046197

Computational Methods in Optimization

HOMEWORK #1

Alexander Shender: 328626114

Eliran Cohen: 204187801

Contents

[Question 1. 2](#_Toc69686948)

[Part a. 2](#_Toc69686949)

[Part b. 3](#_Toc69686950)

[Question 2. 4](#_Toc69686951)

[Part a. 4](#_Toc69686952)

[Part b. 5](#_Toc69686953)

[Part c. 5](#_Toc69686954)

[Part d. 6](#_Toc69686955)

[Question 3. 7](#_Toc69686956)

[Part a. 7](#_Toc69686957)

[Part b. 7](#_Toc69686958)

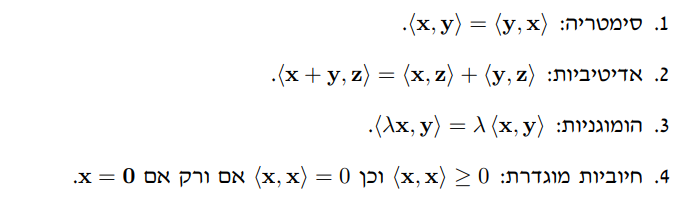
[Part c. 8](#_Toc69686959)

[Question 4-6. 9](#_Toc69686960)

# Question 1.

## Part a.

There are 4 conditions that need to hold that define the inner product, as specified in the tutorial no. 1:



In our case, we have

Condition 1 holds:

Condition 2 holds:

Condition 3 holds:

While conditions 1-3 indeed hold, the condition 4 will hold only if and . This means M matrix should at least PSD. But we don’t have such condition on M.

As basic example let’s take M to be a matrix of all zeros. Then , but .

So this operation is **NOT AN INNER PRODUCT**

## Part b.

The only difference we have here is the new information on the matrix (now denoted Q):

So conditions 1-3 hold as previously in Part a.

Condition 4:

Since we know that Q is a PD matrix, this means that for each ,

Which helps us to prove the condition 4.

So this operation **answers all 4 conditions for the inner product.**

# Question 2.

Given: A is symmetric,

## Part a.

Given: . Prove: A is invertible,

**Invertibility:** If A is symmetric, PD, , then the following hold from the characteristics of the PD matrixes:

Which means:

Thus, A spans over all of its dimensions and has a full rank. **Thus, A is invertible**.

Another way to show this is to remember that all of eigenvalues of A are positive -> A is invertible.

To prove that is PD, we show that all its eigenvalues are strictly positive:

For matrix A, eigenvector and corresponding eigenvalue :

Since , so does . Also, .

**Thus, all the eigenvalues are positive, and .**

## Part b.

Given: , A invertible. Prove:

A is invertible ->

Invertible matrix has all non-zero eigenvalues.

->

Combining two of those constraints together we get:

Which is exactly the property of PD matrix – all its eigenvalues are strictly positive.

**Thus,**

## Part c.

Given: . Prove:

We try to prove the property of the PDS matrix:

Which means

**Thus, .**

**Example for B**: m = 1, n = 2.

We take :

## Part d.

Given: , Prove: .

It if sufficient to show that for some , this holds:

Expanding:

If some , this means this whole expression can be negative too, if we choose , that has all the values equal 0 except for the value. For example:

The condition that A has non-negative diagonal values is **sufficient** to prove it is PSD, Example for the extreme case by having zero values on the diagonals:

By having positive values on a diagonal will still make the inequality hold.

# Question 3.

Given:

continuously differentiable. . is the downhill direction at point if there exists such that for each :

## Part a.

Prove: If direction uphold: for some x, then is the downhill direction.

From the definition:

Thus, this is the downhill direction from the definition given in this exercise.

## Part b.

Prove: if , then the direction is the downhill direction.

From theorem 4 from the tutorial: if is continuously differentiable, then:

If , then , then:

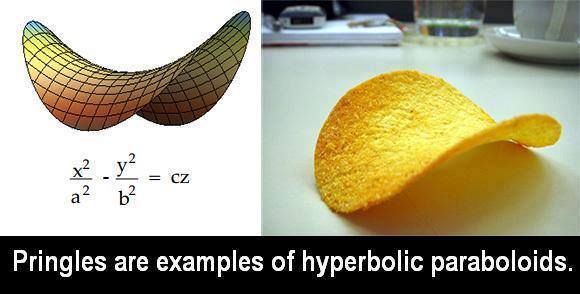
Thus,

From Part a. of this question we know that if this inequality holds, then is the downhill direction.

## Part c.

Find example of a function , a point , directions , s.t. is the downhill direction and is not.

What came on my mind is Pringles (image source – google images):



Let’s take

Let’s take trivial point

Calculating gradient:

From what we have proven:

# Question 4-6.

<continue to the next page>