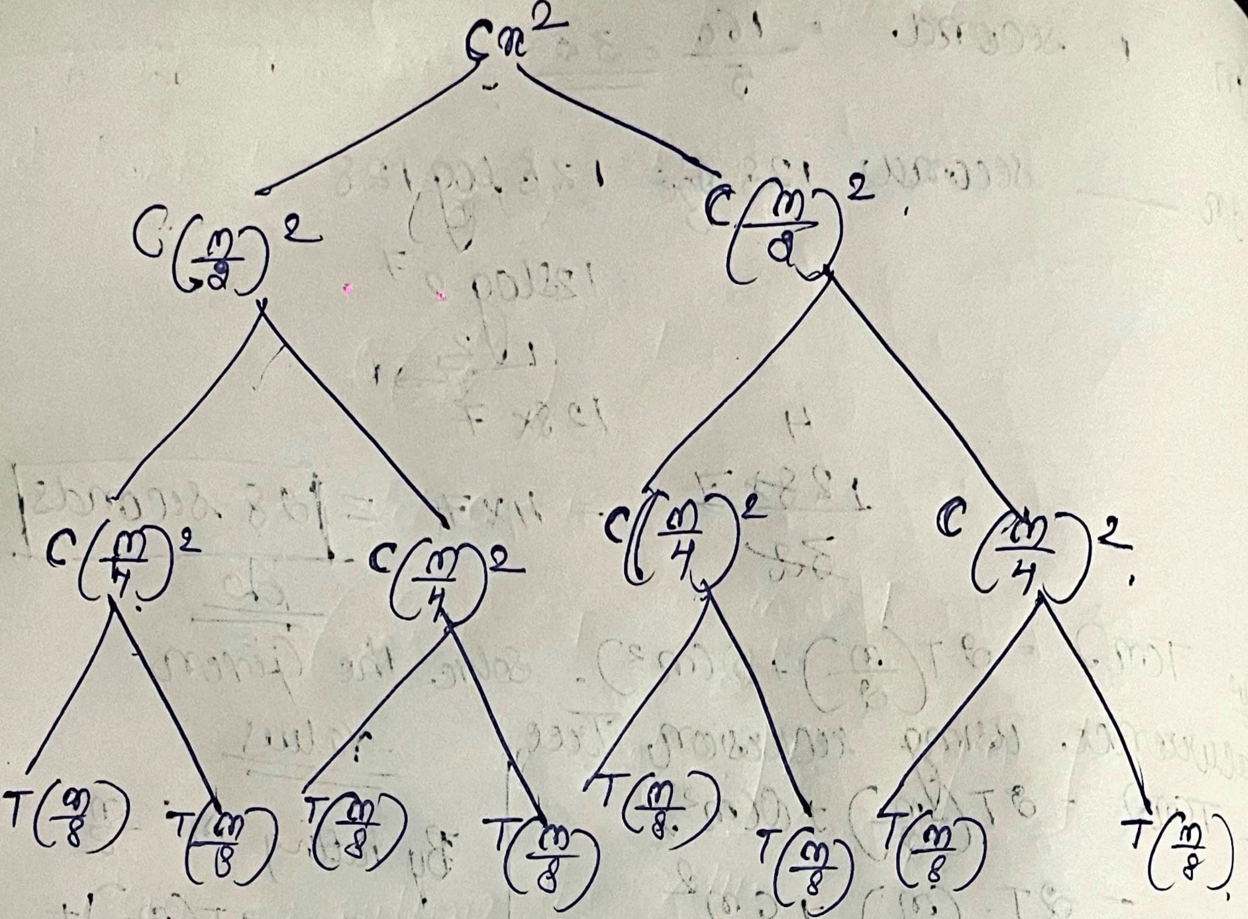
values

By Putting $n = \frac{n}{2}$

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + c\left(\frac{n}{2}\right)^2$.

By Putting $\frac{n}{4}$

$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + c\left(\frac{n}{4}\right)^2$



(c)

cost of a node at level 0 $= cn^2 = c\left(\frac{n}{2^0}\right)^2$

cost of " " " " 1 $= c\left(\frac{n}{2}\right)^2 = c\left(\frac{n}{2^1}\right)^2$

" " " " " 2 $= c\left(\frac{n}{4}\right)^2 = c\left(\frac{n}{2^2}\right)^2$

" " " " " $\boxed{k = c\left(\frac{n}{2^k}\right)^2}$

Level	Total Cost
0	cn^2
1	$c\left(\frac{n}{2}\right)^2$
2	$c\left(\frac{n}{4}\right)^2$
⋮	
K	$c\frac{n^2}{2^k}$

Suppose at level 2

$4 \times \frac{cn^2}{4 \times 4}$

so $\Rightarrow \frac{cn^2}{4}$

Total cost $T(n) = cn^2 + \frac{cn^2}{2} + \frac{cn^2}{2^2} + \dots \frac{cn^2}{2^k}$

$\Rightarrow cn^2\left(1 + \frac{1}{2^1} + \frac{1}{2^2} + \dots + \frac{1}{2^k}\right)$

$\Rightarrow cn^2\left(\frac{1 - \left(\frac{1}{2}\right)^{k+1}}{1 - \frac{1}{2}}\right)$

$\boxed{1 + r + r^2 + \dots r^n = \frac{1 - r^{n+1}}{1 - r}}$

$\Rightarrow 2cn^2\left(\frac{1 - \left(\frac{1}{2}\right)^{k+1}}{\frac{1}{2}}\right)$

$\Rightarrow 2cn^2\left(1 - \frac{1}{2^{k+1}}\right)$

$\Rightarrow 2cn^2\left(1 - \frac{1}{2 \cdot 2^k}\right)$

$\Rightarrow 2cn^2\left(1 - \frac{1}{2 \cdot 2^{\log n}}\right)$

$\Rightarrow 2cn^2\left(1 - \frac{1}{2n}\right)$.

Let the base case of recursion tree arises at level K

$c\left(\frac{n}{2^k}\right)^2 = 1$

$\Rightarrow c\left(\frac{n}{2^k}\right)^2 = 1$

Let $c = 1$

$\left(\frac{n}{2^k}\right)^2 = 1 \Rightarrow \frac{n}{2^k} = 1$

$n = 2^k$

$\boxed{K = \log n}$

$\Rightarrow 2cn^2\left(\frac{2n-1}{2n}\right)$

$2cn^2 - cn$

$\Rightarrow O(cn^2)$